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EVANESCENT WAVE CALCULATING IN THE SCATTERING OF ELECTROMAGNETIC WAVE ON NANO-APERTURE

The searching is about electromagnetic wave scattering through the round nano-aperture. Diameter of the hole is much more smaller, than the wave length. We use the nano-aperture terms from [1]. Features in that case are associated with the Rayleigh criterion. The problem of wave scattering were searching by Bethe [1] at the first time. Bethe derived the formula of radiation power. Problems of that object order is associated with Nano Optics ([1], [3]).

The integral equations method it the most useful method it the diffraction theory [4]. That method were used to calculate fields passed through the hole.

Statement of the Problem

We denote the electromagnetic wave by $u_0(x, y)$ (fig. 1). We are finding E, H satisfied the Maxwell's equations

$$\operatorname{rot} \bar{H} = -i\omega\varepsilon\bar{E} \tag{1}$$

$$\operatorname{rot} \bar{E} = i\omega\mu\bar{H}, \tag{2}$$

border conditions

$$E_{1y}(-x, y, z) = E_{2y}(x, y, z), \tag{3}$$

$$H_{1y}(-x, y, z) = -H_{2y}(x, y, z), \tag{4}$$

$$H_{1x}(-x, y, z) = H_{2x}(x, y, z), \tag{5}$$

$$E_{1x}(-x, y, z) = -E_{2x}(x, y, z). \tag{6}$$

At the second case the evanescent *condition satisfied*

$$e^{i(k_x x + k_y y)} e^{-i|k_z||z|}, \qquad k_x^2 + k_y^2 > k^2. \tag{7}$$

In the case of TE–polarization we find $u(x,y) = E_z(x,y)$ satisfying the Helmholtz equation $\Delta u + k^2 u = 0$. We'll calculate other field components using this formulas:

$$E_x(x,y) = 0, \tag{8}$$

$$E_y(x,y) = 0, \tag{9}$$

$$H_x(x,y) = -\frac{i}{\omega\mu}\frac{\partial u(x,y)}{\partial y}, \tag{10}$$

$$H_y(x,y) = \frac{i}{\omega\mu}\frac{\partial u(x,y)}{\partial x}, \tag{11}$$

$$H_z(x,y) = 0. \tag{12}$$

In the case of TM–polarization we are find $u(x,y) = E_z(x,y)$ satisfying the Helmholtz equation. We'll calculate other electrical field components using this formulas:

$$E_x(x,y) = \frac{i}{\omega\varepsilon}\frac{\partial u(x,y)}{\partial y}, \tag{13}$$

$$E_y(x,y) = -\frac{i}{\omega a}\frac{\partial u(x,y)}{\partial x}, \tag{14}$$

$$E_z(x,y) = 0, \tag{15}$$

$$H_x(x,y) = H_y(x,y) = 0. \tag{16}$$

The function $u(x,y)$ on the board in the first case is satisfy the Dirichlet condition

$$u|_\Gamma = -u_0|_\Gamma, \tag{17}$$

and in the TM–case — function $u(x,y)$ on the board is satisfy the Neumann condition:

$$\frac{\partial u}{\partial n}\Big|_\Gamma = \frac{-\partial u_0}{\partial n}\Big|_\Gamma . \tag{18}$$

We calculate functions $u_1(x,y)$ and $u_2(x,y)$ to find our problem solution (fig. 1).

Fig. 1. The image of incident wave angle θ on the aperture with size a. Determination of functions u_1 and u_2

Functions u_1, u_2 are satisfy the Helmholtz equations with wave numbers k_1 and k_2. Moreover this functions are satisfy the conjugate conditions (3-6) on the hole. Using the technique of generalized potentials from [5] with some methods from [6] the problem is reduced to integral equations system. The approximation solution based on the Galerkin approximation method. Besides the construction of approximation algorithm required the approximation methods theory [7].

We constructed computational scheme, wrote a program on C++. The features of calculation on specialized environments MatLab, Remcom and SemCad are discussed.

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