Elektrochemiczny metali w odniesieniu do kawitacji w szczelinie elektrody

Электрохимическая обработка металлов с учётом кавитации в межэлектрод-ном промежутке

Electrochemical machining of metals with cavitation in the electrode gap

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W pracy rozpatruje problem dwuwymiarowy obróbki elektrochemicznej (ECM) metali. Podczas przetwarzania, w pobliżu punktów narożnych powierzchni katody do anody, w niektórych trybach są utworzone falistych części granicy, która ogólnie rzecz biorąc, nie należy umieszczać w ciągłym strumieniu elektrolitu w szczelinie elektrodowej (IEP). W niniejszej pracy, analityczne rozwiązanie problemu ECHO katody z punktu narożnego przy użyciu modelu matematycznego na podstawie analogii z problemem idealnego płynu przepływa z wolnymi granic. Problem ten jest rozwiązywany przez obecność w pobliżu punktów narożnych wnęki o skończonej długości. Bierze się tu pod uwagę wpływ hydrodynamicznego przepływu na charakterystykę pola elektrycznego. Wyniki obliczeń granicy anody, potwierdzający pojawienie się na jej części fali, którego kształt zależy od parametrów przepływu.

SŁOWA KLUCZOWE: potencjał, elektrochemii, anoda, katoda, gęstość prądu, elektrolitów, metalu.

Two-dimensional problems of electrochemical machining (ECM) metals is studied. The matter is that during processing in some modes we can see on the anode surface near the corner points of the cathode an image-wavy sections of the border, which, generally speaking, should not appear in a continuous flow of electrolyte in interelectrod gap (IEG). In this paper, an analytical solution of the problem ECM by cathode with a corner point is obtained by using mathematical model, based on the analogy with a flow of the ideal fluid with free boundary sections. Then the same problem is solved in the presence of a finite length cavity at the neighborhood of the corner point taking into account the influence of hydrodynamics on the electric field. The submitted results of the calculation of the anode boundary confirm the occurrence of wavy sections of the border, the shape of which depends on the flow parameters.

KEYWORDS: potential, electrochemistry, anode, cathode, current density, electrolyte, metal.

Solution of some two dimensional problems of ECM by polygonal cathode, including cases with the presence of insulation are presented in [1,2,3].

We consider the two-dimensional problem of electrochemical shaping as it is shown in Fig. 1. Fig. 1, in the plane $z = x + iy$, shows the right half part of the electrode gap: AG is the line of symmetry, $ABDE$ is the cathode boundary, GFS is anode boundary, ES is electrically insulated section of the IEG, A and E are the point at infinity. The origin is chosen at the point B. The cathode feed rate $\vec{V}_K$ is orthogonal to x axis directed along the edges BD. The width of the IEG in the neighborhood of A, E, equal H_A and H_E respectively. The condition of stationary electrochemical shaping must be satisfied at the anode border. It is necessary to determine the shape of the anode surface.

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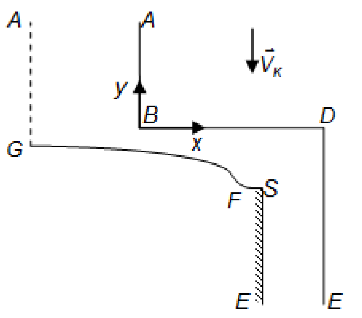

Fig. 1. Area IEG

We assume that the metal removal rate on the anode per unit mass is determined by Faraday's law $V_m = j_A \eta \varepsilon$, where $\eta = \eta(j_A)$ is current efficiency, j_A is current density, ε is electrochemical equivalent of the metal, cathode surface moves with constant velocity $\vec{V}_K$ and linear velocity of points on the surface of the anode is

$$V_A = V_K \cdot \cos\theta \qquad (1)$$

where θ is the angle between the anode velocity vector and the outward unit vector normal to the surface of the anode. In this case, the general scheme of the process does not change with time, and the process can be regarded as steady or stationary [4,5]. Steady current density distribution j_A on the stationary anode border is defined as

$$\eta j_A = \frac{\rho V_K}{\varepsilon} \cos\theta ,$$

where ρ is density of the anode material and on anode boundary the next relation is true[1,6]

$$j_A = \frac{\rho V_K}{\varepsilon}(a_0 + b_0 \cos\theta) \qquad (2)$$

We introduce in IEG a Cartesian coordinate system Ox_1y_1. We assume, neglecting the near-electrode phenomena, that there is the potential ψ_1 of the electric field in IEG, satisfying the Laplace equation (3)

$$\Delta \psi_1 = 0 \qquad (3)$$

and on the electrodes the conditions of constant potential $\psi_{1A} = u_A$, $\psi_{1K} = u_K$ are satisfied. By (3), there exists a harmonically conjugate function φ_1, and we can enter the complex potential of the electrostatic field $W(x_1, y_1) = \varphi_1(x_1, y_1) + i\psi_1(x_1, y_1)$, which is an analytic function in the region $z_1 = x_1 + iy_1$.

The existance of the complex potential makes it possible to consider our problem as a two-dimensional problem of the fictitious flow of ideal fluid with a given section of the boundary (cathode) and an unknown portion of the border (the anode), on which a certain condition, in our case it is the condition (2), is specified. Streamlines in this fictitious flow correspond to the electric field lines, on which $\psi_1 = \text{const}$ [7].

We introduce the characteristic values of current density $j_0 = \rho V_K / \varepsilon$, length $H = \kappa(u_A - u_K)/j_0$ (κ electrical conductivity of the medium) and move on to the dimensionless variables

$$x = \frac{x_1}{H}, \quad y = \frac{y_1}{H}, \quad z = x + iy, \quad W = \varphi + i\psi = \frac{W_1 - iu_K}{u_A - u_K} .$$

Then, in view of (2), the function ψ at interelectrode space satisfies the Laplace equation and the boundary conditions on the surface of the electrodes

$$\psi_A = 1, \quad \psi_K = 0, \quad \frac{\partial \psi_A}{\partial n} = \frac{j_A}{j_0} = a_0 + b_0 \cos\theta ,$$

Where a_0, b_0 are constants, reflected dependence of current efficiency on current density.

To solve the problem, we introduce an auxiliary complex variable $u = \xi + i\eta$, varying in the area $D_u = \{u = \xi + i\eta, 0 \le \xi \le \pi/2, 0 \le \eta \le \pi|\tau|/4\}$, $\tau = i|\tau|$ (Fig. 2), and we will look for a function $z(u)$, conformally maping the area D_u on the flow domain with the points correspondence indicated in Fig. 1, 2, also in the area D_u points B, D, S are corresponding to the points $B(b + \pi\tau/4)$, $D(d + \pi\tau/4)$, $S(\pi/2 + is)$.

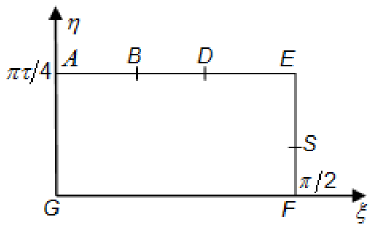

Fig. 2. Area D_u

To construct function $z(u)$, it is enough to find the derivative $dW(u)/du$ of the complex potential of the fictitious flow and Zhukovsky function [7]

$$\chi(u) = \ln\left(\frac{V_0 dz}{dW}\right) = r(u) + i\theta(u) , \qquad (4)$$

wherein $r(u) = \ln(V_0/V)$, V is the fictitious flow velocity magnitude, θ is velocity vector inclination angle to the x axis, $V_0 = a_0 + b_0$ is the speed at the point G.

For the complex potential of the fictitious flow $W(u) = \varphi + i\psi$ we have the following boundary conditions:

$$\begin{aligned}
&\psi(u) = 0, && u = \xi + \frac{\pi|\tau|}{4};\\
&\psi(u) = 1, && u = \xi;\ u = \frac{\pi}{2} + i\eta,\ 0 \le \eta < s;\\
&\varphi(u) = 0, && u = i\eta;\\
&\varphi(u) = \varphi_0, && u = \frac{\pi}{2} + i\eta,\ s < \eta < \frac{\pi|\tau|}{4}.
\end{aligned}$$

The variation area of complex potential W is a rectangle $D_W = \{\varphi + i\psi,\ 0 \le \varphi \le \varphi_0,\ 0 \le \psi \le 1\}$ (Fig. 3).

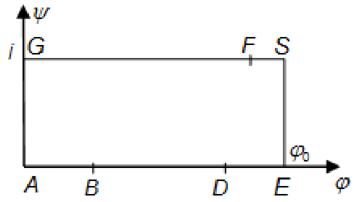

Fig. 3. Area D_W

To determine the derivative dW/du of the complex potential we conformally map area D_u on the upper half-plane D_ω with the points correspondence, indicated in Fig. 3, 4, by conversion

$$\omega(u) = sn\left(2K\left(\frac{2u}{\pi} - \frac{1}{2}\right), k\right) = -\frac{1}{\sqrt{k}}\frac{\vartheta_2(2u)}{\vartheta_3(2u)}, \qquad (5)$$

where $sn(u)$ is Jacobi function, $\vartheta_i(u)$ ($i = \overline{1,4}$) is theta functions for the periods π and $\pi\tau$, $K = \pi\vartheta_3^2(0)/2$, $k = \vartheta_2^2(0)/\vartheta_3^2(0)$ [8].

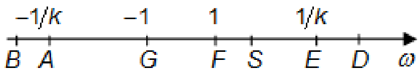

Fig. 4. Area D_ω

With help of the Schwarz-Christoffel formula [9], we find the derivative of the function that maps the area D_ω at the area of function $W(u)$:

$$\frac{dW}{d\omega} = \frac{M}{\sqrt{(\omega+1)(\omega-\sigma)(\omega^2 - 1/k^2)}},$$

$$M = i\left(\int_{-1/k}^{-1} \frac{d\omega}{\sqrt{(\omega+1)(\omega-\gamma)(\omega^2 - 1/k^2)}}\right)^{-1},$$

where $\gamma = \omega(\pi/2 + is)$. Taking into account (5), we obtain:

$$\frac{dW}{du} = N\sqrt{\frac{\omega(u) - 1}{\omega(u) - \gamma}}, \quad N = i\left(\int_{-1/k}^{-1}\sqrt{\frac{\omega - 1}{\omega - \gamma}}d\omega\right)^{-1}.$$

For the Zhukovsky function $\chi(u) = r(u) + i\theta(u)$ we have the following boundary conditions

$$\begin{aligned}
&\theta(u) = 0, && u = i\eta;\ u = \xi + \frac{\pi\tau}{4},\ b < \xi < d;\\
&\theta(u) = 0, && u = \frac{\pi}{2} + i\eta,\ 0 < \eta < s;\\
&\theta(u) = -\frac{\pi}{2}, && u = \xi + \frac{\pi\tau}{4},\ 0 < \xi < b,\ d < \xi < \frac{\pi}{2};\\
&\theta(u) = -\pi, && u = \xi + \frac{\pi\tau}{4}.
\end{aligned}$$

On the anode border GFS the condition satisfied:

$$r(\xi) = \ln\left(\frac{V_0}{a_0 + b_0\cos\theta(\xi)}\right).$$

Zhukovsky function $\chi(u)$ we search in the form $\chi(u) = \chi_0(u) + f(u)$, where the function $\chi_0(u) = r_0(u) + i\theta_0(u)$ satisfies the boundary conditions:

$$\begin{aligned}
&\theta_0(u) = 0, && u = i\eta;\ u = \xi + \frac{\pi\tau}{4},\ b < \xi < d;\\
&\theta_0(u) = 0, && u = \frac{\pi}{2} + i\eta,\ 0 < \eta < s;\\
&\theta_0(u) = -\frac{\pi}{2}, && u = \xi + \frac{\pi\tau}{4},\ 0 < \xi < b,\ d < \xi < \frac{\pi}{2};\\
&\theta_0(u) = -\pi, && u = \xi + \frac{\pi\tau}{4};\\
&r_0(u) = 0, && u = \xi.
\end{aligned}$$

and has in D_u the same features as $\chi(u)$, the function $f(u)$ is analytic in D_u and continuous in $\overline{D_u}$ [10]. We constructe $\chi_0(u)$ by singular points method [7]:

$$\begin{aligned}
\chi_0(u) = &-\ln\left(\frac{\vartheta_1(u - \pi\tau/4)\vartheta_2(u - \pi\tau/4)}{\vartheta_4(u - \pi\tau/4)\vartheta_3(u - \pi\tau/4)}\right) +\\
&+\frac{1}{2}\ln\left(\frac{\vartheta_1(u - b - \pi\tau/4)\vartheta_1(u + b - \pi\tau/4)}{\vartheta_1(u - b + \pi\tau/4)\vartheta_1(u + b + \pi\tau/4)}\right) -\\
&-\frac{1}{2}\ln\left(\frac{\vartheta_1(u - d - \pi\tau/4)\vartheta_1(u + d - \pi\tau/4)}{\vartheta_1(u - d + \pi\tau/4)\vartheta_1(u + d + \pi\tau/4)}\right) +\\
&+\ln\left(\frac{\vartheta_2(u - if)\vartheta_3(u - if)}{\vartheta_2(u + if)\vartheta_3(u + if)}\right) - \frac{\pi}{2}i
\end{aligned}$$

Comparing the boundary conditions for the functions $\chi(u)$ and $\chi_0(u)$, we obtain the boundary conditions for the unknown function $f(u) = \lambda(u) + i\mu(u)$:

$$\mu(u) = 0,\quad u = i\eta;\ u = \frac{\pi}{2} + i\eta; \qquad (6)$$

$$\mu(u) = 0,\quad u = \xi + \frac{\pi\tau}{4}; \qquad (7)$$

$$\lambda(u) = \ln\left(\frac{V_0}{a_0 + b_0\cos(\theta_0(u) + \mu(u))}\right),\quad u = \xi. \qquad (8)$$

To construct the unknown function $f(u)$ we map the area D_u on the semicircle [10] with help of the function

$$t = e^{2i(u - \pi\tau/4)} \qquad (9)$$

Taking into account the boundary conditions (6), (7), we may continue the function $f(u)$ analytically to the ring and write it in the form of a Laurent series:

$$f(u) = \sum_{n=-\infty}^{\infty} c_n t^n(u) = \sum_{n=-\infty}^{\infty} c_n e^{2i(u - \pi\tau/4)n}, \qquad (10)$$

where c_n is real coefficients.

On the basis of the boundary condition (7), we find

$$c_n = c_{-n}\,, \quad c_0 = -2\sum_{n=1}^{\infty} c_n \operatorname{ch}(\pi|\tau|n/2)\,,$$

and by (10) obtain:

$$f(u) = c_0 + 2\sum_{n=1}^{\infty} c_n \cos\bigl(2n(u - \pi\tau/4)\bigr). \qquad (11)$$

Condition (8), in view of the representation of the function $f(u)$ (11), has the form

$$\begin{aligned} & c_0 + 2\sum_{n=1}^{\infty} c_n \cos(2n\xi)\operatorname{ch}(\pi|\tau|n/2) = \\ & \quad = \ln V_0 - \ln\bigl(a_0 + b_0\cos(\theta_0(u) + \mu(u))\bigr) \end{aligned} \qquad (12)$$

Multiplying equation (12) for $\cos(2n\xi)$ and then integrating it by ξ within $0, \pi/2$, we obtain an infinite system of equations for the coefficients c_n :

$$c_n = \frac{-2}{\pi\operatorname{ch}(\pi|\tau|n/2)} \int_0^{\pi/2} \ln\bigl(a_0 + b_0\cos(\theta_0(\xi) + \mu(\xi))\bigr)\cos(2n\xi)d\xi$$

$n = \overline{1,\infty}$.

Dimensionless coordinates of the points of IEG borders we define by (4) according to the formula

$$\frac{z(u)}{H_E} = \frac{1}{V_0 H_E}\int_0^u \frac{dW(u)}{du} e^{\chi(u)} du\,.$$

Finally it is necessary to determine the mathematical parameters τ, b, d. This can be done by setting H_A/H_E and anode sizes: $L = \operatorname{Re}\bigl(z(\pi/2) - z(0)\bigr)/H_E$, $h = -\operatorname{Im}\bigl(z(\pi/2) - z(0)\bigr)/H_E$.

Fig. 5 and Fig. 6 shows the results of calculation of the anode form for $a_0 = 0.1$, $b_0 = 0.9$, $H_A/H_E = 0.5$, and various L and h. One can see there that anode boundaries are monotonic in vicinity of cathode corner point B .

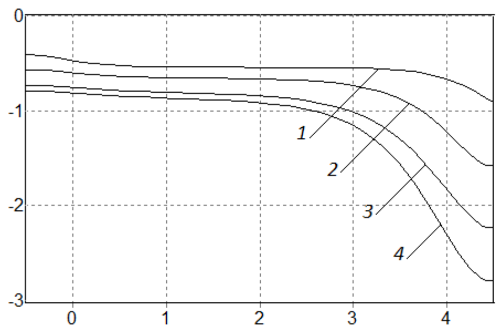

Fig. 5.

$L = 5$, 1) $h = 0.5$, 2) $h = 1$, 3) $h = 1.5$, 4) $h = 2$

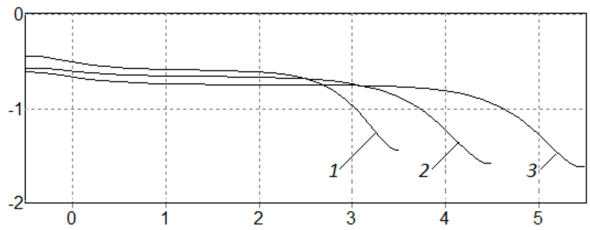

Fig. 6.

$h = 1$, 1) $L = 4$, 2) $L = 5$, 3) $L = 6$

To explain the emerging practice of non-monotonic sites on the anode surface near the cathode critical point, we consider the problem with taking into account the effect of cavitation on the flow characteristics in the IEG.

When the stationary machining of a workpiece at the close vicinity of cathode corner point (B in Fig. 1), the flow velocity increases sharply, and there are conditions to form cavities filled with air bubbles. (Fig. 7). In these areas, the electrical conductivity is broken and they play the role of insulator. This leads to the formation of irregularities on the surface to be treated.

Fig. 7 display in the plane $z = x + iy$ the right half of the IEG: AG is the line of symmetry, $ABDE$ is the boundary of the cathode, GF is anode boundary, EF is electrically insulated section of IEG, A , E are points at infinity. The width of IEG in the neighborhood of A , E , equal to H_A и H_E , velocity of electrolyte flow is V_A и V_E correspondingly. The electrolyte flow in IEG direct from point A to point E . At the point B there is separation of the flow from the cathode surface to form a cavity, closes at a fictitious plate CS , perpendicular to the face of the cathode BD [6]. The boundary of the cavity BC and the plate CS are Insulated sections IEG. The flow velocity at the boundary of the cavity BC is constant and equal V_0 . The origin is at the point B . The feed rate of the cathode $\vec{V}_K$ orthogonal to x-axis directed along the edge BD . On the anode border the condition of stationary electrochemical shaping must be satisfied. It is required to determine the shape of the anode surface and the cavity.

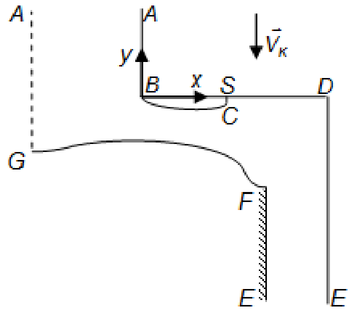

Fig. 7. Area IEG

The potential of the electric field in IEG satisfies the Laplace equation and the boundary conditions on the surface of the electrodes

$$\psi_A = 1,\quad \psi_K = 0\,,\quad \frac{\partial\psi_A}{\partial n} = \frac{j_A}{j_0} = a_0 + b_0\cos\theta\,.$$

On the electrically insulated areas IEG

$$\varphi|_{AG} = 0\,,\quad \varphi|_{BCS} = \varphi_0\,,\quad \varphi|_{EF} = \varphi_1\,.$$

The area D_W of variation for the complex potential of the electrostatic field $W(u) = \varphi + i\psi$ for this problem is shown in Fig. 8. At the point B in the plane $W(u)$ we have a split, corresponding to the cavity border boundary ($\varphi = \varphi_0$), whose position is unknown in advance.

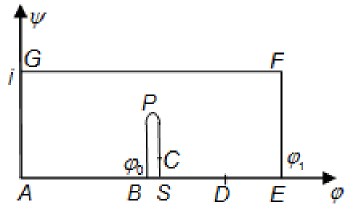

Fig. 8. Area D_W

Let in the plane of the auxiliary complex variable $u = \xi + i\tau$ flow area D_z corresponds to the area $D_u = \{u = \xi + i\eta,\ 0 \le \xi \le \pi/2,\ 0 \le \eta \le \pi|\tau|/4\}$ (Fig. 9) and the function *z(u)* conformally maps the domain D_u to the domain D_z with the points correspondence indicated in Fig. 7, 9, and in the area D_u – $A(ia)$, $E(\pi/2 + ie)$, $D(\pi/2 + id)$, $S(\pi/2 + is)$, $P(p + \pi\tau/4)$-,,,,..

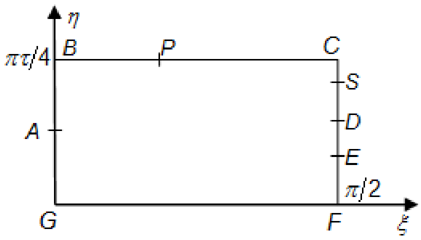

Fig. 9. Area D_u

We represent function dz/du as

$$\frac{dz}{du} = \frac{dz}{dW}\frac{dW}{dW_g}\frac{dW_g}{du}\,, \tag{13}$$

where $W_g(u) = \varphi_g(u) + i\psi_g(u)$ is the complex potential of the electrolyte flow.

To construct $z(u)$ we find the derivative $dW_g(u)/du$ of the complex potential of the electrolyte flow and the Zhukovsky function

$$\chi_g(u) = \ln\left(\frac{V_0 dz}{dW_g}\right) = r(u) + i\theta(u)\,,$$

где – модуль скорости течения электролита, θ – угол наклона вектора скорости к оси x. Where $r(u) = \ln(V_0/V)$, V is vector magnitude of the electrolyte flow, θ is the angle of the velocity vector to x-axis.

Complex potential $W_g(u)$ of electrolyte flow satisfies the boundary conditions

$$\psi_g(u) = 0,\quad u = i\eta,\ 0 \le \eta < a;\ u = \xi;$$
$$\psi_g(u) = 0,\quad u = \frac{\pi}{2} + i\eta,\ 0 \le \eta < e;$$
$$\psi_g(u) = q,\quad u = i\eta,\ a < \eta \le \frac{\pi|\tau|}{4};\ u = \xi + \frac{\pi\tau}{4};$$
$$\psi_g(u) = q,\quad u = \frac{\pi}{2} + i\eta,\ e < \eta \le \frac{\pi|\tau|}{4}$$

The variation range of function $W_g(u)$ is the strip $D_{W_g} = \{\varphi_g + i\psi_g,\ 0 \le \psi_g \le q\}$, $q = V_E H_E$ is the output of the electrolyte.

The boundary conditions for the complex potential $W_g(u)$ give the opportunity to construct function $W_g(u)/du$ by the singular points method:

$$\frac{dW_g(u)}{du} = \frac{N \cdot \vartheta_1(2u)}{\vartheta_1(u-ia)\vartheta_1(u+ia)\vartheta_4(u-ia)\vartheta_4(u+ia)} \times$$
$$\times \frac{\vartheta_4(2u)}{\vartheta_2(u-ie)\vartheta_2(u+ie)\vartheta_3(u-ie)\vartheta_3(u+ie)}$$

Constant N can be determined by the condition that the electrolyte output is equal $q = V_E H_E$. With help of determining the residue of the function $W_g(u)$ at a point E we obtain

$$N = \frac{q}{\pi}\vartheta_1'(0)\vartheta_4(0)\vartheta_2(ia-ie)\vartheta_2(ia+ie)\vartheta_3(ia-ie)\vartheta_3(ia+ie)\,.$$

For Zhukovsky function $\chi_g(u) = r(u) + i\theta(u)$ we have the following boundary conditions:

$$\theta(u) = -\frac{\pi}{2},\quad u = i\eta;\ u = \frac{\pi}{2} + i\eta,\ 0 < \eta < d;$$
$$\theta(u) = \frac{\pi}{2},\quad u = \frac{\pi}{2} + i\eta,\ s < \eta \le \frac{\pi|\tau|}{4};$$
$$\theta(u) = 0,\quad u = \frac{\pi}{2} + i\eta,\ d < \eta < s;$$
$$r(u) = 0,\quad u = \xi + \frac{\pi\tau}{4}.$$

On the anode border GF the imaginary parts of $W(u)$, $W_g(u)$ are constant. With help of the representation (13) for dz/du we can get on the boundary GF next condition^

$$r(\xi) = \ln\left(\frac{V_0}{a_0 + b_0\cos\theta(\xi)}\frac{d\varphi(\xi)}{d\varphi_g(\xi)}\right),$$

what give us the ability to take into account electrolyte flow regime and the variability of the current output.

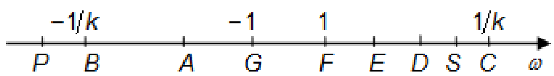

Fig. 10. Area D_ω

To determine the function $d\varphi/d\varphi_g$ we map area D_u to the upper half-plane with the points correspondence, indicated in Fig. 9, 10 by the transformation (5). Then by means of the Schwarz-Christoffel formula, we find the derivatives of functions mapping the area D_ω to variation area for functions W and W_g:

$$\frac{dW}{d\omega}=M\frac{\omega-\delta}{\sqrt{(\omega^2-1)(\omega-\alpha)(\omega+1/k)(\omega-\beta)(\omega-\gamma)}},$$

$$M=i\left(\int_\alpha^{-1}\frac{\omega-\delta}{\sqrt{(\omega^2-1)(\omega-\alpha)(\omega+1/k)(\omega-\beta)(\omega-\gamma)}}d\omega\right)^{-1},$$

$$\frac{dW_g}{d\omega}=\frac{q}{\pi}\frac{\beta-\alpha}{(\omega-\alpha)(\omega-\beta)},$$

where $\alpha=\omega(A)=\omega(ia)$, $\beta=\omega(E)=\omega(\pi/2+ie)$, $\gamma=\omega(S)=\omega(\pi/2+is)$, $\delta=\omega(P)=\omega(p+\pi\tau/4)$.

Taking into account, that $\int_{-1/k}^{\gamma}dW=0$, we obtain an equation, relating the mathematical parameters: α, β, γ, δ:

$$\delta=\frac{\int_{-1/k}^{\gamma}\frac{\omega d\omega}{\sqrt{(\omega^2-1)(\omega-\alpha)(\omega+1/k)(\omega-\beta)(\omega-\gamma)}}}{\int_{-1/k}^{\gamma}\frac{d\omega}{\sqrt{(\omega^2-1)(\omega-\alpha)(\omega+1/k)(\omega-\beta)(\omega-\gamma)}}}.$$

By this formulas plus relation (5), we find

$$\frac{d\varphi(\xi)}{d\varphi_g(\xi)}=\frac{M\pi}{q(\beta-\alpha)}\frac{(\omega(\xi)-\delta)\sqrt{(\omega(\xi)-\alpha)(\omega(\xi)-\beta)}}{\sqrt{(\omega^2(\xi)-1)(\omega(\xi)+1/k)(\omega(\xi)-\gamma)}}.$$

We find Zhukovsky function $\chi_g(u)$ in the form $\chi_g(u)=\chi_0(u)+f(u)$ where the function $\chi_0(u)=r_0(u)+i\theta_0(u)$ satisfies the boundary conditions

$$\begin{aligned}
&\theta_0(u)=-\frac{\pi}{2}, && u=i\eta;\ u=\frac{\pi}{2}+i\eta,\ 0<\eta<d;\\
&\theta_0(u)=\frac{\pi}{2}, && u=\frac{\pi}{2}+i\eta,\ s<\eta\le\frac{\pi|\tau|}{4};\\
&\theta_0(u)=0, && u=\xi;\ u=\frac{\pi}{2}+i\eta,\ d<\eta<s;\\
&r_0(u)=0, && u=\xi+\frac{\pi\tau}{4}.
\end{aligned}$$

and has in D_u the same features as the $\chi_g(u)$, function $f(u)$ is analytic in D_u and continuous in $\overline{D}_u$. We construct $\chi_0(u)$ by the singular points method:

$$\chi_0(u)=\ln\left(\frac{\vartheta_4(u)\vartheta_2(u)}{\vartheta_1(u)\vartheta_3(u)}\right)-$$
$$-\frac{1}{2}\ln\left(\frac{\vartheta_2(u-id)\vartheta_2(u+id)\vartheta_2(u-is)\vartheta_2(u+is)}{\vartheta_3(u-id)\vartheta_3(u+id)\vartheta_3(u-is)\vartheta_3(u+is)}\right)$$

By means of comparing the boundary conditions for the functions $\chi_g(u)$ и $\chi_0(u)$, we obtain the boundary conditions for the unknown function $f(u)=\lambda(u)+i\mu(u)$:

$$\mu(u)=0,\quad u=i\eta;\ u=\frac{\pi}{2}+i\eta; \tag{14}$$

$$\lambda(u)=0,\quad u=\xi+\frac{\pi\tau}{4}; \tag{15}$$

$$\lambda(u)=\ln\left(\frac{V_0}{a_0+b_0\cos\mu(u)}\frac{d\varphi(u)}{d\varphi_g(u)}\right)-r_0(u),\quad u=\xi. \tag{16}$$

To construct the unknown function $f(u)$ we map the area D_u to the semicircle with help of the function (9).

Taking into account the boundary conditions (14), (15), the function $f(u)$ can be analytically continued to ring and be written in the form of a Laurent series (10), where c_n are real coefficients.

On the basis of the boundary condition (15) we find

$$c_0=0,\ c_n=-c_{-n}$$

And obtain

$$f(u)=2i\sum_{n=1}^{\infty}c_n\sin(2n(u-\pi\tau/4)). \tag{17}$$

The boundary condition (16), in view of representation $f(u)$ in the form (17), has the form

$$\begin{aligned}
&2\sum_{n=1}^{\infty}c_n\cos(2n\xi)\,\mathrm{sh}(\pi|\tau|n/2)=\\
&=\ln\left(\frac{V_0}{a_0+b_0\cos\mu(\xi)}\frac{d\varphi(\xi)}{d\varphi_g(\xi)}\right)-r_0(\xi),
\end{aligned} \tag{18}$$

Multiplying equation (18) for $\cos(2n\xi)$ and integrating it by ξ within $0,\pi/2$, we obtain an infinite system of equations for the coefficients c_n:

$$c_n=\frac{2}{\pi\,\mathrm{sh}(\pi|\tau|n/2)}\times$$
$$\times\int_0^{\pi/2}\left(\ln\left(\frac{1}{a_0+b_0\cos\mu(\xi)}\frac{d\varphi(\xi)}{d\varphi_g(\xi)}\right)-r_0(\xi)\right)\cos(2n\xi)d\xi,$$
$$n=\overline{1,\infty}$$

We obtain next formula for dimensionless coordinates of IEG boundaries:

$$\frac{z(u)}{H_E}=\frac{1}{V_0H_E}\int_0^u\frac{dW_g}{du}e^{\chi_g(u)}du.$$

To finish we have to determine the mathematical parameters τ, a, s, d e. This can be done by setting the speed ratio V_A/V_0, V_E/V_A, anode dimensions $L=\mathrm{Re}(z(\pi/2)-z(0))/H_E$, $h=-\mathrm{Im}(z(\pi/2)-z(0))/H_E$, and the condition $\mathrm{Im}(z(\pi/2+is))=\mathrm{Im}(z(\pi\tau/4))$ which fix the position of the cathode boundary.

Fig. 11-13 shows the calculated shapes of anode and cavity surfaces for the different values of parameters. One can see there that there is wave section on the anode surface and its shape depends on the parameters of the electrolyte flow in the IEG even at very small relative size of the cavity. To remove this section we need an additional machining. One method to eliminate the cavity is proposed in [11]

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