

11⁵⁰, МА, 09-712, Kazanul AB

Неопределённый интеграл.

$$1638. \int \frac{\sqrt{x^4 + x^{-4} + 2}}{x^3} dx =$$

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Анатолий

$$\int \frac{\sqrt{(x^2 + x^{-2})^2}}{x^3} dx = \int \frac{x^2 + x^{-2}}{x^3} dx = \int \left(\frac{1}{x} + x^{-5} \right) dx =$$
$$= \ln|x| + \frac{x^{-4}}{-4} + C = \ln|x| - \frac{1}{4x^4} + C$$

$$1650 \int \operatorname{tg}^2 x dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \int \left(\frac{1}{\cos^2 x} - 1 \right) dx =$$
$$= \operatorname{tg} x - x + C$$

← ЭЛ. Адрес группы 09-712

$$1640. \int \frac{x^2 dx}{1-x^2} =$$

$$= \int \left(\frac{1}{1-x^2} - 1 \right) dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| - x + C$$

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8.02.2018 ~т, 1008

$$1652. \int \operatorname{th}^2 x \, dx =$$

$$\operatorname{th} x = \frac{\operatorname{sh} x}{\operatorname{ch} x}$$

$$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$$

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Heongede

$$\int \frac{\operatorname{sh}^2 x}{\operatorname{ch}^2 x} \, dx = \int \frac{\operatorname{ch}^2 x - 1}{\operatorname{ch}^2 x} \, dx =$$

$$= \int \left(1 - \frac{1}{\operatorname{ch}^2 x}\right) \, dx = x - \operatorname{th} x + C$$

$$1661. \int \frac{dx}{2+5x^2} = \frac{1}{\sqrt{10}} \frac{dx}{1+t^2}$$

$$\int \frac{dt}{1+t^2} = \operatorname{arctg} t + C$$

$$= \frac{1}{2} \cdot \frac{y''}{11} + C =$$

1656.

$$\int (2x-3)^{10} \, dx$$

$$y = 2x-3$$

$$x = \frac{1}{2}(y+3)$$

$$dx = \frac{1}{2} dy$$

$$\frac{1}{2} \int y^{10} \, dy = \frac{1}{2} \cdot \frac{y^{11}}{11} + C =$$

$$dx = d\left(\frac{1}{2}(y+3)\right) = \frac{1}{2} d(y+3)$$
$$\int \frac{dx}{2+3x^2} = \frac{1}{2} \int \frac{dx}{1+(\sqrt{\frac{3}{2}}x)^2}$$

11⁵⁰, МА, 09-712, Казанск. АВ

$$\int \frac{\sin x}{\cos^2 x} dx = \int \frac{\cos^2 x - 1}{\cos^2 x} dx =$$

$$= \int \left(1 - \frac{1}{\cos^2 x}\right) dx = x - \operatorname{th} x + C$$

1656. $\int (2x-3)^{10} dx$ $y=2x-3$

$$\frac{1}{2} \int y^{10} dy = \frac{1}{2} \cdot \frac{y^{11}}{11} + C =$$

$$x = \frac{1}{2}(y+3)$$

$$dx = d\left(\frac{1}{2}(y+3)\right) = \frac{1}{2} d(y+3)$$

$$\frac{1}{2} \int (2x-3)^{10} d(2x-3) \quad dx = \frac{1}{2} dy$$

$$\int \frac{dx}{2+3x^2} = \frac{1}{2} \int \frac{dx}{1+\left(\sqrt{\frac{3}{2}}x\right)^2}$$

Неопреде

лённый Сан.

$$\frac{\sqrt{\frac{3}{2}x}}{\left(\sqrt{\frac{3}{2}x}\right)^2} = \frac{1}{\sqrt{6}} \operatorname{arctg}\left(\sqrt{\frac{3}{2}x}\right) + C$$

$$= \frac{1}{22}(2x-3)^{11} + C$$

у ж а с

$$= \frac{1}{2}(dy + d3) = \frac{1}{2} dy$$

$$\sqrt{\frac{3}{2}}x = t$$

$$\frac{1}{2} \sqrt{\frac{2}{3}} \int \frac{dt}{1+t^2} = \frac{1}{\sqrt{6}} \operatorname{arctg} t + C$$
$$= \frac{1}{\sqrt{6}} \operatorname{arctg} \sqrt{\frac{3}{2}}x + C$$

8.02.2018 ~т, 1008

11⁵⁰

$$1668. \int \frac{dx}{1 + \cos x} =$$

$$1 + \cos x = 2 \cos^2 \frac{x}{2}$$

$$= \frac{1}{2} \int \frac{dx}{\cos^2 \frac{x}{2}} = \operatorname{tg} \frac{x}{2} + C$$

$$\int \frac{dy}{\cos^2 y} = \operatorname{tg} y + C$$

$$\frac{e^{3x}+1}{e^x+1} = \frac{(e^x)^3+1}{e^x+1} = (e^x)^2+e^x+1 = e^{2x}+e^x+1$$

$$\text{"1646"} = \frac{1}{2}e^{2x}+e^x+x+C$$

~~$$1648 \quad \left[\frac{x}{\pi} \right]$$

$$-\sin \alpha = \cos\left(\frac{\pi}{2}+\alpha\right)$$~~

$$1642. \quad \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{dx}{\sqrt{1+x^2}}$$

$$1-\sin 2x = 1 + \cos\left(\frac{\pi}{2}+2x\right) = 2\cos^2\left(\frac{\pi}{4}+x\right)$$

$$\sqrt{2} \int |\cos\left(\frac{\pi}{4}+x\right)| dx$$

$$1648. \quad \int \sqrt{1-\sin 2x} dx = \int \sqrt{\cos^2 x - 2\cos x \sin x + \sin^2 x} dx = \int |\cos x - \sin x| dx =$$

$$= \int (\cos x - \sin x) dx, \quad \text{sgn}(\cos x - \sin x) = \frac{(\sin x + \cos x)^2}{\sqrt{2} \sin\left(\frac{\pi}{4}+x\right)} \text{sgn}(\cos x - \sin x) + C$$

$$= \sqrt{2} \sin\left(\frac{\pi}{4}+x\right)$$

$$1670. \int \frac{dx}{1+\sin x} = \left(\int \frac{dx}{(\cos \frac{x}{2} + \sin \frac{x}{2})^2} \right) = \int \frac{dx}{1 - \cos(x + \frac{\pi}{2})} = \int \frac{dx}{2 \sin^2(\frac{x}{2} + \frac{\pi}{4})} = \int \frac{d(\frac{x}{2} + \frac{\pi}{4})}{\underbrace{\sin^2(\frac{x}{2} + \frac{\pi}{4})}_{\cos^2(\frac{x}{2} - \frac{\pi}{4})}} =$$

$$= -\operatorname{ctg}(\frac{x}{2} + \frac{\pi}{4}) + C$$

$$\sin x = -\cos(x + \frac{\pi}{2})$$

$$\int \frac{dx}{1+\sin x} = \int \frac{dx}{1+\cos(\frac{\pi}{2}-x)} = \int \frac{dx}{2 \cos^2(\frac{\pi}{4}-\frac{x}{2})} = \int \frac{d(\frac{\pi}{4}-\frac{x}{2})}{\cos^2(\frac{\pi}{4}-\frac{x}{2})} =$$

$$= \int \frac{d(\frac{\pi}{4}-\frac{x}{2})}{\cos^2(\frac{\pi}{4}-\frac{x}{2})} = \operatorname{tg}(\frac{\pi}{4}-\frac{x}{2}) + C$$

$$-\operatorname{ctg}(\frac{\pi}{4}-\frac{x}{2}) = +\operatorname{tg}(\frac{x}{2}-\frac{\pi}{4}) = +\operatorname{tg}(\frac{x}{2} + \frac{\pi}{4} - \frac{\pi}{2})$$

$$-\operatorname{ctg} \alpha = \operatorname{tg}(\alpha - \frac{\pi}{2})$$

$$\sin \alpha = \cos(\frac{\pi}{2} - \alpha) = \cos(\alpha - \frac{\pi}{2})$$

$$\cos \alpha = \sin(\frac{\pi}{2} - \alpha) = -\sin(\alpha - \frac{\pi}{2})$$

$$\operatorname{ctg} \alpha = -\operatorname{tg}(\alpha - \frac{\pi}{2})$$

$$1681. \int \sin \frac{1}{x} \frac{dx}{x^2} = - \int \sin \frac{1}{x} d\left(\frac{1}{x}\right) \stackrel{\frac{1}{x}=t}{=} - \int \sin t dt = \cos t + C = \cos \frac{1}{x} + C$$

$$\frac{dx}{x^2} = d\left(-\frac{1}{x}\right)$$

$$\int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + C = -\frac{1}{x} + C$$

$$d f(x) = f'(x) dx$$

$$1690. \int \frac{e^x dx}{e^x + 2} = \int \frac{d(e^x + 2)}{e^x + 2} = \ln(e^x + 2) + C$$

$$1695. \int \sin^5 x \cos x dx = \int \sin^4 x \underbrace{\cos x dx}_{d \sin x} = \frac{\sin^6 x}{6} + C$$

$$1679. \int \frac{x^3 dx}{x^8 - 2} = \frac{1}{4} \int \frac{dx^4}{(x^4)^2 - 2} \stackrel{x^4=t}{=} \frac{1}{4} \int \frac{dt}{t^2 - 2}$$

$$= \frac{1}{8\sqrt{2}} \frac{d\frac{t}{\sqrt{2}}}{\left(\frac{t}{\sqrt{2}}\right)^2 - 1}$$

1702

$$\int \frac{\sin^2 x dx}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} = \frac{1}{2} \int \frac{d \sin^2 x}{\sqrt{(a^2 - b^2) \sin^2 x + b^2}} \quad t = \sin^2 x$$

$$= \frac{1}{2} \int \frac{dt}{\sqrt{a^2 - b^2} t + b^2} \quad |a| \neq |b| = \frac{1}{2(a^2 - b^2)} \int \frac{d((a^2 - b^2)t + b^2)}{\sqrt{(a^2 - b^2)t + b^2}} =$$

$$= \frac{1}{a^2 - b^2} \sqrt{(a^2 - b^2)t + b^2} + C$$

$$1677. \int \frac{d(1+x^2)}{(1+x^2)^2} = \frac{1}{2} \int \frac{dt}{t^2} = -\frac{1}{2} \frac{1}{t} + C$$

5.02.18 45 1010 II ⁵⁰ Казанцев А.В. 09-712

704 $\int \frac{dx}{\cos x} = \int \frac{\frac{1}{\cos^2 x} + \frac{tgx}{\cos x}}{\frac{1}{\cos x} + tgx} dx = \int \frac{t = \frac{1}{\cos x} + tgx}{\frac{1}{\cos^2 x} + \frac{tgx}{\cos x}} dx$

$dt = \left(\frac{1}{\cos^2 x} + \frac{tgx}{\cos x} \right) dx$

$= \int \frac{1}{t} dt = \ln t + C = \ln \left(\frac{1}{\cos x} + tgx \right) + C =$

$\int \frac{\cos x dx}{\cos^2 x} = \int \frac{d \sin x}{1 - \sin^2 x} = \int \frac{dt}{1-t^2} = \frac{1}{2} \ln \frac{1+t}{1-t} + C$

$\frac{\sin x}{\cos x} = \frac{1 - \cos x}{2} \cdot \frac{2}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x}$

$\ln \frac{1 + \sin x}{\cos x} = \ln \frac{2 \sin^2(\frac{\pi}{4} + \frac{x}{2})}{-2 \sin(\frac{\pi}{2} + x)} = \ln \frac{1 + \sin x}{1 - \sin x} = \ln \frac{1 + \sin(\frac{\pi}{2} + x)}{1 - \sin(\frac{\pi}{2} + x)} = \ln \left| \frac{1 + \sin(\frac{\pi}{2} + x)}{1 - \sin(\frac{\pi}{2} + x)} \right| = \ln \left| \frac{1 + \cos(\frac{\pi}{2} + x)}{1 - \cos(\frac{\pi}{2} + x)} \right| = \ln \left| \frac{1 + \cos(\frac{\pi}{2} + x)}{1 - \cos(\frac{\pi}{2} + x)} \right|$

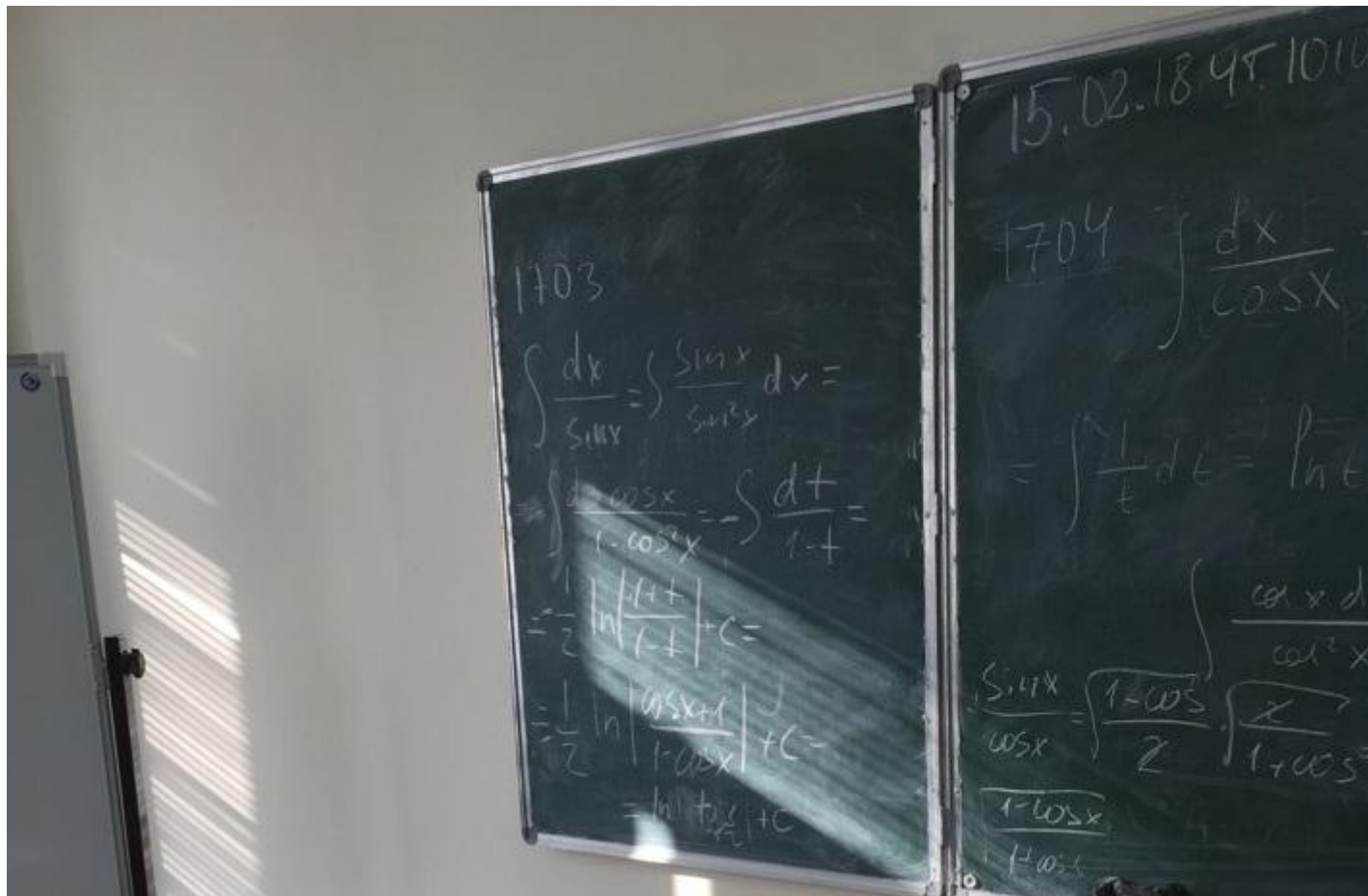
$\Rightarrow dt = \left(\frac{\sin x}{\cos^2 x} + \frac{1}{\cos^2 x} \right) dx$

$= \frac{1}{\cos x} + dx$

$\int \frac{dt}{t} = \ln \left| \frac{1+t}{1-t} \right| + C$

$\sin x = \cos(\frac{\pi}{2} + x)$

$\frac{1 - \cos(\frac{\pi}{2} + x)}{1 + \cos(\frac{\pi}{2} + x)} = \frac{1 - \sin(\frac{\pi}{2} + x)}{1 + \sin(\frac{\pi}{2} + x)} = \frac{1 - \cos(\frac{\pi}{4} + \frac{x}{2})}{1 + \cos(\frac{\pi}{4} + \frac{x}{2})} = \frac{2 \sin^2(\frac{\pi}{8} + \frac{x}{4})}{2 \cos^2(\frac{\pi}{8} + \frac{x}{4})} = \tan^2(\frac{\pi}{8} + \frac{x}{4})$



1703

$$\int \frac{dx}{\sin x} = \int \frac{\sin x}{\sin^2 x} dx =$$

$$= \int \frac{-\cos x}{1 - \cos^2 x} = \int \frac{dt}{1-t^2}$$

$$= \frac{1}{2} \ln \left| \frac{1+t}{1-t} \right| + C =$$

$$= \frac{1}{2} \ln \left| \frac{\cos x + 1}{1 - \cos x} \right| + C =$$

$$= \ln \left| \frac{1 + \cos x}{1 - \cos x} \right| + C$$

15.02.18 9T 1010

1704

$$\int \frac{dx}{\cos x} =$$

$$= \int \frac{1}{t} dt = \ln t$$

$$= \ln |\sin x| + C$$

$$\frac{\sin x}{\cos x} = \sqrt{\frac{1-\cos x}{2}} \sqrt{\frac{2}{1+\cos x}}$$

$$= \frac{1-\cos x}{1+\cos x}$$

16.02.2018, вт, 17⁰⁰ (1710) 805 (вместо 117 - таи теңер. 41ТАБ!), МА, 09-712,

Kazanul AB.

(Зинормур асбо кате!)

1791. $\int \ln x dx = x \ln x - \int x \frac{1}{x} dx = x \ln x - x + C$

$\int u dv = uv - \int v du$

1795 $\int x e^{-x} dx = -\int x d(e^{-x}) = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + C$

1793. $\int \left(\frac{\ln x}{x}\right)^2 dx = -\int (\ln x)^2 d\left(\frac{1}{x}\right) = -\frac{1}{x} (\ln x)^2 + \int \frac{1}{x} \ln x dx$
 $u = (\ln x)^2 \Rightarrow du = 2 \ln x \cdot \frac{1}{x} dx = -\frac{2}{x} (\ln x) dx$
 $dv = \frac{1}{x^2} dx \Rightarrow v = -\frac{1}{x}$
 $= -\frac{1}{x} (\ln x)^2 - 2 \int \ln x d\left(\frac{1}{x}\right) = -\frac{1}{x} (\ln x)^2 - 2 \int \frac{1}{x} \ln x dx$

$2 \ln x \cdot \frac{1}{x} dx =$
 $2 \int \frac{1}{x} \ln x dx + 2 \int \frac{1}{x} \frac{1}{x} dx =$
 $-2 \int \frac{1}{x} dx + C$

16.02.2018, nr, 17⁰⁰

1791. $\int \ln x dx = x \ln x - x + C$

$\int u dv = uv - \int v du + C$

1795. $\int x e^{-x} dx = -\int x d(e^{-x}) = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + C$

(1710) 805 (вместо 17-ти тепер ЧИТАБ!) МА 09-712

1806. $-\int \arcsin x d(\frac{1}{x}) = -\frac{1}{x} \arcsin x + \int \frac{dx}{x \sqrt{1-x^2}}$

1797. $\int x^5 e^{-x^2} dx = \frac{1}{2} \int x^2 e^{-x^2} dx^2 = \frac{x^2 + \frac{1}{2}}{2}$

1804. $\int x \operatorname{arctg} x dx = \frac{1}{2} \int \operatorname{arctg} x dx^2 = \frac{x^2}{2} \operatorname{arctg} x - \frac{x^2}{2}$

1805. $\frac{1}{3} \int \operatorname{arctg} dx^3 = \frac{x^3+1}{2} \operatorname{arctg} x - \frac{x^3}{2} + C$
 $= \frac{x^3}{3} \operatorname{arctg} x + \frac{1}{3} \int \frac{x^3 dx}{\sqrt{1-x^2}} = \frac{1}{3} \int \frac{t dt}{\sqrt{1-t}}$
 $= \frac{x^3}{3} \operatorname{arctg} x + \frac{1}{6} \int \frac{x^2 dx^2}{\sqrt{1-x^2}} = \int \frac{t dt}{\sqrt{1-t}}$
 $\sqrt{1-t} = y$
 $t = 1-y^2$

Kazanuel AB. $\int \frac{du}{\sqrt{u^2-1}}$
 $= -\int \frac{dt}{\sqrt{1-t^2}}$ (аналогично като habe!)

$t e^{-t} dt$

$\int x^2 \frac{dx}{1+x^2} =$

$\int dx - \int \frac{dx}{1+x^2} = x - \operatorname{arctg} x + C$

22.02.2018, мн, 1010, 11⁵⁰, МА, 09-712, Козанцев А.В.

$$\int x^2 \arccos x dx = \frac{1}{3} \int \arccos x d x^3 = \frac{1}{3} x^3 \arccos x - \int \frac{1}{3} x^3 d \arccos x =$$

$$= \frac{x^3}{3} \arccos x + \frac{1}{3} \int \frac{x^2 x dx}{\sqrt{1-x^2}}$$

$$+ \frac{1}{3} \int \frac{x^2 dx}{\sqrt{1-x^2}}$$

$$\int \frac{x^2 dx}{\sqrt{1-x^2}} \rightarrow !$$

$$\sqrt{1-x^2} = t$$

1 марта 2018 г., чт, 11⁵⁰, 1010, МА, 09-712, Казанцев А.В.
 В начале занятия студенты/ок. Наступил ОПРОС!

$$\begin{aligned}
 1828. \int e^{ax} \cos bx \, dx &= \frac{1}{b} \int e^{ax} d \sin bx = \frac{1}{b} e^{ax} \sin bx - \frac{1}{b} \int \sin bx d e^{ax} = \frac{1}{b} e^{ax} \sin bx - \\
 &= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} \int e^{ax} d \cos bx = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a}{b^2} \int \\
 &= \frac{e^{ax}}{b^2} (b \sin bx + a \cos bx) - \frac{a^2}{b^2} \int e^{ax} \cos bx \, dx \quad (+C) \\
 X &= \frac{e^{ax}}{a^2 + b^2} (b \sin bx + a \cos bx) + C
 \end{aligned}$$

$$\begin{aligned}
 \frac{a}{b} \int e^{ax} \sin bx \, dx &= \\
 \cos bx d e^{ax} &=
 \end{aligned}$$

$$\begin{aligned}
1813. \quad \int x (\operatorname{arctg} x)^2 dx &= \frac{1}{2} \int (\operatorname{arctg} x)^2 dx^2 = \\
&= \frac{1}{2} x^2 (\operatorname{arctg} x)^2 - \frac{1}{2} \int x^2 \underbrace{d(\operatorname{arctg} x)^2}_{\frac{2 \operatorname{arctg} x}{1+x^2}} = \\
&= \frac{1}{2} (x \operatorname{arctg} x)^2 - \int \frac{1+x^2-1}{1+x^2} \operatorname{arctg} x dx = \\
&= \frac{1}{2} (x \operatorname{arctg} x)^2 - \int \operatorname{arctg} x dx + \int \frac{\operatorname{arctg} x}{1+x^2} dx = \\
&= \frac{1}{2} (x \operatorname{arctg} x)^2 - x \operatorname{arctg} x + \int \frac{x}{1+x^2} dx + \int \operatorname{arctg} x d(\operatorname{arctg} x) = \\
&= \frac{1}{2} (x \operatorname{arctg} x)^2 - x \operatorname{arctg} x + \frac{1}{2} \ln(1+x^2) + \frac{1}{2} (\operatorname{arctg} x)^2 + C.
\end{aligned}$$

1 марта 2018 г., 11⁵⁰,
 В начале занятия

11⁵⁰, 1010, МА, 09-7.12, Казанцев А.В.
 Справочник/ок. Начини ОПОРОС!

$$1877 \int \frac{dx}{(x+1)(x^2+1)} =$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{a}{x+1} + \frac{bx+c}{x^2+1}$$

$$= \frac{\frac{1}{2}}{x+1} + \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1}$$

$$\frac{1}{2} \int \frac{dx}{x+1} - \frac{1}{2} \int \frac{x dx}{x^2+1} + \frac{1}{2} \int \frac{dx}{x^2+1} = \frac{1}{2} \ln|x+1| - \frac{1}{4} \ln|x^2+1| + \frac{1}{2} \arctan x + C =$$

$$\Rightarrow 1 = a(x^2+1) + (bx+c)(x+1) = x^2(a+b) + x(b+c) + a+c$$

$$\begin{cases} a+b = 0 \\ b+c = 0 \\ a+c = 1 \end{cases}$$

$$\begin{cases} a+b = 0 \\ a-b = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{2} \\ b = -\frac{1}{2} \end{cases}$$

$$\begin{cases} a+b+c = 1 \\ a+b = 0 \end{cases} \Rightarrow c = 1$$

$$\begin{cases} a = \frac{1}{2} \\ b = -\frac{1}{2} \\ c = \frac{1}{2} \end{cases}$$

$$= \frac{1}{2} \ln \frac{(x+1)^2}{x^2+1} + \frac{1}{2} \arctan x + C$$

Метод неопр-х коэффициентов

1866. $\int \frac{2x+3}{(x-2)(x+5)} dx =$

$$\frac{2x+3}{(x-2)(x+5)} = \frac{a}{x-2} + \frac{b}{x+5} \Rightarrow \frac{2x+3}{(x-2)(x+5)} = \frac{a(x+5)+b(x-2)}{(x-2)(x+5)} = \frac{(a+b)x + 5a - 2b}{(x-2)(x+5)}$$
$$\begin{cases} a+b=2 \\ 5a-2b=3 \end{cases} \Rightarrow \begin{cases} a+b=2 \\ 7a=7 \end{cases} \Rightarrow a=1, b=1$$
$$= \frac{1}{x-2} + \frac{1}{x+5}$$

$$\rightarrow = \int \frac{dx}{x-2} + \int \frac{dx}{x+5} = \ln|x-2| + \ln|x+5| + C = \ln|(x-2)(x+5)| + C$$

"ИЗГАЛЕНИЕ": $(x-2)(x+5) = x^2 + 3x - 10$

$$= \int \frac{d(x^2 + 3x - 10)}{x^2 + 3x - 10} = \ln|x^2 + 3x - 10| + C$$

получился др. способ,
оставил на баланском
вернем в комп. зурге!

1884

$$\int \frac{dx}{x^4+1} = \int \frac{dx}{(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)}$$

by partial + casus b) = $\frac{a+b}{1-dx}$



$$x^2-\sqrt{2}x+1 = x^2 - 2x \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{1}{2} = (x-\frac{1}{\sqrt{2}})^2 + \frac{1}{2} = \frac{1}{2}((\sqrt{2}x-1)^2+1)$$

$$x^2+\sqrt{2}x+1 = (x+\frac{1}{\sqrt{2}})^2 + \frac{1}{2} = \frac{1}{2}((\sqrt{2}x+1)^2+1)$$

$$x^4+1 = () ()$$

$$x^4+1 = x^4+2x^2+1-2x^2 = (x^2+1)^2-2x^2 = (x^2+1-\sqrt{2}x)(x^2+1+\sqrt{2}x)$$

$$\frac{1}{x^4+1} = \frac{1}{(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)} = \frac{ax+b}{x^2-\sqrt{2}x+1} + \frac{cx+d}{x^2+\sqrt{2}x+1} = \frac{-\frac{1}{2\sqrt{2}}x + \frac{1}{2}}{x^2-\sqrt{2}x+1} + \frac{\frac{1}{2\sqrt{2}}x + \frac{1}{2}}{x^2+\sqrt{2}x+1}$$

$$1 = (ax+b)(x^2+\sqrt{2}x+1) + (cx+d)(x^2-\sqrt{2}x+1) = x^3(a+c) + x^2(a\sqrt{2}+b-c\sqrt{2}+d) + x(a+b\sqrt{2}+c-d\sqrt{2}) + b+d$$

$c = -a$ $2a\sqrt{2} + 1 = 0 \Rightarrow a = -\frac{1}{2\sqrt{2}}$
 $d = b \Rightarrow b = d = \frac{1}{2}$

$$= -\frac{1}{2\sqrt{2}} \int \frac{2x-\sqrt{2}}{x^2-\sqrt{2}x+1} dx + \frac{1}{2\sqrt{2}} \int \frac{2x+\sqrt{2}}{x^2+\sqrt{2}x+1} dx =$$

$$= -\frac{1}{4\sqrt{2}} \int \frac{2x-\sqrt{2}}{x^2-\sqrt{2}x+1} dx + \frac{1}{4\sqrt{2}} \int \frac{\sqrt{2}}{x^2-\sqrt{2}x+1} dx + \frac{1}{4\sqrt{2}} \int \frac{2x+\sqrt{2}}{x^2+\sqrt{2}x+1} dx + \frac{1}{4\sqrt{2}} \int \frac{\sqrt{2}}{x^2+\sqrt{2}x+1} dx$$

$$= -\frac{1}{4\sqrt{2}} \ln|\sqrt{2}-\sqrt{2}x+1| + \frac{1/2}{4\sqrt{2}} \int \frac{\sqrt{2} dx}{1+(\sqrt{2}x-1)^2} + \frac{1}{4\sqrt{2}} \ln|x^2+\sqrt{2}x+1| + \frac{1/2}{4\sqrt{2}} \int \frac{\sqrt{2} dx}{1+(\sqrt{2}x+1)^2}$$

