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BEYOND ENTROPY: THE ZETA FUNCTION

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Abstract: Can the concept of "arrow of time" make sense in standard physical framework? It will be presumed that, for an arguing only backed on ZFC axioms and set theory, and except in a very particular case (2-metric and "thermal time" in quantum mechanics) the answer is undecidable. The use of category theory, in particular of the concept of Grothendieck's topos and monad, allows an access to a new stand point about this issue. As illustration, and using the links with the Riemann zeta function, this note analyses the breakthrough offered, in temporal matter, by the open fractional dynamics (in fractal fold structure 1<d<2). The analysis highlights the link between a "proper time" based on zeta function and the concept of entanglement and entropy. Taking priority over Noetherian energy in the physics of open systems, thus dual (internal-external/direct-anti), entropy appears related to a coarse graining bundled by arithmetic properties expressed through zeta Riemann function.

keywords: Fractal. Zeta function. Entropy. Topos. Monads. Arrow of time

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1 Introduction

The concept of "arrow of time" is a concept open to many speculations. Since consciousness is a macroscopic characteristic, would it be possible to prove its existence using standard and non-quantum physical problems? If such an arrow exists, it is very probably related to the concept of entropy, to date the only oriented function in physics. Its foundation is combinatorial, which too much naturally links its expression to probabilities and therefore to a theory of additive and normalized measures (opposed to non-additive and none able to be normalized). In the spirit of the Maxwellian paradoxes about global irreversibility emerging from a local reversibility through a change of scales, and although quantum dynamics appeals an absolute reversible Newtonian time, Connes and Rovelli were able to design an arrow of so-called Thermal Time [1]. Its existence is based on the fact that quantum mechanics involves so-called von Neumann algebras (algebra attached to the laws governing the transitions between quantum states). In this algebraic framework, automorphisms only depend on a single parameter (Tomita Takesaki theorem). This parameter is such that if we consider the conjunction of two idempotent operators applied over a basis of canonical states of representation (cycles) a non-commutativity arises. It is only attached to the single control parameter. A link can then be established with the KMS principle (Kubo, Martin and Schwinger) which, in physics, leads the same type of non-commutativity for Hamiltonian systems subject to thermal uncertainty [2 see many references inside]. The conservation of the identity of the structures is then expressed as a time shift, namely an “arrow of time” (for A. Connes linked to the temperature of the fossil radiation). Since, the non-trivial zeros of the zeta function are related to the transitions matrices between stable states [3] and the path integrals of Quantum Mechanics reveal that the underlying synthetic metric is of dimension $d=2$ (Peano metric: r,t), the authors decided to focus the analysis on the links between the functional properties of the zeta function and of Extensity Transfers on Fractal Interfaces of lower dimension $d<2$, for open irreversible systems. The canonical dynamics are then controlled by fractional differential equations and give rise to universal transfer functions involving power laws and a paradoxical delegitimization of the notion of energy (due to a revocation of Noether principles). The most emblematic of these universal transfer functions is the Cole and Cole whose singular dispersions properties and causality are linked to « $\alpha=1/d$ » [4].

2 Results and discussion

Among the most interesting results obtained, we note firstly the fact that Cole and Cole transfer function $Z_\alpha(\iota\omega)$, as pure hyperbolic geodesic in frequency space, is a stacking basis giving birth to the Riemann zeta function $\zeta_\alpha(s)$. The stalking parameter « θ » is none other than the one involved in the complex parameterization of the zeta function: $s=\alpha+\iota\theta$.

The second result relates to the incompleteness (of the representation backed on the only fractional transfer function with "α" parameter $Z_\alpha(\omega\tau)$. This function has no inverse Fourier transform so there is no one-parameter automorphism for this kind of fractional dynamics. Time as a usual dynamic deployment over an order parameter is not able to be locally relevant. Automorphisms, -because there are obviously some, for instance via renormalization-, are backed on sheaves of arithmetic correlations which destroy the absolute external order of the dynamics. These sheaves are based on a dual function stated: $Z_{1-\alpha}(\iota\tau)$. Itself is the basis of a stalking giving birth to $\zeta_{1-\alpha}(1-s)$. But, the stalk is now parametrized by « -θ » then $s=\alpha-\iota\theta$. This change in the signature is without consequence if $d=2$, but if $d<2$, the breaking of symmetry $\alpha \rightleftharpoons (1-\alpha)$, leads radical upshots; namely $\zeta^*_{1-\alpha}(s) \neq \zeta_{1-\alpha}(s^*)$. However, the categorical adjunction $\alpha \rightleftharpoons (1-\alpha)$ gives rise to a Grothendieck topos. This topos allows to understand why and how $\zeta_\alpha(s)$ and $\zeta_{1-\alpha}(1-s)$ unfold over dual universal varieties; all degeneracies involved by the internal arithmetic correlations are hidden behind $Z_\alpha(\omega\tau)$.

The third important result is the following: we confirm that the usual *Space-Time* coupling either linear ($v=Lt^{-1}$), diffusive ($D=L^2t^{-1}$), Lorentzian or other flat dispersions, comes from a more general form based on *Scale-Frequency* monad: namely based on $[C=(1/\Lambda)\,N^\alpha]_{Z\alpha(\omega)}$ characterized by $N: N=\iota\omega\tau=\iota\Pi p_i^{r_i}$ where r_i are integers and p_i are prime numbers. If we add that Scale-Frequency structure is dual with $[C'=(1/\Lambda')\,\tau^{1-\alpha}]_{Z1-\alpha(\tau)}$ the role of arithmetic correlations (sheaves) in a structure appears and then the zeta functions, as a representation of a covering (stalking) of the dual monads. Ultimately, this approach leads the functional relation on zeta functions $\zeta_\alpha(s)=F[\zeta_{1-\alpha}(1-s)]$, relation whose analytic complexity comes from the symmetry breaking between $Z_\alpha(\omega\tau)$ and $Z_{1-\alpha}(\tau)$ over the hyperbolic geodesic. While joining discrete and continuous structure, duality which structures the Grothendick topos restore the final role of the energy through the completion or the dual amalgam.

The fourth result brings us back to the arrow of time. In above approach, the parameter « θ » plays the role of a "proper time". It can be shown that the asymmetry between $+\theta$ and $-\theta$ is due to the appearance of a phase angle which two folds the complex plane of representation. This angle may be associated physically with an increase in internal entropy and a production of external anti-entropy aimed at balancing the duality. From this new dynamic site, any creative process requires an increase of dissipation and an increase of the "time constants" of the processes implemented (downing of frequencies namely an increasing of the uncertainty), approach opposite to any local optimization. In the case $d=2$ the process is only controlled by chance and the creative process is reduced to serendipity. The important point is that this is not so if $d<2$ and thought then become a world of human consciousness and representation, subject to the will of creation.

3 Conclusions and Future work

Above approach could appear purely theoretical but it is not so. Indeed historically this approach based in fine on category theory finds its origin at the end of the seventies in the design of the early lithium batteries [5]. Far from quantum mechanics framework [1], the mathematical foundations of our approach remain to be strengthened, in particular concerning the role of the zeta function as the result of a stalking of a fractional transfer function (via sheaves) , and as “bridging category” between discrete and continuous representations based not only on fractal geometries [6-7] but on number and measure theory. For instance, it can be proved that the Riemann hypothesis is undecidable in the framework of set theory under ZFC axioms. Nevertheless, David A. Martin axiom, extended to a so call Martin-Riemann axiom, guarantees that this passage is carried out in a regular way by respecting the Baire category theorem and zero dimension of Lebesgue measure for the countable approach of the continuous [8]. The link with non-standard analysis must be considered in this frame.
Moreover, the work of physical origins about the link between the arrow of time and the extension of Kan, highlighted by the authors, should be the subject of in-depth mathematical work in relation with non-commutative geometries [9]. In addition, the current works are centered on the metric, therefore the curvatures; nevertheless, beyond this aspect, arises the question of the singular topologies of open systems related to stalked fractal geometries [7] in relation with multi-connex graphs. So a considerable amount of mathematical studies to support new approaches.
In addition, this work could lead to physical applications (in particular in the field of materials with paradoxical properties and in quantum devices) but also in the fields of pedagogy and management (disappearance of optimization principles through figures, double quality plan, relationship to working time, new principles of econo-physics etc). Morover and finally analysis opens new perspectives between mathematic and philosophy [10,11,12].

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