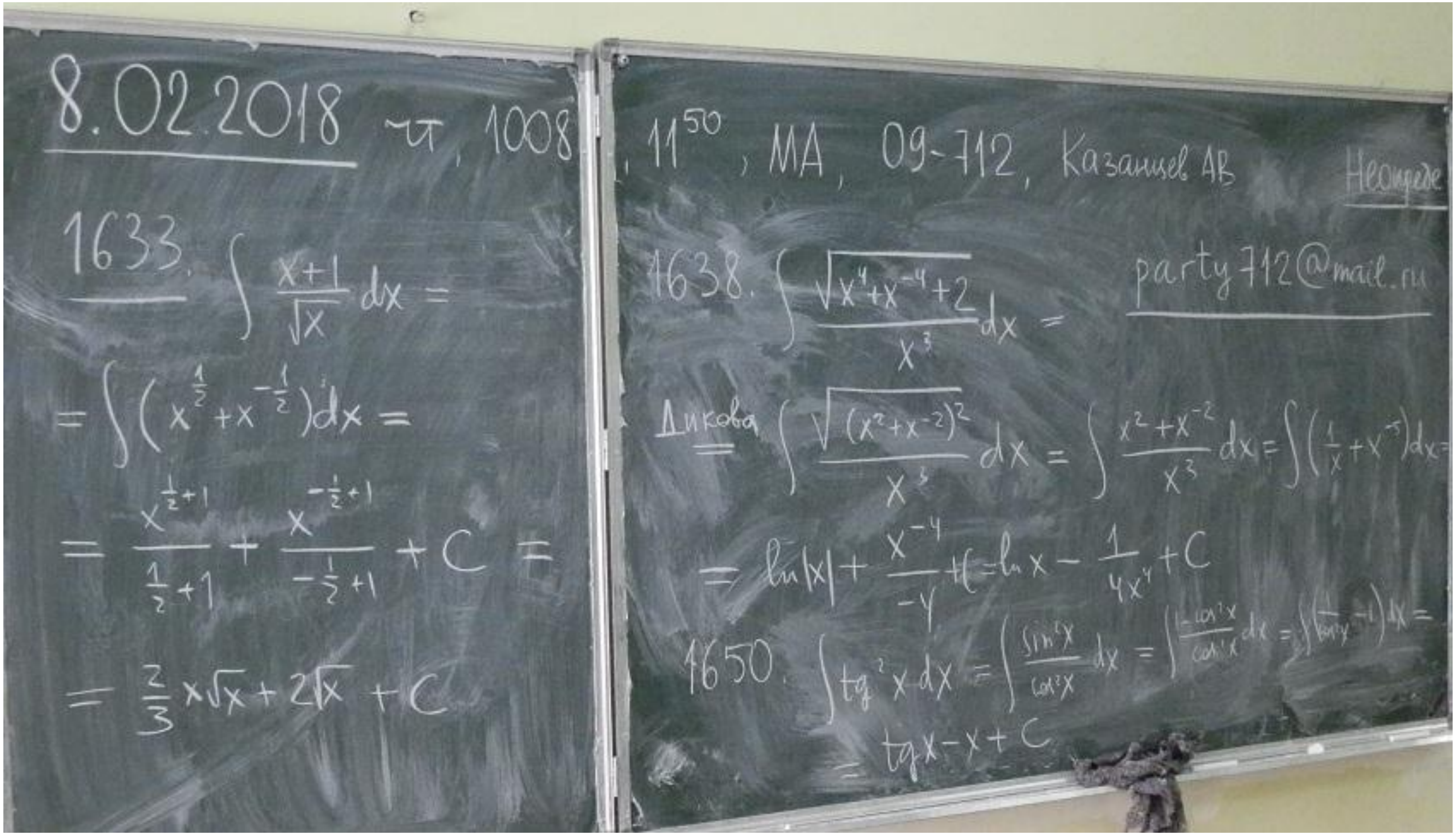
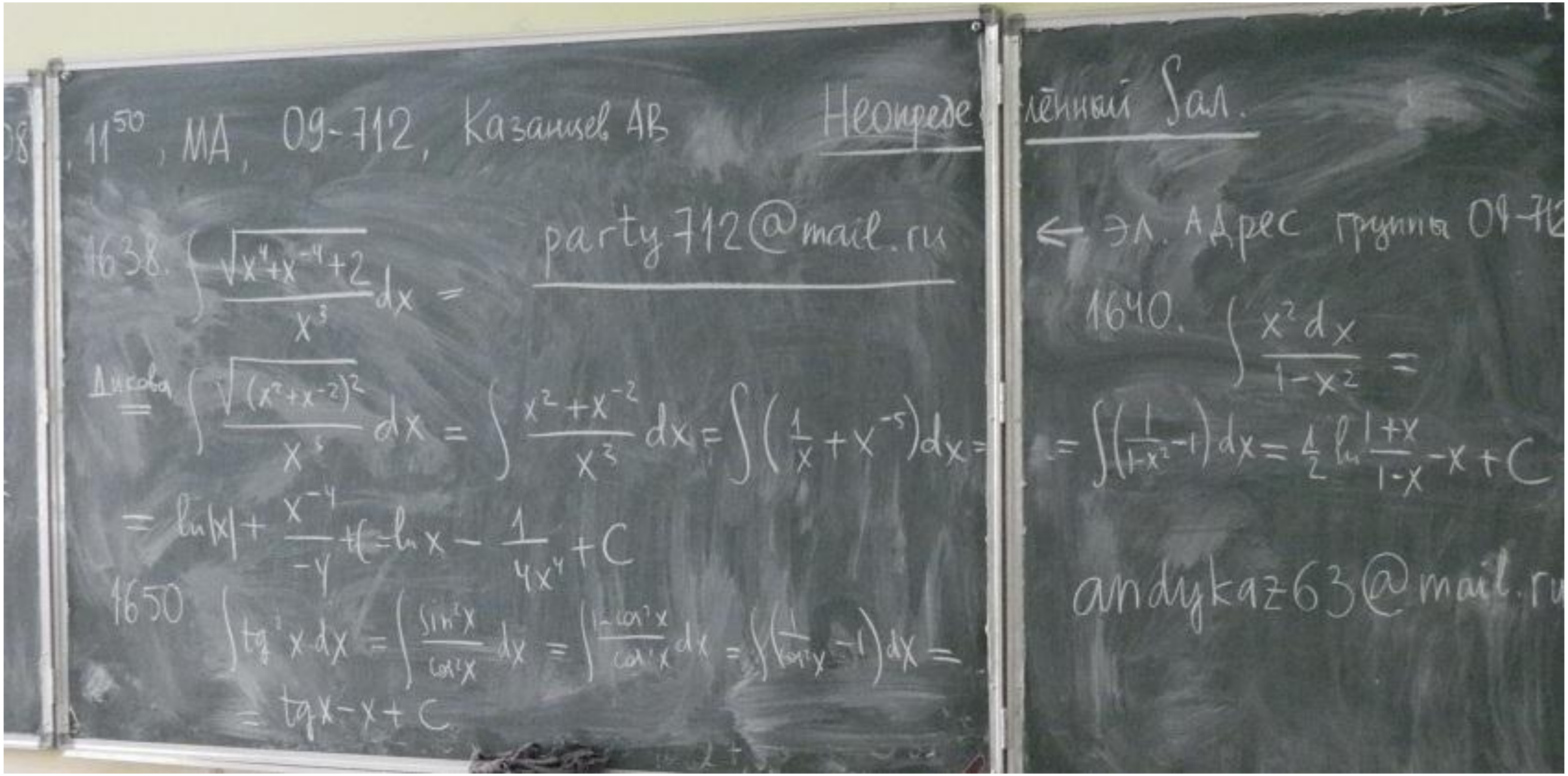
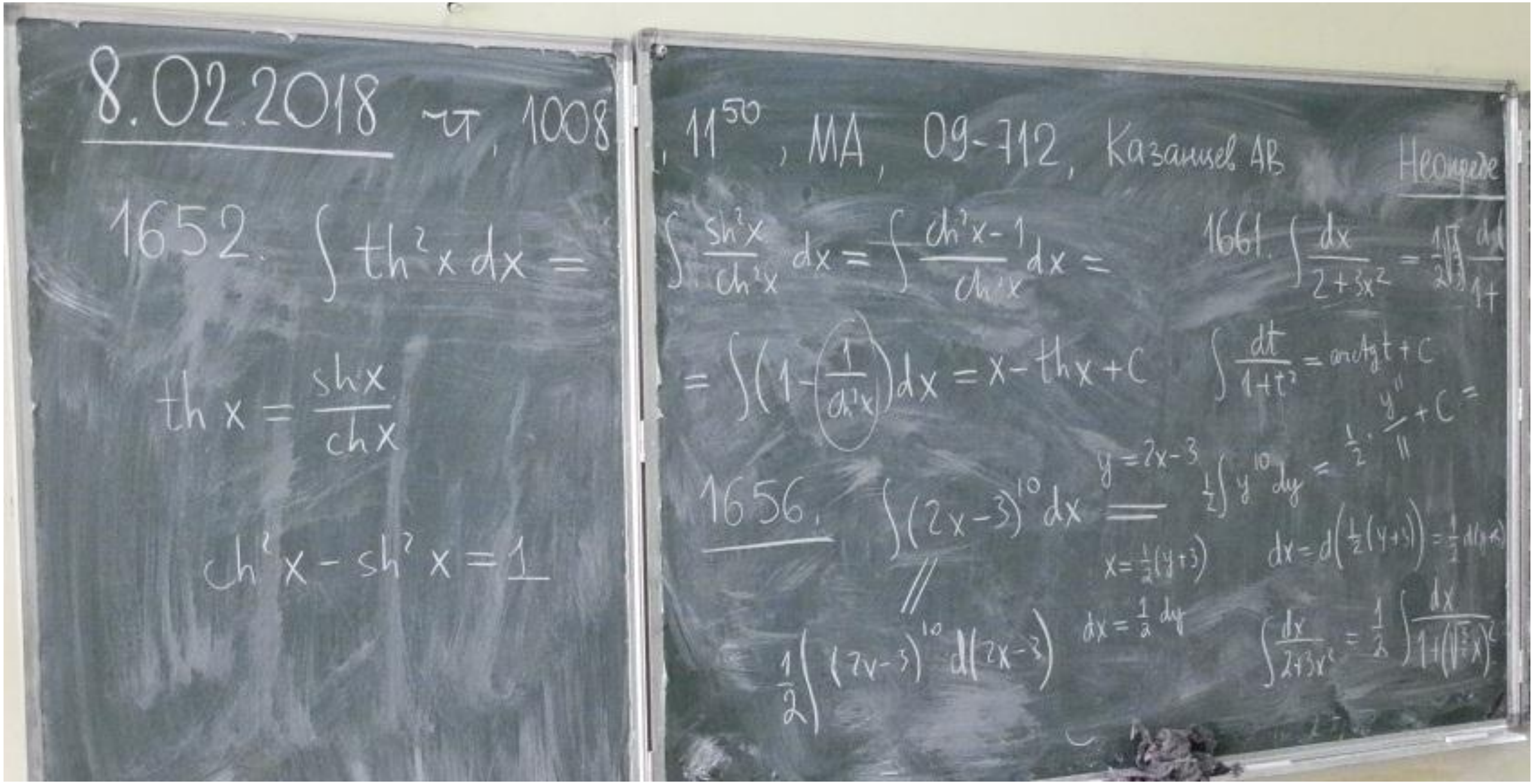
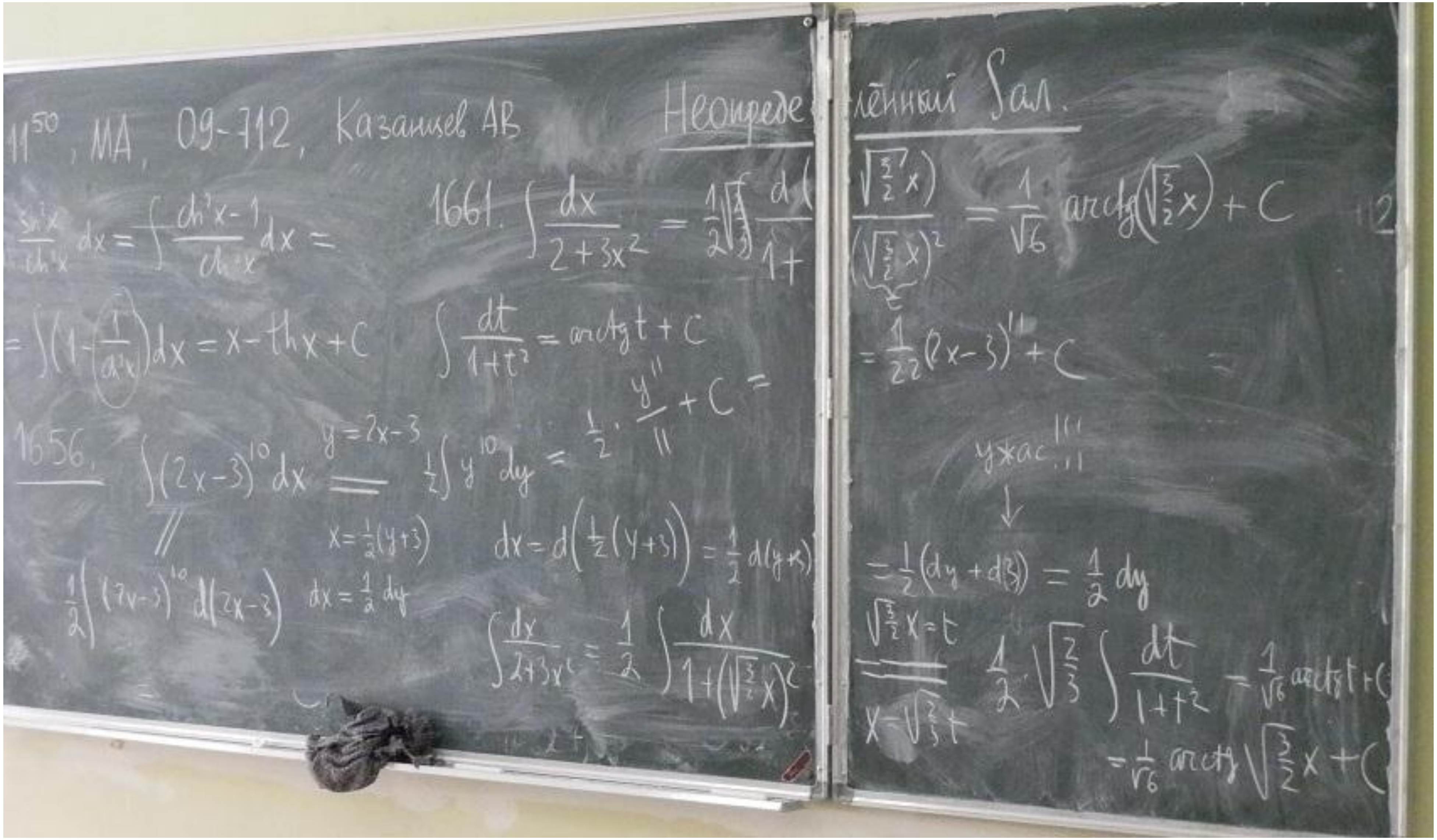
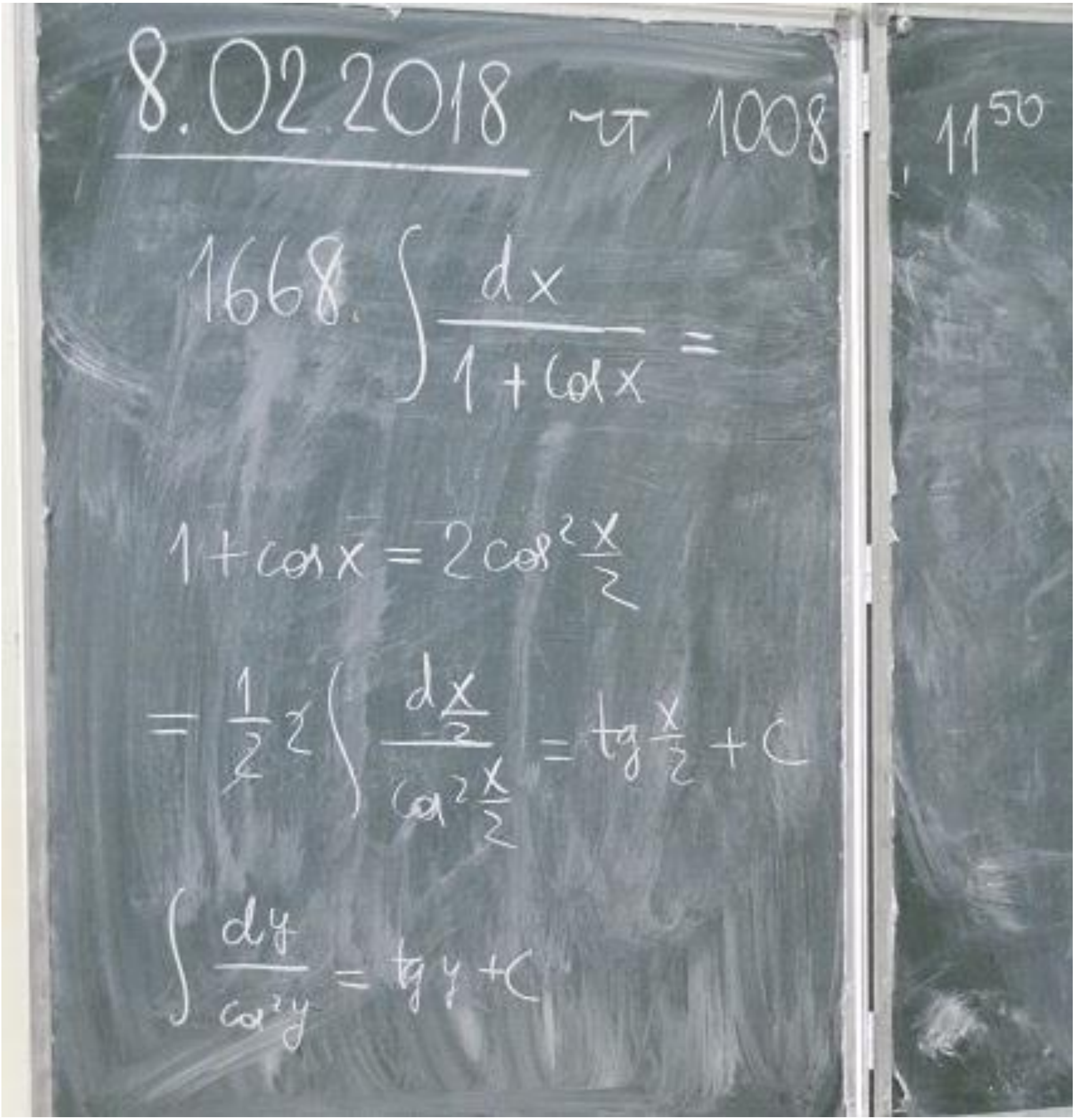

8.02.2018 чт, 1008, 11^50, МА, 09-712, Казанцев АВ
Неопреде
1633. ∫ (x+1)/√x dx =
= ∫ (x^{1/2} + x^{-1/2}) dx =
= x^{1/2+1}/(1/2+1) + x^{-1/2+1}/(-1/2+1) + C =
= 2/3 x√x + 2√x + C
1638. ∫ √(x^4+x^{-4}+2)/x^3 dx =
party712@mail.ru
Дикова
= ∫ √((x^2+x^{-2})^2)/x^3 dx = ∫ (x^2+x^{-2})/x^3 dx = ∫ (1/x + x^{-5}) dx =
= ln|x| + x^{-4}/(-4) + C = ln x − 1/(4x^4) + C
1650. ∫ tg^2 x dx = ∫ sin^2x/cos^2x dx = ∫ (1−cos^2x)/cos^2x dx = ∫ (1/cos^2x − 1) dx =
= tg x − x + C

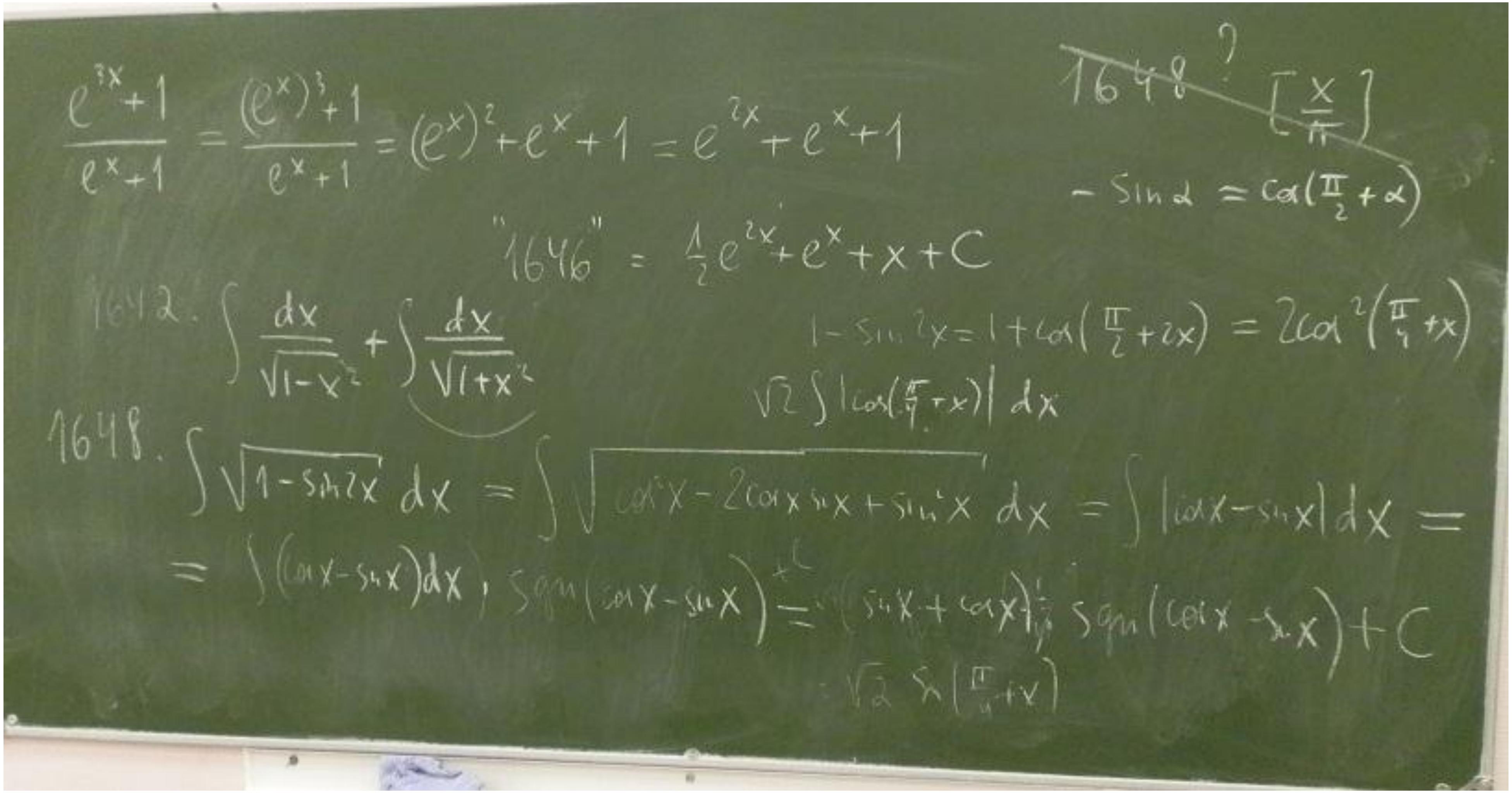

11^50, МА, 09-712, Казанцев АВ
Неопределённый ∫ал.
party712@mail.ru
← эл. адрес группы 09-712
1638.
1640.
1650
andykaz63@mail.ru

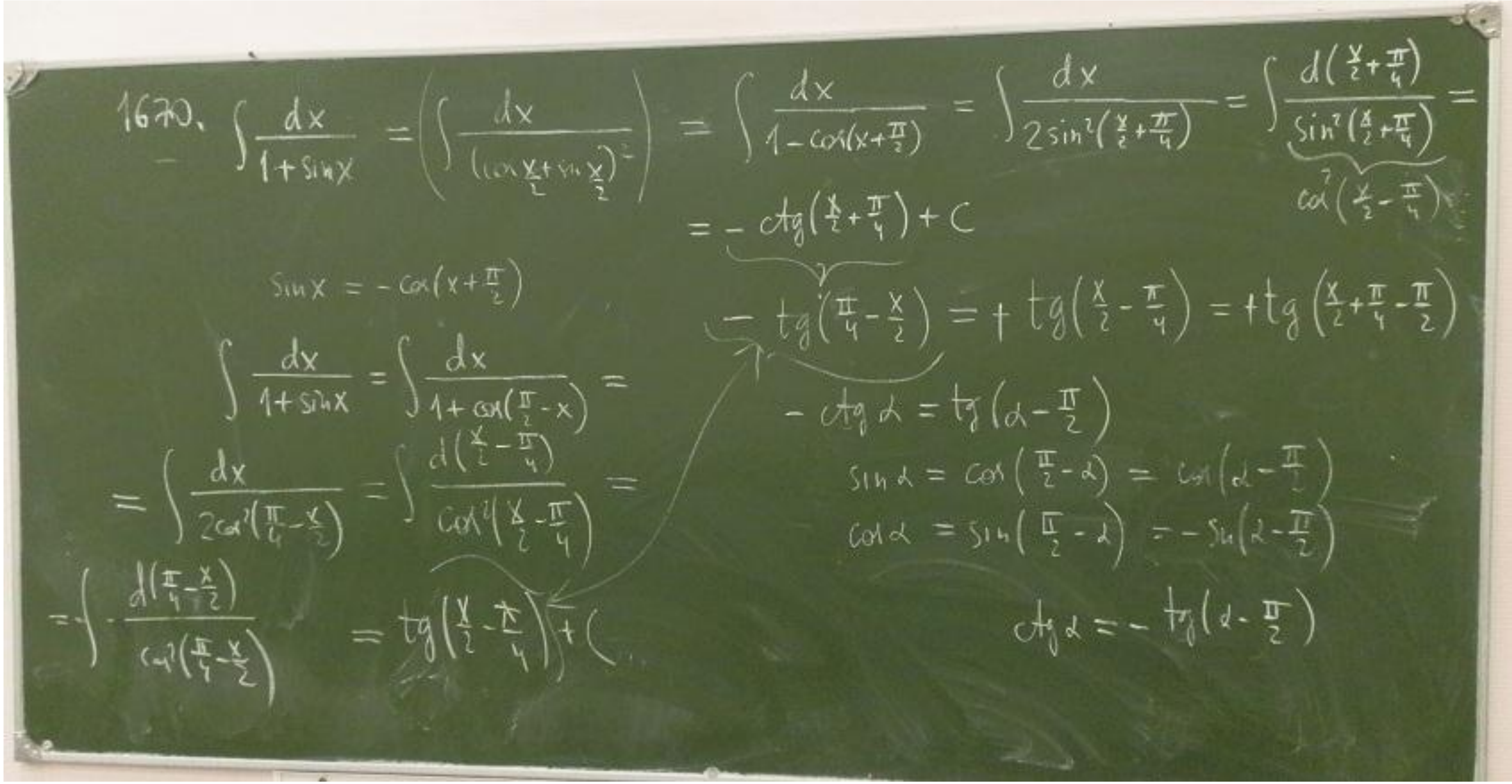

8.02.2018 чт, 1008, 11^{50}, МА, 09-712, Казанцев АВ
Неопреде
1652. $\int th^2 x\, dx = \int \frac{sh^2 x}{ch^2 x} dx = \int \frac{ch^2 x - 1}{ch^2 x} dx =$
$= \int \left(1 - \frac{1}{ch^2 x}\right) dx = x - th\, x + C$
$th\, x = \frac{sh\, x}{ch\, x}$
$ch^2 x - sh^2 x = 1$
1661. $\int \frac{dx}{2+3x^2} =$
$\int \frac{dt}{1+t^2} = arctg\, t + C$
1656. $\int (2x-3)^{10} dx \overset{y=2x-3}{=} \frac{1}{2}\int y^{10} dy = \frac{1}{2} \cdot \frac{y^{11}}{11} + C =$
$x = \frac{1}{2}(y+3)$
$dx = d\left(\frac{1}{2}(y+3)\right) = \frac{1}{2} d(y+3)$
$dx = \frac{1}{2} dy$
$\frac{1}{2}\int (2x-3)^{10} d(2x-3)$
$\int \frac{dx}{2+3x^2} = \frac{1}{2}\int \frac{dx}{1+(\sqrt{\frac{3}{2}}x)^2}$



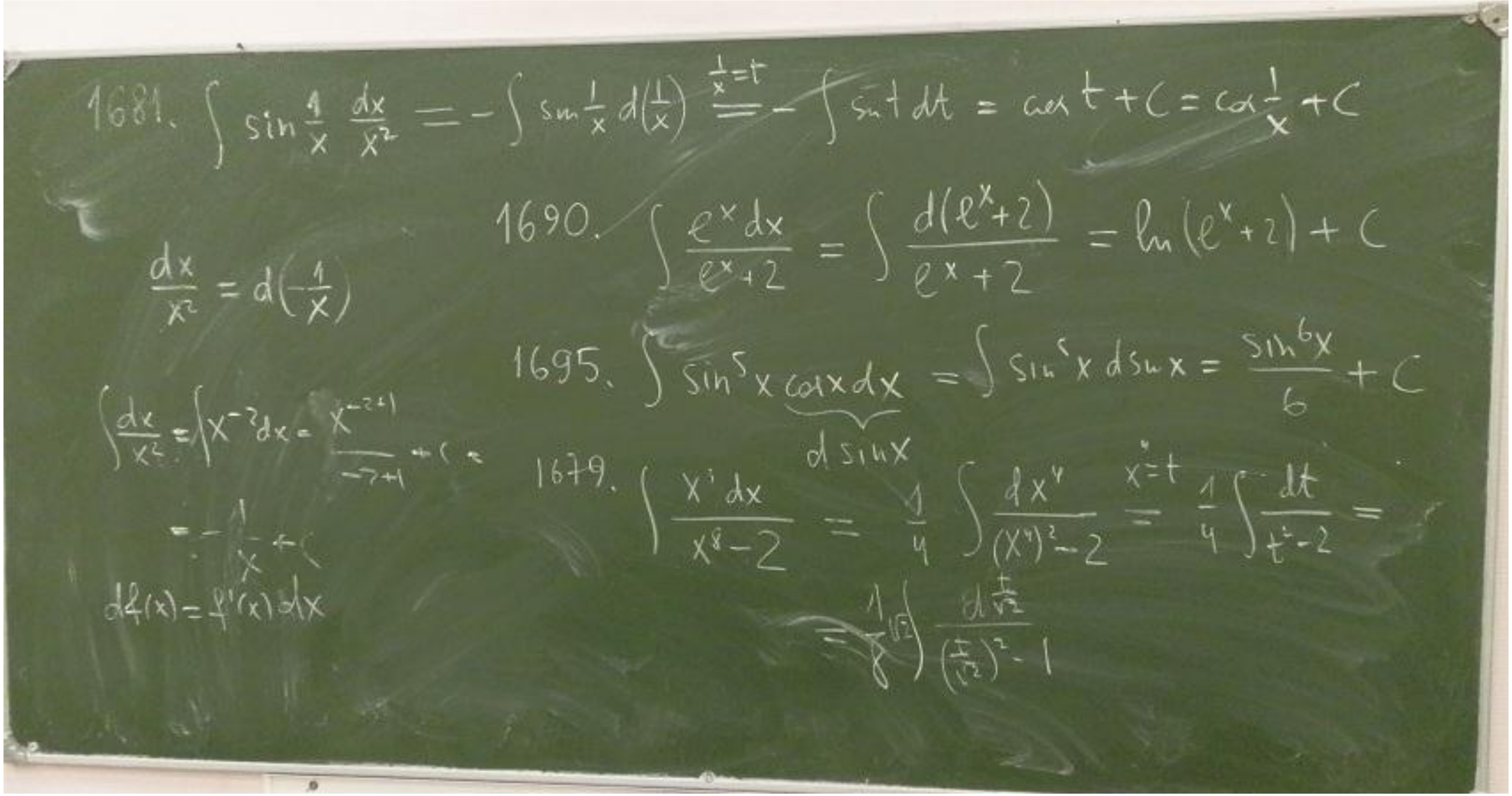
11^50, МА, 09-712, Казанцев АВ
Неопределённый ∫ал.
1661.
1656.
y = 2x − 3
x = 1/2(y+3)
ужас!!!

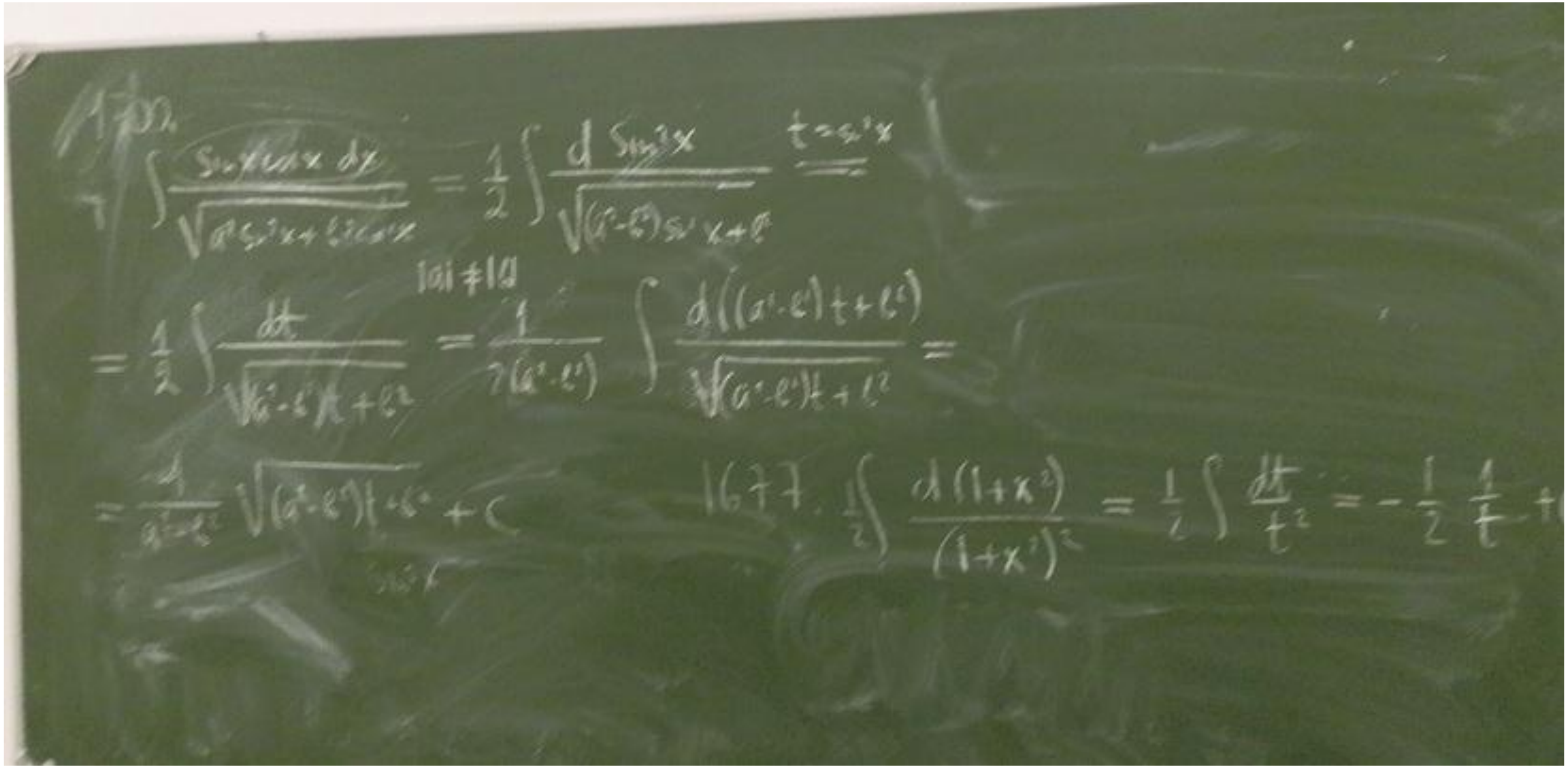

8.02.2018 чт, 1008 , 11^{50}
1668. $\int \frac{dx}{1+\cos x} =$
$1+\cos x = 2\cos^2\frac{x}{2}$
$= \frac{1}{\cancel{2}}\cancel{2}\int \frac{d\frac{x}{2}}{\cos^2\frac{x}{2}} = tg\frac{x}{2} + C$
$\int \frac{dy}{\cos^2 y} = tg\, y + C$

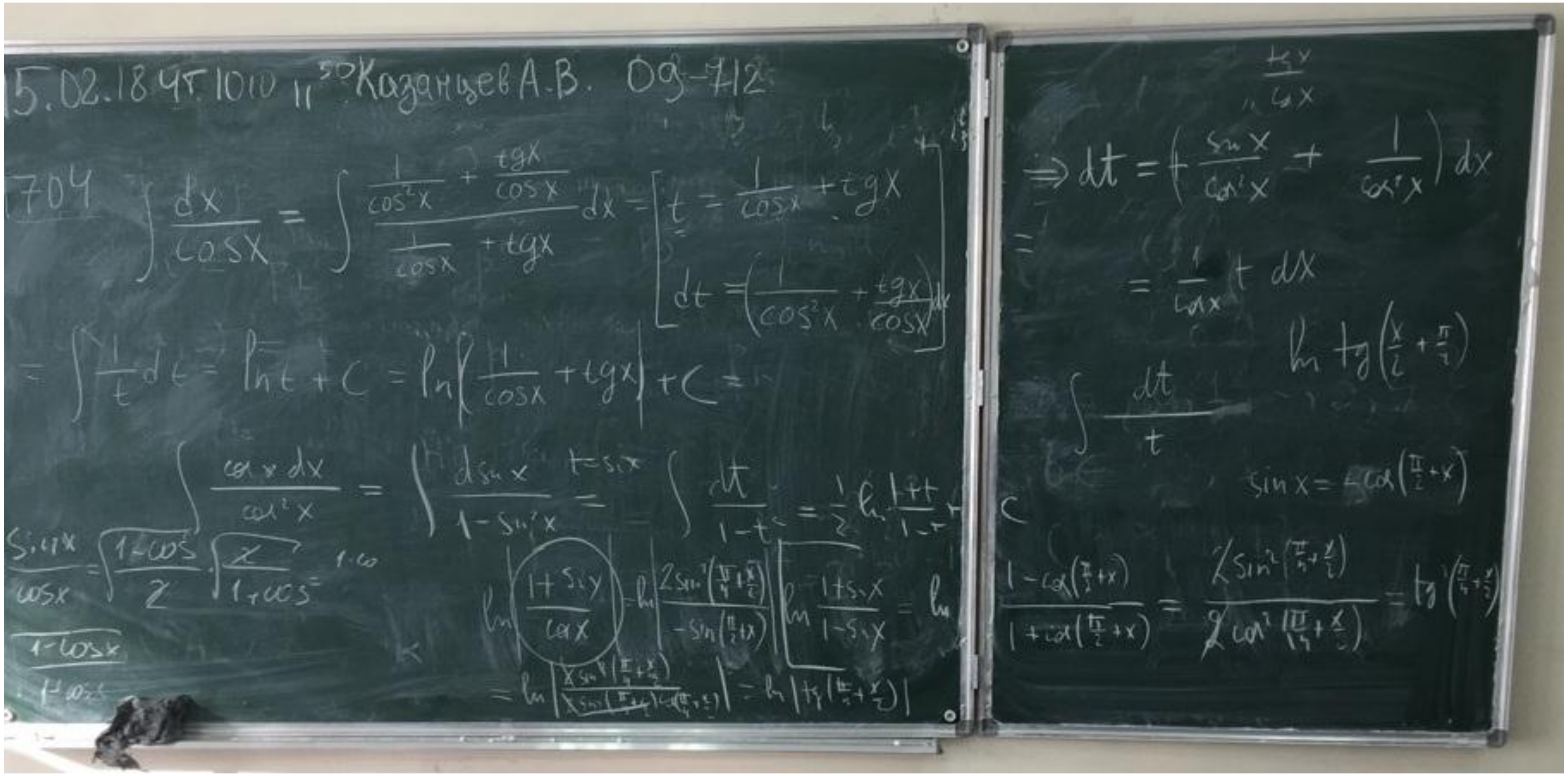

$\frac{e^{3x}+1}{e^x+1} = \frac{(e^x)^3+1}{e^x+1} = (e^x)^2+e^x+1 = e^{2x}+e^x+1$
~~1648~~ ? $[\frac{x}{\pi}]$
$-\sin\alpha = \cos(\frac{\pi}{2}+\alpha)$
"1646" $= \frac{1}{2}e^{2x}+e^x+x+C$
1642. $\int\frac{dx}{\sqrt{1-x^2}}+\int\frac{dx}{\sqrt{1+x^2}}$
$1-\sin 2x = 1+\cos(\frac{\pi}{2}+2x) = 2\cos^2(\frac{\pi}{4}+x)$
$\sqrt{2}\int|\cos(\frac{\pi}{4}+x)|\,dx$
1648. $\int\sqrt{1-\sin 2x}\,dx = \int\sqrt{\cos^2x-2\cos x\sin x+\sin^2x}\,dx = \int|\cos x-\sin x|\,dx =$
$= \int(\cos x-\sin x)dx \cdot \operatorname{sgn}(\cos x-\sin x) = (\sin x+\cos x)\cdot\operatorname{sgn}(\cos x-\sin x)+C$
$= \sqrt{2}\sin(\frac{\pi}{4}+x)$



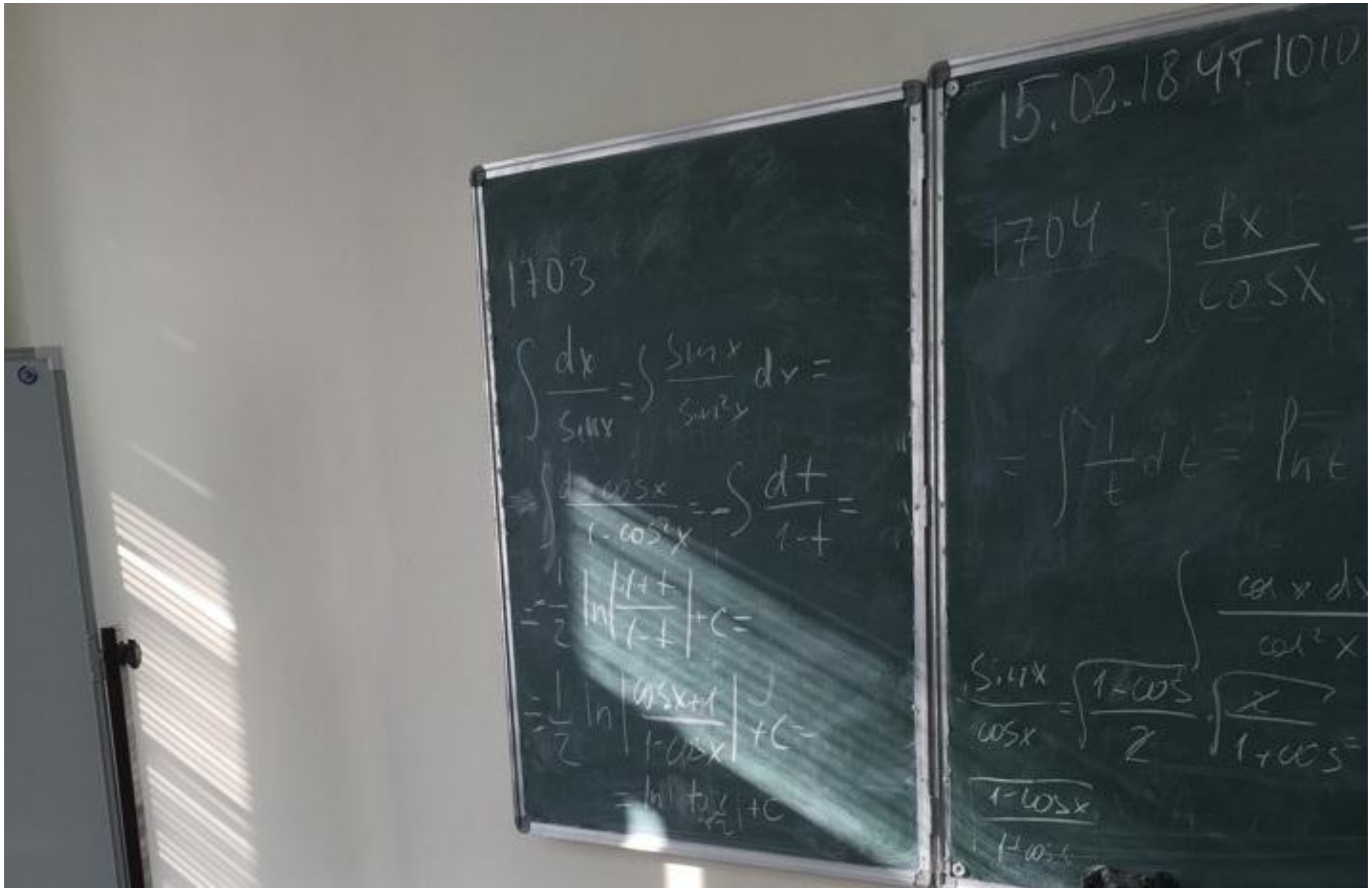
1670. $\int \frac{dx}{1+\sin x} = \left(\int \frac{dx}{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}\right) = \int \frac{dx}{1-\cos(x+\frac{\pi}{2})} = \int \frac{dx}{2\sin^2(\frac{x}{2}+\frac{\pi}{4})} = \int \frac{d(\frac{x}{2}+\frac{\pi}{4})}{\underbrace{\sin^2(\frac{x}{2}+\frac{\pi}{4})}_{\cos^2(\frac{x}{2}-\frac{\pi}{4})}} =$
$= -\underbrace{\operatorname{ctg}(\frac{x}{2}+\frac{\pi}{4})}+C$
$-\operatorname{tg}(\frac{\pi}{4}-\frac{x}{2}) = +\operatorname{tg}(\frac{x}{2}-\frac{\pi}{4}) = +\operatorname{tg}(\frac{x}{2}+\frac{\pi}{4}-\frac{\pi}{2})$
$\sin x = -\cos(x+\frac{\pi}{2})$
$\int \frac{dx}{1+\sin x} = \int \frac{dx}{1+\cos(\frac{\pi}{2}-x)} =$
$= \int \frac{dx}{2\cos^2(\frac{\pi}{4}-\frac{x}{2})} = \int \frac{d(\frac{x}{2}-\frac{\pi}{4})}{\cos^2(\frac{x}{2}-\frac{\pi}{4})} =$
$= \int -\frac{d(\frac{\pi}{4}-\frac{x}{2})}{\cos^2(\frac{\pi}{4}-\frac{x}{2})} = \operatorname{tg}(\frac{x}{2}-\frac{\pi}{4})+C$
$-\operatorname{ctg}\alpha = \operatorname{tg}(\alpha-\frac{\pi}{2})$
$\sin\alpha = \cos(\frac{\pi}{2}-\alpha) = \cos(\alpha-\frac{\pi}{2})$
$\cos\alpha = \sin(\frac{\pi}{2}-\alpha) = -\sin(\alpha-\frac{\pi}{2})$
$\operatorname{ctg}\alpha = -\operatorname{tg}(\alpha-\frac{\pi}{2})$

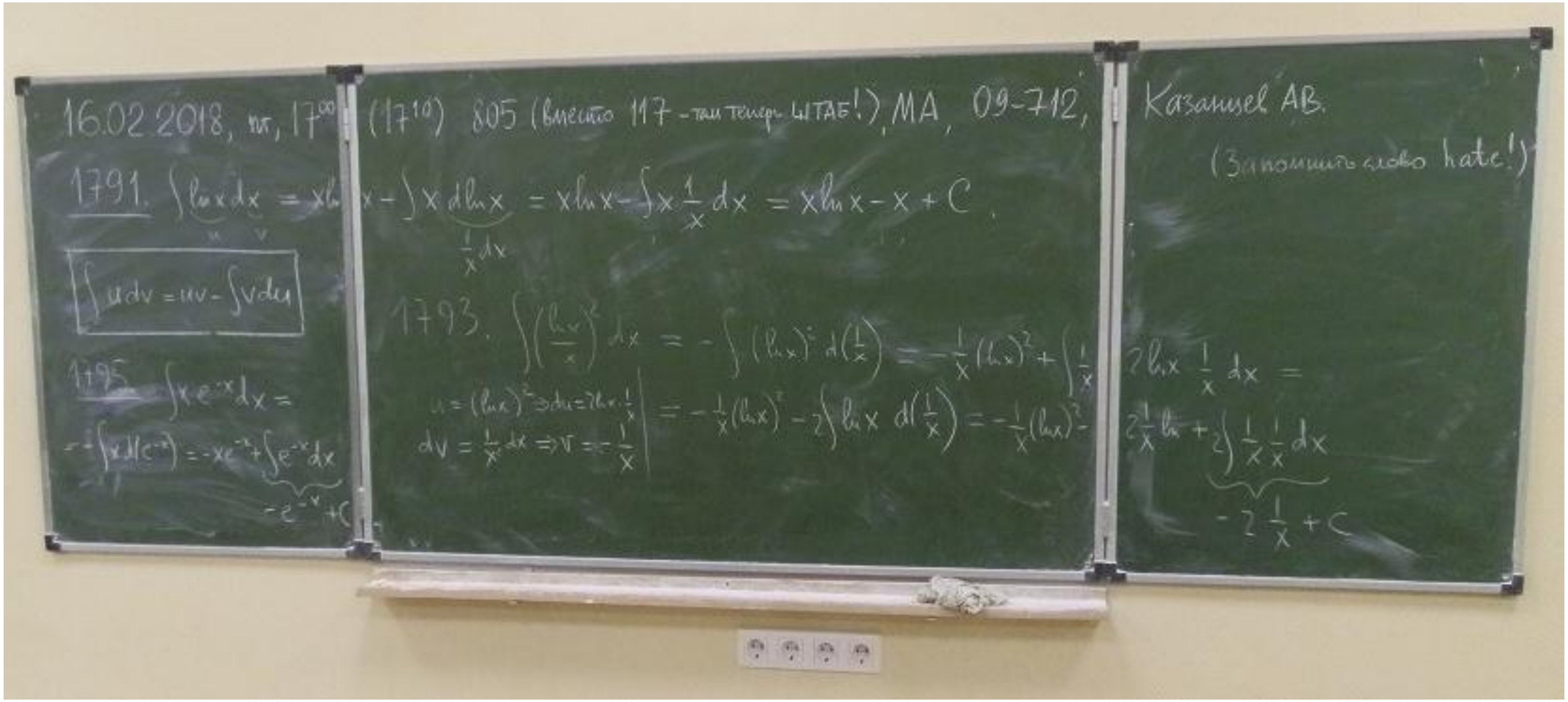

1681. $\int \sin\frac{1}{x}\frac{dx}{x^2} = -\int \sin\frac{1}{x}\, d\left(\frac{1}{x}\right) \overset{\frac{1}{x}=t}{=} -\int \sin t\, dt = \cos t + C = \cos\frac{1}{x} + C$
$\frac{dx}{x^2} = d\left(-\frac{1}{x}\right)$
$\int\frac{dx}{x^2} = \int x^{-2}dx = \frac{x^{-2+1}}{-2+1} + C = -\frac{1}{x} + C$
$df(x) = f'(x)dx$
1690. $\int \frac{e^x dx}{e^x+2} = \int \frac{d(e^x+2)}{e^x+2} = \ln(e^x+2) + C$
1695. $\int \sin^5 x \underbrace{\cos x\, dx}_{d\sin x} = \int \sin^5 x\, d\sin x = \frac{\sin^6 x}{6} + C$
1679. $\int \frac{x^3 dx}{x^8-2} = \frac{1}{4}\int\frac{dx^4}{(x^4)^2-2} \overset{x^4=t}{=} \frac{1}{4}\int\frac{dt}{t^2-2} =$
$= \frac{1}{8}\sqrt{2}\int \frac{d\frac{t}{\sqrt{2}}}{\left(\frac{t}{\sqrt{2}}\right)^2-1}$

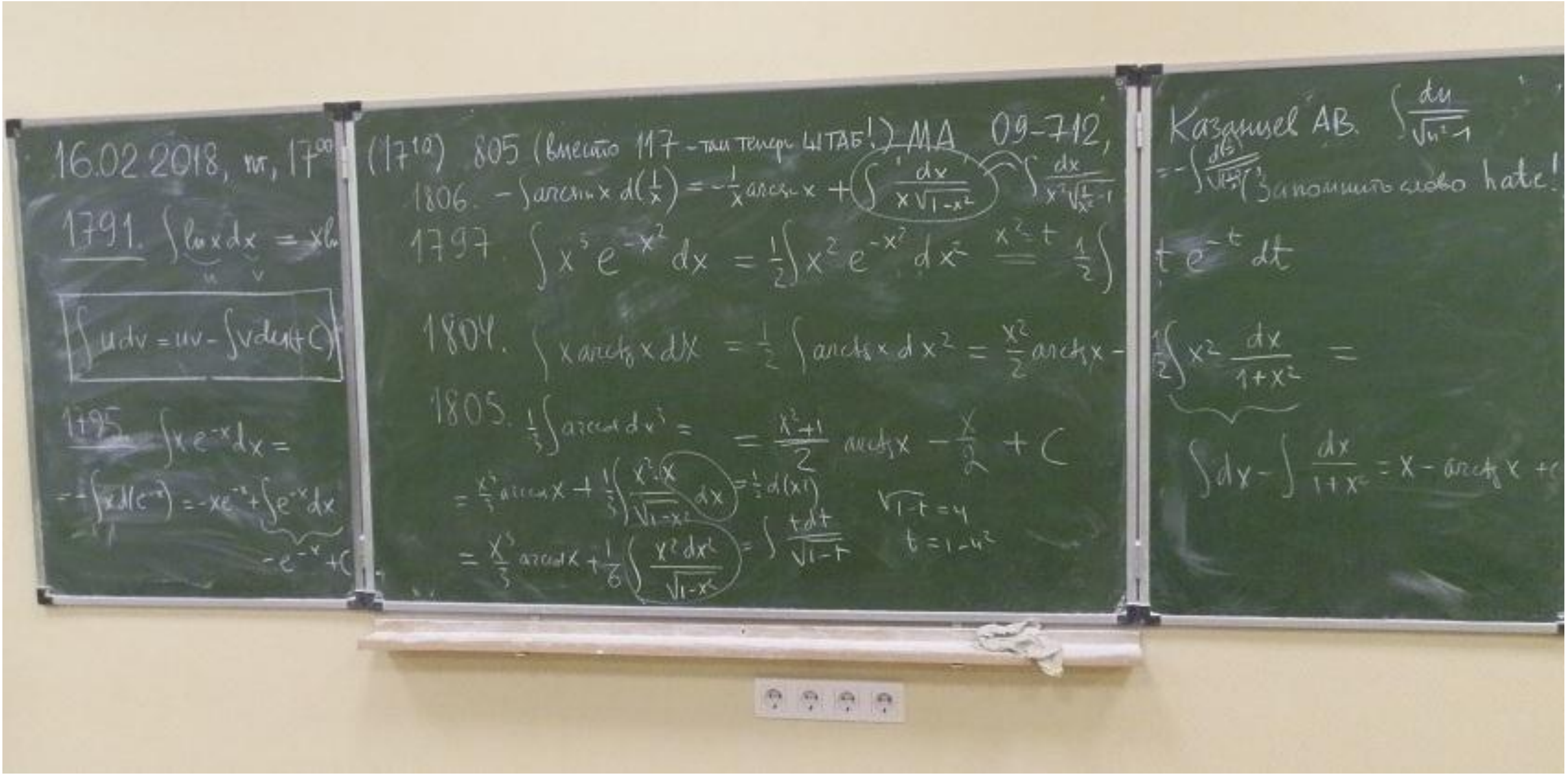

$\int \frac{\sin x \cos x \, dx}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} = \frac{1}{2} \int \frac{d \sin^2 x}{\sqrt{(a^2-b^2)\sin^2 x + b^2}} \underset{=}{t = \sin^2 x}$
$|a| \neq |b|$
$= \frac{1}{2} \int \frac{dt}{\sqrt{(a^2-b^2)t + b^2}} = \frac{1}{2(a^2-b^2)} \int \frac{d((a^2-b^2)t+b^2)}{\sqrt{(a^2-b^2)t+b^2}} =$
$= \frac{1}{a^2-b^2} \sqrt{(a^2-b^2)t+b^2} + C$
1677. $\frac{1}{2} \int \frac{d(1+x^2)}{(1+x^2)^2} = \frac{1}{2} \int \frac{dt}{t^2} = -\frac{1}{2} \frac{1}{t} + C$



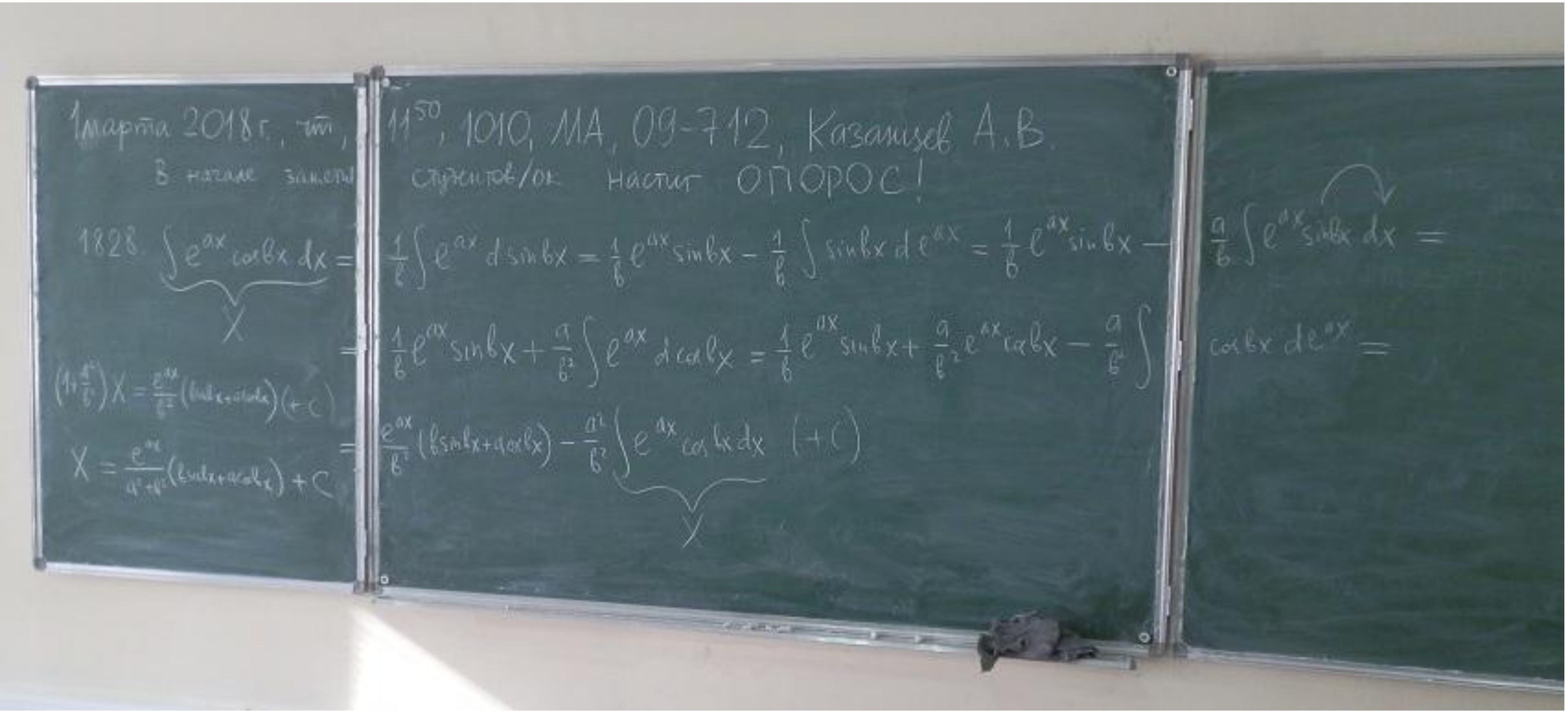
5.02.18 чт 10¹⁰ 11⁵⁰ Казанцев А.В. 09-712
704 ∫ dx/cosx = ∫ (1/cos²x + tgx/cosx)/(1/cosx + tgx) dx = [t = 1/cosx + tgx, dt = (1/cos²x + tgx/cosx)dx]
= ∫ 1/t dt = ln t + C = ln|1/cosx + tgx| + C
∫ cosx dx/cos²x = ∫ dsinx/(1−sin²x) = ∫ dt/(1−t²) = 1/2 ln (1+t)/(1−t) + C
ln tg(x/2 + π/4)
sin x = −cos(π/2 + x)


15.02.18 ЧТ 1010
1703
1704


16.02.2018, пт, 17^{00} (17^{10}) 805 (вместо 117 – там теперь ШТАБ!), МА, 09-712, Казанцев А.В.
(Запомнить слово hate!)
1791. $\int \ln x\,dx = x\ln x - \int x\,d\ln x = x\ln x - \int x\frac{1}{x}dx = x\ln x - x + C$.
$\boxed{\int u\,dv = uv - \int v\,du}$
1793. $\int\left(\frac{\ln x}{x}\right)^2 dx = -\int(\ln x)^2 d\left(\frac{1}{x}\right) = -\frac{1}{x}(\ln x)^2 + \int\frac{1}{x}\,2\ln x\,\frac{1}{x}dx =$
$u=(\ln x)^2 \Rightarrow du = 2\ln x\cdot\frac{1}{x}$
$dv = \frac{1}{x^2}dx \Rightarrow v = -\frac{1}{x}$
$= -\frac{1}{x}(\ln x)^2 - 2\int \ln x\,d\left(\frac{1}{x}\right) = -\frac{1}{x}(\ln x)^2 - 2\frac{1}{x}\ln x + 2\underbrace{\int\frac{1}{x}\frac{1}{x}dx}_{-2\frac{1}{x}+C}$
1795. $\int x e^{-x}dx = -\int x\,d(e^{-x}) = -xe^{-x} + \underbrace{\int e^{-x}dx}_{-e^{-x}+C}$


16.02.2018, пт, 17⁰⁰
(17¹⁰) 805 (вместо 117 – там теперь ШТАБ!) МА 09-712,
Казанцев А.В.
Запомнить слово hate!
1791.
1795.
1797.
1804.
1805.
1806.


22.02.2018, чт, 1010, 11 50, МА, 09-712, Казанцев А.В.
$\int x^2 \arccos x\,dx = \frac{1}{3}\int \arccos x\,dx^3 = \frac{1}{3}x^3 \arccos x - \frac{1}{3}\int x^3\,d\arccos x =$
$= \frac{x^3}{3}\arccos x + \frac{1}{3}\int \frac{x^2 x\,dx}{\sqrt{1-x^2}}$
$\frac{1}{2}\int \frac{x^2\,dx^2}{\sqrt{1-x^2}}$
$\sqrt{1-x^2} = t$


1 марта 2018 г., чт, 11 50, 1010, МА, 09-712, Казанцев А.В.
студентов/ок настиг ОПРОС!
1828.

$$1813.\quad \int x(\operatorname{arctg} x)^2\,dx = \frac{1}{2}\int (\operatorname{arctg} x)^2\,dx^2 =$$

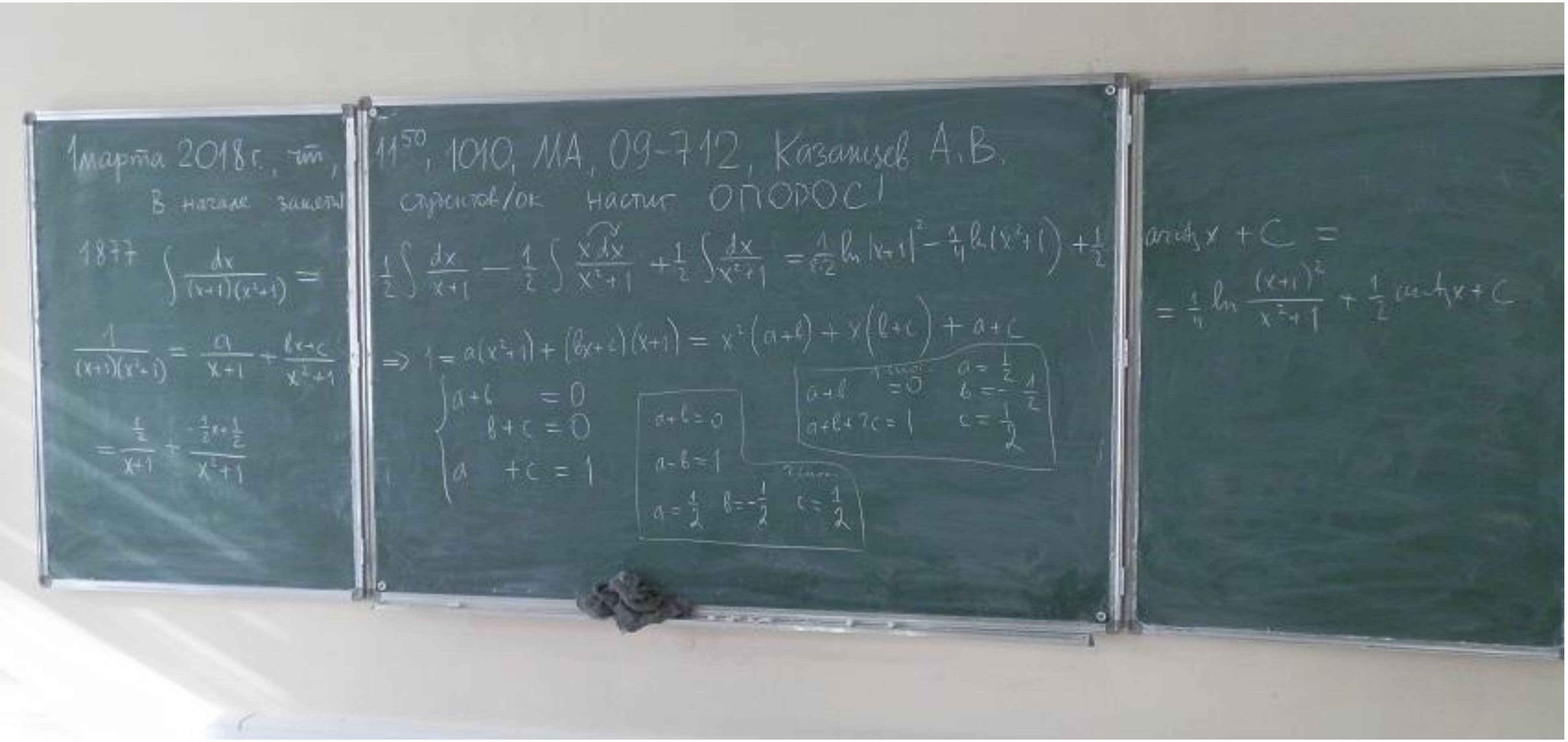
$$= \frac{1}{2}x^2(\operatorname{arctg} x)^2 - \frac{1}{2}\int x^2\,\underbrace{d(\operatorname{arctg} x)^2}_{\frac{2\operatorname{arctg} x}{1+x^2}} =$$

$$= \frac{1}{2}(x\operatorname{arctg} x)^2 - \int \frac{1+x^2-1}{1+x^2}\operatorname{arctg} x\,dx =$$

$$= \frac{1}{2}(x\operatorname{arctg} x)^2 - \int \operatorname{arctg} x\,dx + \int \frac{\operatorname{arctg} x}{1+x^2}\,dx =$$

$$= \frac{1}{2}(x\operatorname{arctg} x)^2 - x\operatorname{arctg} x + \int \frac{x}{1+x^2}\,dx + \int \operatorname{arctg} x\,d(\operatorname{arctg} x) =$$

$$= \frac{1}{2}(x\operatorname{arctg} x)^2 - x\operatorname{arctg} x + \frac{1}{2}\ln(1+x^2) + \frac{1}{2}(\operatorname{arctg} x)^2 + C$$


1марта 2018г., чт, 11^50, 1010, МА, 09-712, Казанцев А.В.
ОПОРОС!
1877

Метод неопр-х коэффициентов

1866. $\int \frac{2x+3}{(x-2)(x+5)}\,dx =$

$\frac{2x+3}{(x-2)(x+5)} = \frac{a}{x-2} + \frac{b}{x+5} \Rightarrow 2x+3 = a(x+5)+b(x-2) = (a+b)x + 5a - 2b$

$\begin{cases} a+b=2 \\ 5a-2b=3 \end{cases} \quad \begin{cases} a+b=2 \\ 7a=7 \end{cases} \Rightarrow a=1,\ b=1$

$= \frac{1}{x-2} + \frac{1}{x+5}$

$= \int \frac{dx}{x-2} + \int \frac{dx}{x+5} = \ln|x-2| + \ln|x+5| + C = \ln|(x-2)(x+5)| + C$

"Изгаление": $(x-2)(x+5) = x^2+3x-10$

$= \int \frac{d(x^2+3x-10)}{x^2+3x-10} = \ln|x^2+3x-10| + C$

получился др. способ, основанный на банальном везении в конкр. случае!

1884. $\displaystyle\int \frac{dx}{x^4+1} = \int \frac{dx}{(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)} =$

$\operatorname{tg}(\underbrace{\operatorname{arctg}\alpha + \operatorname{arctg}\beta}) = \frac{\alpha+\beta}{1-\alpha\beta}$

$x^2-\sqrt{2}x+1 = x^2 - 2x\frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{1}{2} = \left(x-\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2} = \frac{1}{2}\left((\sqrt{2}x-1)^2+1\right)$

$x^2+\sqrt{2}x+1 = \left(x+\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2} = \frac{1}{2}\left((\sqrt{2}x+1)^2+1\right)$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$x^4+1 = (\quad)(\quad)$

$x^4+1 = \underbrace{x^4+2x^2+1}-2x^2 = (x^2+1)^2-2x^2 = (x^2+1-\sqrt{2}x)(x^2+1+\sqrt{2}x)$

$$\frac{1}{x^4+1} = \frac{1}{(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)} = \frac{ax+b}{x^2-\sqrt{2}x+1} + \frac{cx+d}{x^2+\sqrt{2}x+1} = \frac{-\frac{1}{2\sqrt{2}}x+\frac{1}{2}}{x^2-\sqrt{2}x+1} + \frac{\frac{1}{2\sqrt{2}}+\frac{1}{2}}{x^2+\sqrt{2}x+1}$$

$$1 = (ax+b)(x^2+\sqrt{2}x+1) + (cx+d)(x^2-\sqrt{2}x+1) = x^3\underbrace{(a+c)}_{0} + x^2\underbrace{(a\sqrt{2}+b-c\sqrt{2}+d)}_{0} + x\underbrace{(a+b\sqrt{2}+c-d\sqrt{2})}_{0} + \underbrace{b+d}_{1}$$

$c=-a \qquad 2a\sqrt{2}+1=0 \qquad \Downarrow\ d=b \Rightarrow b=d=\frac{1}{2}$

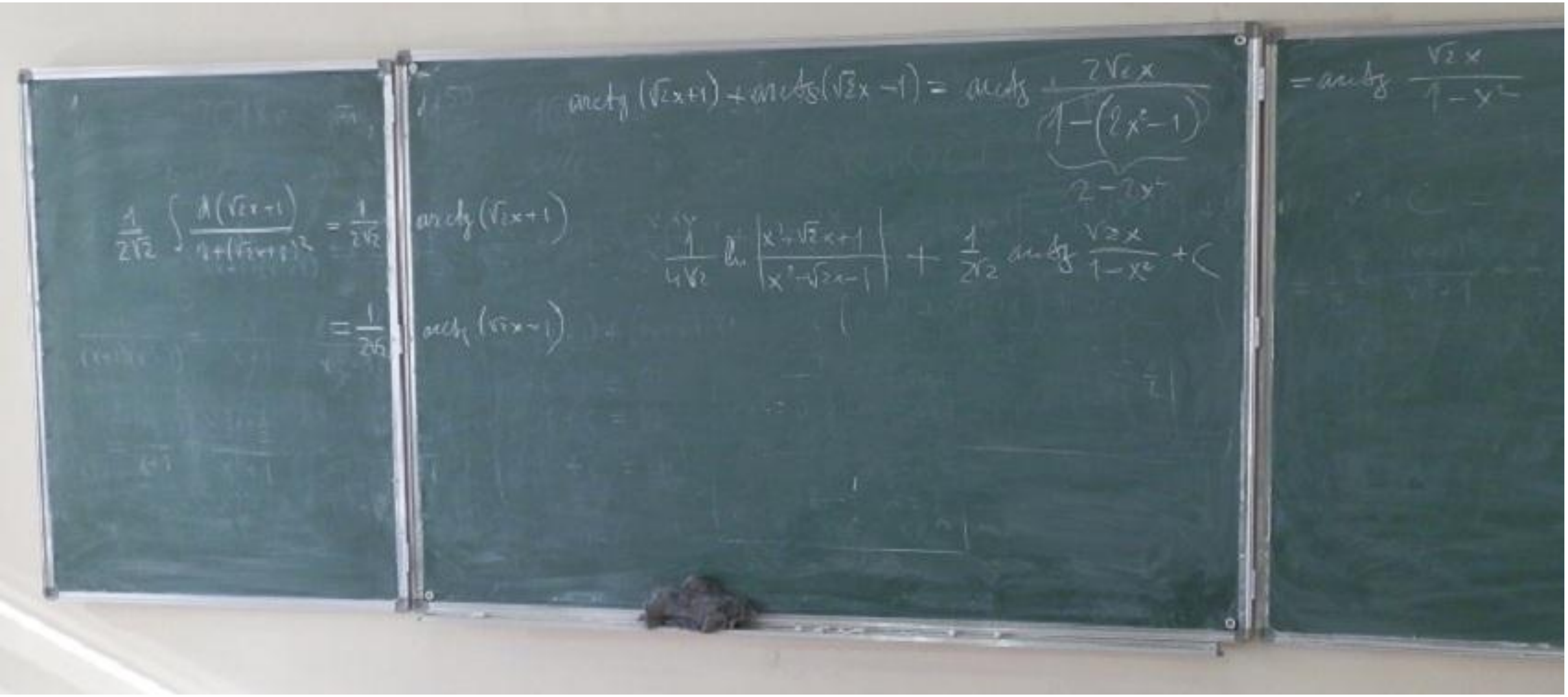
$a = -\frac{1}{2\sqrt{2}}$

$c = \frac{1}{2\sqrt{2}}$

$$= -\frac{1}{2\cdot 2\sqrt{2}}\int\frac{2x-2\sqrt{2}}{x^2-\sqrt{2}x+1}dx + \frac{1}{2\cdot 2\sqrt{2}}\int\frac{2x+2\sqrt{2}}{x^2+\sqrt{2}x+1}dx =$$

$$= -\frac{1}{4\sqrt{2}}\int\frac{2x-\sqrt{2}}{x^2-\sqrt{2}x+1}dx + \frac{1}{4\sqrt{2}}\int\frac{\sqrt{2}\,dx}{x^2-\sqrt{2}x+1} + \frac{1}{4\sqrt{2}}\int\frac{2x+\sqrt{2}}{x^2+\sqrt{2}x+1}dx + \frac{1}{4\sqrt{2}}\int\frac{\sqrt{2}\,dx}{x^2+\sqrt{2}x+1}$$

$$= -\frac{1}{4\sqrt{2}}\ln|x^2-\sqrt{2}x+1| + \frac{1\cdot 2}{4\sqrt{2}}\int\frac{\sqrt{2}\,dx}{1+(\sqrt{2}x-1)^2} + \frac{1}{4\sqrt{2}}\ln|x^2+\sqrt{2}x+1| + \frac{1\cdot 2}{4\sqrt{2}}\int\frac{\sqrt{2}\,dx}{1+(\sqrt{2}x+1)^2}$$


$\frac{1}{2\sqrt{2}}\int\frac{d(\sqrt{2}x+1)}{1+(\sqrt{2}x+1)^2}=\frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}x+1)$
$=\frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}x-1)$
$\operatorname{arctg}(\sqrt{2}x+1)+\operatorname{arctg}(\sqrt{2}x-1)=\operatorname{arctg}\frac{2\sqrt{2}x}{1-\underbrace{(2x^2-1)}_{2-2x^2}}$
$=\operatorname{arctg}\frac{\sqrt{2}x}{1-x^2}$
$\frac{1}{4\sqrt{2}}\ln\left|\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right|+\frac{1}{2\sqrt{2}}\operatorname{arctg}\frac{\sqrt{2}x}{1-x^2}+C$