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STATES ON SYMMETRIC LOGICS: EXTENSIONS

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Dedicated to Professor Anatolij Dvurečenskij

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ABSTRACT. We continue the study of *symmetric logics*, i.e., collections of subsets generalizing Boolean algebras and closed under the symmetric difference. We contribute to several open questions. One of them is whether there is a non-Boolean symmetric logic such that all states on it are $\triangle$-subadditive.

1. Motivation

Orthomodular posets and, in particular, orthomodular lattices appear as algebraic structures of events in quantum mechanics, cf. [6, 7, 15, 16]. The natural requirement that the event system must allow "sufficiently many" states leads (in its stronger form) to orthomodular posets which can be represented as collections of subsets of a set generalizing σ-algebras [6]. In such collections, the set-theoretical symmetric difference can be introduced as an additional operation [13] which cannot be derived from the lattice-theoretical operations and orthocomplementation [8]. Thus we arrive at the notion of a symmetric logic.

During the study of symmetric logics, many questions remained open (cf. [1,2]). Here we answer some of them. We introduce necessary additional constructions with symmetric logics in Section 3. In Section 4, we clarify under which conditions a symmetric logic becomes a Boolean algebra.

2. Basic notions

2.1. Concrete logics

Let Ω be a non-empty set. By 2^Ω we denote the set of all subsets of Ω. For $n \in \mathbb{N}$, we define $\Omega_n = \{1, 2, \dots, n\}$.

Let us recall [6] that a collection $\mathcal{E} \subseteq 2^\Omega$ of subsets of Ω is called a *concrete (quantum) logic* if the following conditions hold true:

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