Fluid Dynamics of Nonaqueous Phase Contaminants in Groundwater: Analytical Solutions and Analogy with Zhukovsky's Trochoid

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***Abstract*—An exact solution to a free-boundary, potential, 2-D flow of a Darcian fluid (mathematically equivalent to flow of a heavy irrotational ideal fluid) past a barrier is obtained by the theory of holomorphic functions. A volume of liquid contaminant contrasting in density with the ambient flowing groundwater makes a lens attached to the stoss or lee side of the barrier. The shape of the interface morphs in response to a pressure-velocity field in the dynamic and static liquid phases. The flow net and interface are plotted from explicit expressions found for the complex potential and complex velocity. As a particular case, we obtain a famous Zhukovsky's gas-bubble contour belonging to the class of trochoids. Serious caveats for remediation projects and artificial recharge of groundwater are inferred: more intensive descending seepage of ponded surface water through a heterogeneous aquifer may worsen the groundwater quality, contrary to what would occur in homogeneous porous media.**

***Index Terms*— environmental engineering, Darcian flow, groundwater contamination and clean-up, free surface, holomorphic functions, conformal mappings, hodograph transform.**

I. Introduction

Light and dense nonaqueous phase liquids (so-called LNAPLs-DNAPLs, e.g. crude oil, diesel, trichloroethylene, bitumen, [1]) contaminate groundwater/aquifers/vadose zone soil and require costly and protracted environmental engineering techniques in remediation, monitoring and controls (e.g., [2]). Often, NAPLs in a porous/fractured subsurface make macrovolumes (lenses, hydrocarbon traps, ganglia, blobs, streaks, etc. [3]-[6]) separated from groundwater by sharp interfaces. The locus and shape of the interface between two liquid-saturated domains of contrasting density depends on the ambient groundwater flow and the geological boundaries (e.g., bed-,cap-rock unconformities, troughs, bulges, anticline flanks, etc.). Detection of "alien entities" obstructing flow in such aquifers due to their contrast in texture with the main porous medium is problematic due to imprecision of geophysical tools and limited core samples in study areas. Simulation based on analytical and numerical models of multiphase flow and transport in the subsurface [7] is a vital step in many surveys, containment projects and engineering measures aimed at decontamination by pump-and-treat, air-sparging, permeable reactive barriers, LNAPL skimming, DNAPL scraping, etc.

In some NAPL-treatment approaches an intensive groundwater gradient is artificially generated by injection of fresh water through wells or infiltration ponds. In these hydrodynamically agitated clean-up schemes, NAPL can be flushed away or the interface can take a non-trivial (even counter-intuitive) shape. In this paper we study analytically the case of a NAPL attached as a static macrovolume to a solid barrier (an impermeable "alien" inclusion, e.g. a salt dome, constructed silo or tunnel) with a strong influence of the conjugated dynamic groundwater. We utilize the theory of holomorphic functions, in particular, conformal mappings and the hodograph method to analytically find the shape of NAPL interfaces, whose very existence is puzzling.

II. Mathematical Model

We consider a homogeneous porous matrix of hydraulic conductivity k with an impermeable barrier baffling a descending fully saturated Darcian seepage with velocity V_0 far away from the obstacle (Fig.1a presents a vertical cross-section).

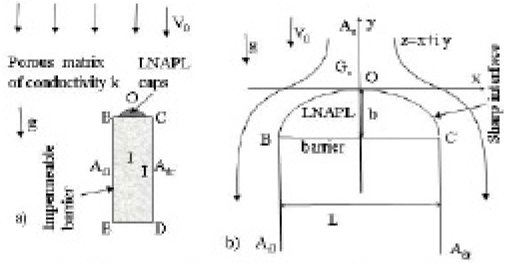

Fig.1 Vertical cross-section of a 2-D barrier with a cap of liquid contaminant

A curve-shaped cap BOC of LNAPL (Fig.1a) sits on the flat horizontal roof BC of the baffle as a static lens, morphed by the ambient seepage flow. The flowing water pushes the LNAPL volume to the stoss side of the barrier. If no ambient flow in Fig.1a, then LNAPL would rise to the

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