Approximate dimension applied to criteria for monogenicity on fractal domains

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Abstract

Suppose that Ω is a bounded domain of $\mathbb{R}^n$ with a fractal boundary Γ and let $\mathbb{R}_{0,n}$ be the real Clifford algebra constructed over the quadratic space $\mathbb{R}^n$. Replacing the fractal dimensions of Γ with conditions of approximating character we will characterize the monogenicity of a $\mathbb{R}_{0,n}$-valued function F in the interior and exterior of Ω, in terms of its boundary value $f = F|_\Gamma$. Moreover, our geometric perspective allows for generalizations of certain monogenic extension results to a wide class of domains.

1 The basics of Clifford analysis

Traditionally, Clifford analysis is regarded as a broadly accepted branch of classical analysis and usually described as a higher dimensional function theory offering a successful generalization of the classical complex analysis. It relies heavily on results on functions defined on domains in Euclidean spaces with values in a Clifford algebra. For the standard reference see [**?**].

The real Clifford algebra associated with $\mathbb{R}^n$ endowed with the Euclidean metric is the minimal enlargement of $\mathbb{R}^n$ to a real linear associative algebra $\mathbb{R}_{0,n}$ with identity such that $x^2 = -|x|^2$, for any $x \in \mathbb{R}^n$.

Let $\{e_j\}_{j=1}^n$, be basis elements for $\mathbb{R}^n$ satisfying $e_ie_j + e_je_i = -2\delta_{ij}$, where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise, $i, j = 1 \dots n$. For a set $A = \{i_1, \dots, i_h\} \subset N = \{1, \dots, n\}$ with $1 \le i_1 < i_2 < \cdots < i_h \le n$, let $e_A = e_{i_1}e_{i_2}\dots e_{i_h}$. Any element $a \in \mathbb{R}_{0,n}$ is of the form $a = \sum_{A\subseteq N} a_Ae_A$, $a_A \in \mathbb{R}$, where $e_\emptyset = e_0 = 1$. Its conjugation is defined by $\overline{a} := \sum_A a_A\overline{e_A}$, where $\overline{e_A} = (-1)^h e_{i_h}\dots e_{i_2}e_{i_1}$. By means of the conjugation, a norm $|a|$ may

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