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Spline-interpolation solution of 3D Dirichlet problem for a certain class of solids

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We present a spline-interpolation approximate solution of the Dirichlet problem for the Laplace equation in axisymmetric solids, cones and cylinders. Our method is based on reduction of the 3D problem to a sequence of 2D Dirichlet problems. The main advantage of the spline-interpolation solution of the 3D Dirichlet problem is its continuity in the whole domain up to the boundary even for the case of linear spline.

Keywords: Laplace equation; spline; boundary-value problem; approximate solution.

1. Introduction

We present a new—spline-interpolation—approximate solution of the Dirichlet problem for the Laplace equation. This equation has wide applications to electric and seismic prospecting, compare Backus (1968), Cunderlk *et al.* (2004), Ho & Lesnic (2000), Keller (1979), Neiman (1986) and Wengui (1986). The method presented here is applicable for axisymmetric solids and also for the cylinders or cones with the sections being some known conformal map of the unit disc. There exist many approximate solutions of this problem. The classical approximate method is the method of finite elements. The spline-interpolation method is the analogue of the finite-element method (FEM) where instead of the discrete boundary nodes we consider the closed boundary curves and introduce the cells as the layers between two parallel planes. Our method is based on reduction of the 3D problem to a sequence of 2D Dirichlet problems. Note that one can apply functions of complex variable in the case of a 2D Dirichlet problem. There also exists another spline solution technique, but the one presented here is substantially different from the one given, e.g. in the book Cottrell *et al.* (2009). The main advantage of the spline-interpolation solution of the 3D Dirichlet problem is its continuity in the whole domain up to the boundary even for the case of the linear spline. The piece-wise analytic form of the spline-interpolation solution is also convenient for applications.

The first and often non-trivial step in the execution of the ordinary FEM is the meshing of the solid (Lu, 2011). The mesh procedure in our case is the simplest possible since the finite element is the layer between two planes orthogonal to one of the coordinate axes. Suppose that the finite-element step equals δ. Then each layer, that would be in our method represented by just one analytical function, would be subdivided by the classical FEM in δ^2 mesh knots. Therefore, we obtain the finite-element set of the size $[1/\delta^2]$ times smaller than the one for the classical FEM. So the purely computational error in our case is smaller than in the ordinary case.