Injection-Abstraction Well Pair in a Porous Formation with an Eccentric Skin Effect of a Gravel Pack and Damaged Zone

Kasimova R.G., Obnosov Yu.V.

***Abstract*—** Steady, 2-D, saturated, one-phase flow of a fluid (water, oil) from an injection to production well is analyzed by the methods of complex variables. The aquifer (formation) is homogenous and of a constant thickness but the vicinity of the well consists of two circular zones of much higher and lower permeability than that of the aquifer. Refraction of streamlines takes place on the two circular interfaces. The conjugation conditions (pressure and normal flux continuity) are exactly met and three fields of Darcian velocity in the three zones are exactly written in terms of well-converging series. The flow net and isotachs are reconstructed by computer algebra routines.

***Index Terms*—** **Pumping-injection well, skin-factor, heterogeneous permeability, topology of refraction,**

I. Introduction

In reservoir engineering/groundwater hydrology formation/aquifer rock adjacent to a borehole is often damaged by mud liquids during well drilling or stimulated by various mechanical or chemical agents during well completion/development/rehabilitation (fracturing, acid injection, jetting, etc. - aimed at increasing the permeability of the vicinity of the well [1],[2]). As result, an originally homogeneous or almost homogeneous bulk porous medium has a sink/source (well) surrounded by a sheath of a porous (or fractured) material, whose permeability is significantly (sometimes, orders of magnitude) higher or lower than that in the undamaged (pristine) pre-stimulation conditions. Standard engineering formulae used in assessing well productivity in terms of injected-abstracted volumes and pressure/velocity fields either consider no well sheath or assume a simple "skin-factor" for a sheath placed concentrically with the sink. Obnosov *et al.* [3] obtained explicit analytical solutions which take into account an eccentric heterogeneity close to the well and illustrated that the flow refracted in such zones becomes intricately complicated. Obnosov [4] obtained an explicit solution for a Darican flow induced by arbitrary singularities (multipoles) and refraction by an eccentric annulus, which models the well sheath.

In this paper we consider a production well placed eccentrically with its gravel pack, which is also eccentrically located with respect to a formation damage, both embedded into a rock matrix. The corresponding zones are denoted by S_3, S_2 and S_1. Fig.1 shows a cross-section of the formation perpendicular to the well axis.

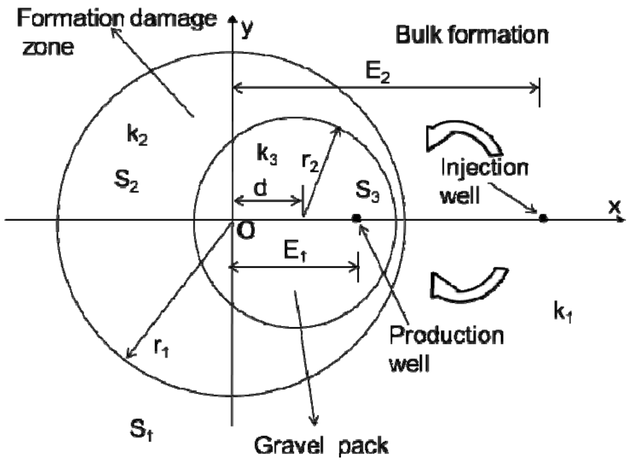

Fig.1 Formation cross-section perpendicular to the well axis

We assume that the well is perfect, i.e. a barefoot one is completely filled with a production fluid, there is no entrance resistance into the borehole due to smallness of perforations (if the well has casing), there is no pressure drop along the well axis and the inflow rate (per unit length) into the well, Q, is a constant value. The damaged zones and gravel pack are assumed to be perfectly circular in cross-sections of Fig.1a and the radii of the circles are r_1 and r_2, respectively, $r_1>r_2$. The origin O of Cartesian coordinates (x,y) coincides with the centre of the damage circle (Fig.1). Fluid flow is assumed to be one-phase, 2-D, Darcian and – owing to the full saturation of all three zones in Fig.1 capillarity is ignored. Phase permeabilities (and hydraulic conductivities k_3, k_2, and k_1) are constant within each zone but jump across the circular interfaces in Fig.1. Fluid injection takes place through another well placed in zone S_1. From mass conservation the injection rate of this well is –Q, i.e. of the same magnitude but opposite sign compared to the abstraction well. Far away from the dipole in Fig.1 the fluid is quiescent.

The coordinates of the wells in Fig.1 are (x_a,y_a) and (x_i,y_i). Schematically, the flow direction is indicated by arrows in Fig.1. The objective of this paper is to analyze the refraction pattern which depends on r_1, r_2, eccentricity e_1, (x_a, y_a) and (x_i, y_i), Q, and hydraulic conductivities.

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Kasimova Rouzalia is with the German University of Technology in Oman. Tel: (968) 99251302 (e-mail: rouzalia.kasimova@gutech.edu.om).
Obnosov Yurii is with Kazan Federal University, Russia (e-mail: yobnosov@ksu.ru).

II. Mathematical Model and solution

We introduce a complex physical coordinate $z = x + iy$ and use overbars for denoting complex conjugations. We will use the terminology of groundwater hydrology [5] and drop obvious equivalent expressions relevant to reservoir engineering.

The Darcian velocities $\vec{V}_j(u_j, v_j)$ within all three zones obey the Darcy law: $\vec{V}_j = -k_j \nabla h_j$ where u_i, v_i are the horizontal and vertical components of the Velocity vector, and ∇h_j are the gradients of hydraulic heads $h_i(x, y)$, $j = 1,2,3$ in each zone. Here h_j is harmonic function and the complexified Darcian velocity $V_j = u_j + i v_j$ is an antiholomorphic one. Complex-conjugated with $V_j(z)$ is the holomorphic function $v_j(z) = u_j - i v_j \ (j = 1,2,3)$.

The refraction conditions along the circles in fig.1 consists of the continuity of the corresponding heads h_i and of the normal components of velocity (continuity of pressure and flux).

The wells in Fig.1 act as a line sink and source (simple poles) i.e. in their vicinity the velocities behave as $\mp Q/[2\pi(z - E_{1,2})]$, where $E_1 \in S_3$, $E_2 \in S_1$.

The final solution ([4]), adapted to our specific case and with slightly changed designations, is:

$$v_j(z) = v_{1j}(z) + v_{2j}(z), \quad j = 1,2,3$$

with

$$v_{11}(z) = -\frac{Q}{2\pi}\left(\frac{\Delta_{12}(1+\Delta_{13})}{(1+\Delta_{13})z} + (1+\Delta_{21})(1-\Delta_{23})\omega_1(z)\right),$$

$$v_{12}(z) = -\frac{Q}{2\pi}(1-\Delta_{23})\left(\omega_1(z) - \Delta_{21}\omega_2(z)\right),$$

$$v_{13}(z) = -\frac{Q}{2\pi}\left(\frac{1}{z-E_1} + \frac{\Delta_{23}}{z-E^*_{1,-1}} - \Delta_{21}(1-\Delta^2_{23})\omega_2(z)\right),$$

$$v_{21}(z) = \frac{Q}{2\pi}\left(\frac{1}{z-E_2} - \Delta_{21}\left(\frac{1}{z} - \frac{1}{z-E^*_{2,0}}\right) + \frac{1-\Delta^2_{21}}{\Delta_{21}}\Omega_1(z)\right),$$

$$v_{22}(z) = \frac{Q}{2\pi}(1-\Delta_{21})\left(\frac{1}{z-E_2} + \frac{1}{\Delta_{21}}\Omega_1(z) - \Omega_2(z)\right),$$

$$v_{23}(z) = \frac{Q}{2\pi}(1-\Delta_{21})(1+\Delta_{23})\left(\frac{1}{z-E_2} - \Omega_2(z)\right),$$

where

$$\omega_1(z) = \sum_{j=0}^{\infty}\delta^j\left(\frac{c_1}{z-E_{1,j}} + \frac{\Delta_{23}(1-\Delta_{21})}{(1-\Delta_{23})(1+\Delta_{13})}\,\frac{\bar{c}_1 + \Delta_{13}c_1}{z - z_{j+1}}\right),$$

$$\omega_2(z) = \sum_{j=0}^{\infty}\delta^j\left(\frac{c_1}{z-E^*_{1,j}} + \frac{\Delta_{23}(1-\Delta_{21})}{(1-\Delta_{23})(1+\Delta_{13})}\,\frac{c_1 + \Delta_{13}\bar{c}_1}{z - z^*_{j+1}}\right),$$

$$\Omega_1(z) = \sum_{j=1}^{\infty}\delta^j\left(\frac{1}{z-z_j} - \frac{1}{z-E^*_{2,-j}}\right), \qquad (1)$$

$$\Omega_2(z) = \sum_{j=1}^{\infty}\delta^j\left(\frac{1}{z-z^*_j} - \frac{1}{z-E_{2,-j}}\right),$$

and

$$z_j = r^2/T(\lambda^{-j}), \quad z^*_j = T(\lambda^{-j}),$$

$$E_{i,j} = T\left(\lambda^j\,\frac{a-E_i}{b-E_i}\right), \qquad i = 1,2,$$

$$E^*_{i,j} = r^2/T\left(\lambda^j\,\frac{a-\bar{E}_i}{b-\bar{E}_i}\right), \quad j = 0,\pm1,\pm2,\dots$$

$$T(z) = \frac{bz-a}{z-1}, \quad \lambda = \frac{(a-d)b}{(b-d)a} < 1,$$

$$a,b = \frac{d^2 + R^2 - r^2 \mp \sqrt{\left(d^2 - R^2 - r^2\right)^2 - 4r^2R^2}}{2d},$$

$$\Delta_{ij} = \frac{k_j - k_i}{k_j + k_i}, \ \delta = \Delta_{21}\Delta_{23}.$$

We wrote a code in Wolframs' *Mathematica* [6] with summation of the four series in (1). The truncation criterion was selected based on the conjugation conditions of dimensionless velocity V_i/k_1 on the two circles, i.e. the discrepancy in the normal and in the linear relation between the tangential components of the velocity, as the circles in Fig.1 are crossed, should be less than a given constant ε (in most computations we selected $\varepsilon = 10^{-6}$. This condition we tested at 13 angularly equidistant points on each circle. Depending on the problem parameters we retained 10-150 terms in the series.

III. Results

Fig.2a-b shows the flow net for k_1=1, k_2=0.1, k_3=10, r_1=1, r_2=0.2, d=−0.7, E_1=−0.7, Q=1 and two locations of the injection well E_2=10 and E_2=1.5.

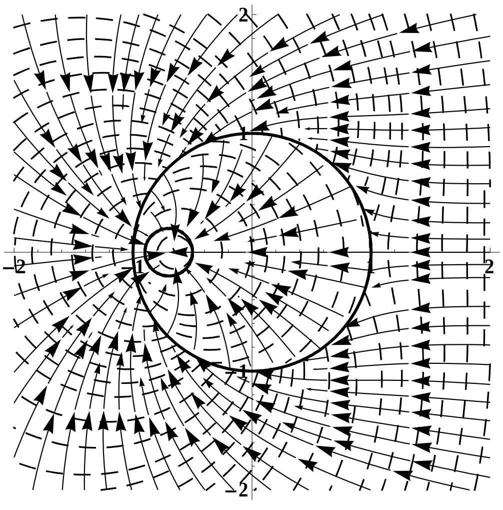

Fig.2a. Flow net for for k_1=1, k_2=0.1, k_3=10, r_1=1, r_2=0.2, d=−0.7, E_1=−0.7, Q=1 and E_2=10

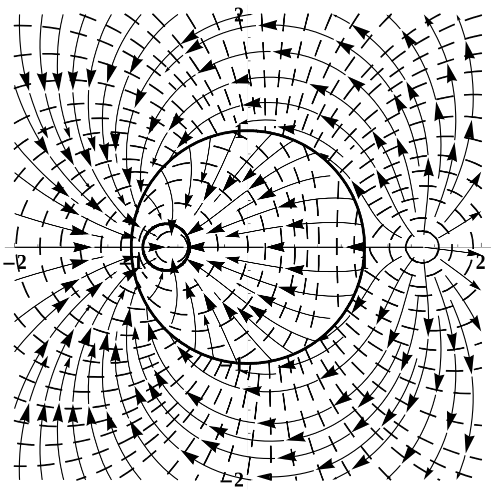

Fig.2b Flow net for for k_1=1, k_2=0.1, k_3=10, r_1=1, r_2=0.2, d=−0.7, E_1=−0.7, Q=1 and E_2=1.5

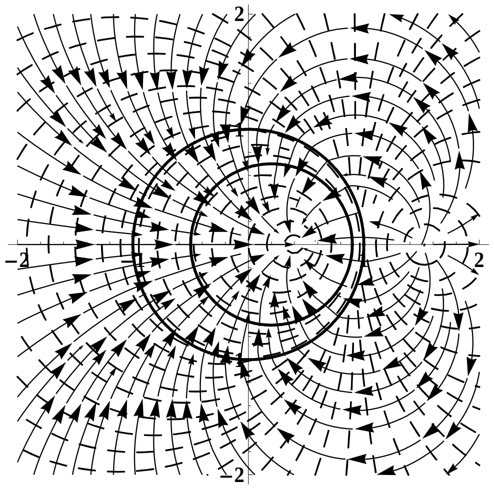

Fig.2c Flow net for k_1=1, k_2=0.1, k_3=10, r_1=1, E_2=1.5, Q=1, d=0.2, E_1=0.4, and r_2=0.7.

Fig.2c presents the results for k_1=1, k_2=0.1, k_3=10, r_1=1, E_2=1.5, Q=1, positive eccentricity of the gravel pack and sink towards the injection well with d=0.2, E_1=0.4, and relatively thin damage zone with r_2=0.7. The stream lines and equipotential lines are reconstructed by *Mathematica* using its **StreamPlot** routine in solid (arrowed) and dashed lines, correspondingly.

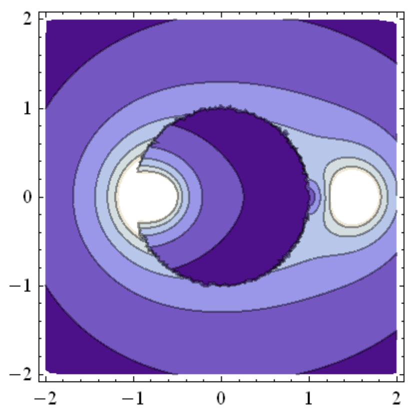

Fig.3a. Contour plot of isotachs for the example in Fig.2a

Fig.3a illustrates the isotachs obtained by the **ContourPlot** routine and the same values of parameters as in Fig.2a. In light-coloured zones of Fig.3a velocities are high and in dark zones – low. Fig.3b presents the isotach contours for the same parameters, but very mild damage of k_2=0.9. As is clear from comparisons of Fig.3a and Fig.3b, the formation damage induces a low-velocity zone between the two wells.

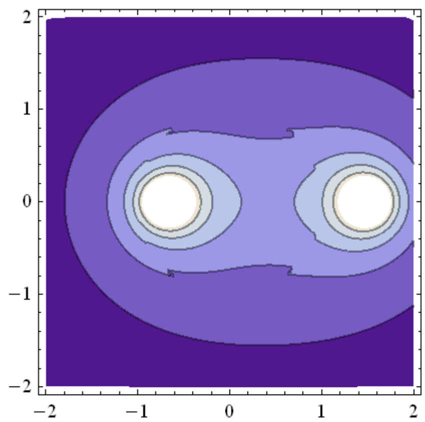

Fig.3b Contour plot of isotachs for the example in Fig.3a but k_2=0.9.

The lines of constant velocity are important in evaluation of suffusion (internal erosion) induced by flow [5]. If the magnitudes of the velocity (hydraulic gradient) exceed a certain limit, then fine solid particles start to be dislodged and transported to the gravel pack and eventually to the borehole of the production well.

There is no limitation on the position of the singularities on the x-axis in Fig.1. In this paper we used these loci of the dipole in order to compute the velocity distribution and travel time of marked particles along the "shortest" path between the wells (a segment of the x-axis). For arbitrary position of the wells the shortest streamline in Fig.1 is not known in advance.

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