# Changes of Velocity of Rotation of the Earth and its Figure's Deformation Associated with Them

Vasily Yu. Belashov

*Kazan (Volga region) Federal University, Kazan, Russia*
E-mail: vybelashov@yahoo.com

**Abstract**

It is shown, that continuous changes of angular velocity of rotation of the is plastically-elastic Earth should cause the continuous coupled deformation of a crustal layer, redistribution of masses in a sub-crustal layer and the change of density associated with it, and also, as consequence of these phenomena, a polar pulsation of a figure when polar diameter of the Earth increases and decreases with time. The mechanism of occurrence of deformations of a body of a planet under action of a deforming (centrifugal) variable force is found, the tensors of deformations and pressure are written out, and on the basis of the rheological equations the equations of balance are deduced, and also calculation of the module of change of polar compression and radial displacements is made at real fluctuations of angular velocity of rotation of the Earth. The calculated values have given the quite real changes of compression and radial displacements of the Earth's crust and its other shells. The opposite process is also shown, namely: observed fluctuations of amplitude of the polar compression, leading to respective alterations of the moment of inertia of the Earth, quite correspond to real fluctuations of duration of a day.

## 1 Introduction

As you know, the rotation of the Earth, characterized by angular velocity, determines its ellipticity, which is the main consequence resulting from the rotation of the figure itself. The shape of the planet – its eccentricity $e$ (or compression $\alpha$) – depends only on two parameters: the angular velocity of rotation $\omega$ and the law of density distribution over depth $d\rho/dr$, as well as, as follows from numerous studies, and latitude $\phi$, i.e. $e = F(\omega, d\rho/dr)$ where $d\rho/dr = f(r,\phi)$.

Considering the polar compression of the planets of the Solar System, their angular velocities of rotation and average densities, we can conclude that the degree of compression of a planet mainly depends on its rotation speed, and therefore, a change of the planet's rotation regime[1] should, first of all, affect the change of polar compression.

From the law of conservation of angular momentum of the Earth, which is written as

$$J\omega = \text{const}, \tag{1}$$

it follows that a change of the angular velocity of the Earth's rotation should inevitably cause a change of the moment of inertia,

$$\frac{\delta\omega}{\omega} = -\frac{\delta\tau}{\tau} = -\frac{\delta J}{J},$$

[1] The question of the reasons for the change of the Earth's rotation velocity is not considered in this paper; rather complete reviews of the hypotheses explaining this phenomenon was published by Belashov (1978, 1984) and Belashov et al. (2018).

where $\tau$ and $\delta\tau$ are the length of the day and its change. Moreover, according to the simplest calculations, the change of the moment of inertia corresponding to real changes of the day duration ($\Delta\tau \approx 0.0034$ s) should reach

$$\frac{\delta J}{J} = \frac{0.0034}{86400} = 4 \cdot 10^{-8}. \tag{2}$$

According to Pariisky (1948), such a change can occur as a result of a change of the density of the subcrustal layer, its "bulging" (as a result of which deformations arise in the crustal layer). Moreover, according to the calculations given by Pariisky (1948), if we take the thickness of the subcrustal layer where the density redistribution takes place, as 80 km, and the thickness of the outer layer that only deforms but does not change its density, as 1 km, then to change the moment the inertia of the Earth corresponding to (2), a vertical displacement of 6-7 m is sufficient.

Note that, as a result of the deformation of the Earth's figure resulting from a change of $\omega$, as it was found by Pariisky (1948), Melchior (1976) and Belashov (1987), the density redistribution in the subcrustal layer actually occurs. Let $\rho_P^0$ is the density at the point $P$ in the initial state. After deformation due to radial displacement, the density at point $P$ becomes equal

$$\rho_P = \left( \rho_P^0 - \zeta \frac{d\,\rho_P^0}{dr} \right)(1-\Theta) = \rho_P^0 - \zeta \frac{d\,\rho_P^0}{dr} - \rho_P^0\,\Theta \tag{3}$$

where $\zeta$ is the displacement, and $\Theta = \frac{1}{Q}\frac{dQ}{dt}$ where $\Theta = dx\,dy\,dz$ is the volume. In our case (we believe that the Earth is deformed conjugatedly, without changing the volume), $\Theta = 0$ and equation (3) takes form

$$\rho_P = \rho_P^0 - \zeta \frac{d\rho_P^0}{dr}.$$

Since $\zeta$ is positive, the substance at the point $P$ becomes denser (since $d\rho_P^0 / dr < 0$, and therefore $-\zeta\, d\rho_P^0 / dr > 0$). This corresponds to the above noted considerations [see also statement of Pariisky (1948)].

The "bulging" of the subcrustal layer should be accompanied by a redistribution of the internal masses (that is, their overflow into this region from the regions in which the negative radial displacement occurs). As it is known (Pariisky, 1948), for any internal point $P$ in the initial state, the Poisson equation is written in form

$$\Delta V_P^0 = -4\pi G\,\rho_P^0,$$

where $\Delta V_P^0$ is the Laplacian of gravitational potential in the initial state, $G$ is the gravitational constant. For a deformed state we have

$$\Delta\left(V_P^0 + V_P\right) = -4\pi G \left( \rho_P^0 - \zeta \frac{d\rho_P^0}{dr} - \rho_P^0\Theta \right).$$

After differentiation, we find

$$\Delta V_P = 4\pi G \left( \rho_P^0\Theta + \zeta \frac{d\rho_P^0}{dr} \right)$$

or, if we again assume that the substance of the subcrustal layer is incompressible,

$$\Delta V_P = 4\pi G \zeta \frac{d\rho_P^0}{dr} . \tag{4}$$

Equation (4) shows that when ω decreases, that is, when the deforming centrifugal force is removed, the Earth will tend to return to its original undeformed state due to the occurrence of gravitational effects [which are given by formula (4)] caused by the new distribution of masses in the Earth's body. However, it should be noted that the change of density and the redistribution of masses can, by virtue of equation (1), affect the change of the angular velocity of the Earth's rotation, that is, the opposite effect can occur (and really occurs).

To find out the effect of the rotation velocity on the change of the shape of the Earth, we consider the relationship of deformations (and displacements) with stresses applied to the volume. For this, as is customary in rheology, we first write down the tensors of the resulting strains and stresses, and then investigate their relationship with each other.

## 2 Strain and Stress Tensors

Let $u$ is the displacement of the particle $P_1(x, y, z)$, and $u + du$ is displacement of particle $P_2(x + dx, y + dy, z + dz)$. We have

$$\left.\begin{array}{c} du_x = \dfrac{\partial u_x}{\partial x} dx + \dfrac{\partial u_y}{\partial y} dy + \dfrac{\partial u_z}{\partial z} dz \\ dx = ds\cos(ds, x) \\ \dfrac{du_x}{ds} = \dfrac{\partial u_x}{\partial x}\cos(ds, x) + ... \end{array}\right\} \quad \frac{du}{ds} = \left\| \frac{du_x}{ds} \quad \frac{du_y}{ds} \quad \frac{du_z}{ds} \right\| = \left| \begin{array}{ccc} \dfrac{\partial u_x}{\partial x} & \dfrac{\partial u_x}{\partial y} & \dfrac{\partial u_x}{\partial z} \\ \dfrac{\partial u_y}{\partial x} & \dfrac{\partial u_y}{\partial y} & \dfrac{\partial u_y}{\partial z} \\ \dfrac{\partial u_z}{\partial x} & \dfrac{\partial u_z}{\partial y} & \dfrac{\partial u_z}{\partial z} \end{array} \right| . \tag{5}$$

The deformation is characterized by a tensor $du/ds$ because it consists of three elongations and three angular deformations. Elongation is expressed through the components

$$\frac{\partial u_x}{\partial x}, \quad \frac{\partial u_y}{\partial y}, \quad \frac{\partial u_z}{\partial z},$$

and shear is expressed through $\mathrm{tg}\phi = \dfrac{\partial u_x}{\partial y} \approx \phi$ for small strains.

To study the deformation proper, we extract the symmetric tensor from the tensor $du/ds$, then we write the deformation tensor in form

$$e = \left| \begin{array}{ccc} \dfrac{\partial u_x}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial z} + \dfrac{\partial u_z}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_y}{\partial x} + \dfrac{\partial u_x}{\partial y}\right) & \dfrac{\partial u_y}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial u_y}{\partial z} + \dfrac{\partial u_z}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_z}{\partial x} + \dfrac{\partial u_x}{\partial z}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_z}{\partial y} + \dfrac{\partial u_y}{\partial z}\right) & \dfrac{\partial u_z}{\partial z} \end{array} \right| . \tag{6}$$

We denote

$$e_{xx} = \frac{\partial u_x}{\partial x}, \quad e_{xy} = e_{yx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right),$$

etc. The strain rates in this case are expressed by the relations

$$\dot{e}_{xx} = \frac{\partial v_x}{\partial x}, \quad \dot{e}_{xy} = \dot{e}_{yx} = \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right).$$

Similarly to (5), the stress tensor is written, it is also a symmetric tensor, that is $p_{ij} = p_{ji}$, and it can also be represented in a form similar to (6).

Both the strain tensor and the stress tensor can be decomposed into the main tensor and the deviator tensor (deviator). In the case of the strain tensor, these concepts are easy to come by considering the most general deformation of a cube: a change of length (three components) and changes of angles (three components). Then the main tensor is written in the form $(e_v)_{rs} = e_{ii}\delta_{rs}$ where $\delta_{rs}$ is the unit axial tensor. This tensor corresponds to cubic isotropic expansion without changing shape, when all sides change proportionally, the angles do not change, and, therefore, the cube remains a cube $e_{xx} = e_{yy} = e_{zz} = e;\ e_v = 3e$.

The deviator tensor which corresponds to the cube skew, that is, changing its shape without changing the volume, has the following form: $(e_0)_{rs} = e_{rs} - \frac{1}{3}e_{ii}\delta_{rs}$. Similarly, the stress tensor can be decomposed into the main one: $p_m = p_{\alpha\alpha}/3$ and the deviator $p_0 = p_{rs} - p_m\delta_{rs}$.

Now we need to find a rheological equation that relates the strain tensor to the stress tensor. We restrict ourselves only to deviator tensors, that is, we accept our physical body (Earth) as incompressible, which is usually done. We choose the rheological function that defines the solid body of Hooke. Such a model is an approximation to the real Earth from a side of a solid. In this case:

$$\alpha = \Theta / p = k, \quad \beta = 2\mu, \quad p_m = \alpha e_v / 3, \quad p_0 = \beta e_0$$

where $\alpha$ is the compression modulus, $\beta$ is the shear modulus, $\mu = E/[2(1+\nu)]$ is the Lame coefficient, $E$ is the Young's modulus, and $\nu$ is the Poisson's coefficient.

The mechanical analogue of Hooke's body is an ideal spring. Such a spring is really a solid in the sense that we defined it, because if it is in a state of constant deformation ($e_0 = c$), a constant stress appears $p_0 = \beta c$.

As one can see from formula (4), in the case of the real Earth, a similar picture is observed, that is, an analog representing the Hooke's solid corresponds in some approximation to the problem of describing the deformation of a planet under the action of an applied stress from a tensile (centrifugal) force.

We now derive the so-called equilibrium equations that specify the relationship between the deformation (through displacements) and the stress arising from the deformation that has occurred. The cause of the initial deformations will be considered an increasing (with increasing angular velocity of the Earth's rotation) centrifugal force. Then the stresses arising from the deformation of the figure will be due to the action of gravitational forces and surface tension forces. As a result of these three forces, the figure of the Earth will tend to some equilibrium state. This means that with a decrease of the angular velocity of rotation, the ellipticity of the Earth will decrease and vice versa.

## 3 Equilibrium Equations

As it is known, the balance of the body is described by the following three fundamental equations:

$$\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}+\rho X=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial Y_x}{\partial x}+\frac{\partial Y_y}{\partial y}+\frac{\partial Y_z}{\partial z}+\rho Y=\rho\frac{\partial^2 v}{\partial t^2}, \tag{7}$$
$$\frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}+\frac{\partial Z_z}{\partial z}+\rho Z=\rho\frac{\partial^2 w}{\partial t^2}$$

where *X*, *Y*, and *Z* are the components of gravitational forces, and $X_x, ..., Z_z$ are the components of the surface tension forces, that is, the elements of the stress tensor. Thus, the set of three equations (7) contains nine unknowns: $X_x, ..., Z_z$. If now to use the three equations of equilibrium of moments, then three unknowns can be excluded, since the stress tensor $p_{ij}$ is symmetric, that is $X_y = Y_x;\ Y_z = Z_y;\ Z_x = X_z$. Now three equations with six unknowns remained:

$$\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}+\rho X=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial X_y}{\partial x}+\frac{\partial Y_y}{\partial y}+\frac{\partial Y_z}{\partial z}+\rho Y=\rho\frac{\partial^2 v}{\partial t^2}, \tag{8}$$
$$\frac{\partial X_z}{\partial x}+\frac{\partial Y_z}{\partial y}+\frac{\partial Z_z}{\partial z}+\rho Z=\rho\frac{\partial^2 w}{\partial t^2}.$$

If now we will can express the stress tensor through the strain tensor using the rheological law obtained in the previous section (Sect. 2) for the Hook solid, we obtain a system of three equations with three unknowns, since the strains can be expressed through the components of the displacement vector *u*, *v* and *w* (Melchior, 1976). This is the simple method proposed by the theory of elasticity.

We first consider Hooke's law when applying the uniaxial compression tensor to the body. Since only one component in this case is not equal to zero (we denote it as $P_1$), we have deformations

$$e_1=\frac{1}{E}P_1,\quad e_2=-\frac{\nu}{E}P_1,\quad e_3=\frac{\nu}{E}P_1 \tag{9}$$

and volume expansion $\Theta=\frac{1-2\nu}{E}P_1$. Here ν is the Poisson's coefficient, and *E* is the Young's modulus. If $P_1$ is the tensile stress, then $\Theta>0$, and $(1-2\nu)/E\ge 0 \Rightarrow \nu\le 0.5$. In the case when $\nu=0.5$ then $\Theta=0$, and incompressibility takes place.

Generalize (9) assuming that there are three uniaxial and normal stresses. At this, suppose that the substance is isotropic. Then we get

$$e_1=\frac{1}{E}P_1-\frac{\nu}{E}(P_2+P_3)=-\frac{\nu}{E}\Sigma+\frac{1+\nu}{E}P_1,$$
$$e_2=-\frac{\nu}{E}\Sigma+\frac{1+\nu}{E}P_2,\quad e_3=\frac{\nu}{E}\Sigma+\frac{1+\nu}{E}P_3 \tag{10}$$

where $\Sigma = P_1 + P_2 + P_3$. It is easy to find that, on the contrary,

$$P_1 = \lambda\Theta + 2\mu e_1, \quad P_2 = \lambda\Theta + 2\mu e_2, \quad P_3 = \lambda\Theta + 2\mu e_3 \tag{11}$$

where

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$

($\lambda$ and $\mu$ are the Lame coefficients). If $\nu = 0.5$ then $\lambda = \infty$ and, therefore, $\lambda$ is called incompressibility modulus.

If now to consider the Hooke law in an arbitrary basis and, accordingly, denote the mixed components of two tensors as $P_i^j$, $e_i^j$, then by virtue of equations (10) and (11) we obtain in tensor notations

$$P_i^j = \delta_i^j \lambda\Theta + 2\mu e_i^j, \quad e_i^j = -\delta_i^j \frac{\nu}{E}\Sigma + \frac{1+\nu}{E} P_i^j .$$

Now six unknowns in equations (8) will be expressed in terms of the displacements *u*, *v* and *w* as follows:

$$X_x = \lambda\Theta + 2\mu\frac{\partial u}{\partial x}, \quad Y_y = \lambda\Theta + 2\mu\frac{\partial v}{\partial y}, \quad Z_z = \lambda\Theta + 2\mu\frac{\partial w}{\partial z},$$

$$Y_z = Z_y = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}\right), \quad Z_x = X_z = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),$$

$$X_y = Y_x = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \tag{12}$$

$$\Theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}.$$

Substituting equations (12) into (8), we finally obtain three equilibrium equations with three unknowns expressed in terms of displacements (equations of motion in displacements in the Lame form):

$$(\lambda+\mu)\frac{\partial\Theta}{\partial x} + \mu\Delta u + \rho X = \rho\frac{\partial^2 u}{\partial t^2},$$

$$(\lambda+\mu)\frac{\partial\Theta}{\partial y} + \mu\Delta v + \rho Y = \rho\frac{\partial^2 v}{\partial t^2}, \tag{13}$$

$$(\lambda+\mu)\frac{\partial\Theta}{\partial z} + \mu\Delta w + \rho Z = \rho\frac{\partial^2 w}{\partial t^2}.$$

Thus, as mentioned above, we obtained the equations expressing the equilibrium state of the Earth's ellipsoid of revolution, which is under the action of a centrifugal force (which causes a change of the figure and, therefore, deformation of the Earth's body), and gravitational and surface tension forces, tending to return the Earth to its initial shape, from which, under the influence of a change of centrifugal force (of its increase with growth of ω), it passed to a stressed state.

Equations (13) are second-order partial differential equations, therefore, it is difficult to give a quantitative estimate of deformations (and displacements) when a conjugated change of the Earth's shape occurs due to non-uniform rotation of the planet (Belashov, 1987). To clarify the reality of the effect of periodic changes of the Earth's rotation velocity on its deformation, we will use further the technique proposed by Stovas (1957, 1961).

### 4 Conjugate Deformation of the Ellipsoid at Changing the Angular Velocity of Rotation

The potential of the deforming forces *V*, which directly determines the polar compression of the ellipsoid, can be represented in the following form:

$$V_0 = \frac{\omega^2 a^2 (1-\alpha)[1-2\,\mathrm{tg}^2\,\phi]}{6[(1-\alpha)^2 + \mathrm{tg}^2\,\phi]}$$

where *a* is the equatorial semi-axis of rotation, α is the polar compression of the ellipsoid, and ϕ is the geocentric latitude.

We now turn to the deforming forces. The radial deforming force is determined by the following formula:

$$F_r = \frac{1}{3}\omega^2 a(1-\alpha)\frac{[(1-\alpha)^2\cos^2\phi+\sin^2\phi]^{1/2}[(1-\alpha)^2\cos^2\phi-2\sin^2\phi]}{[(1-\alpha)^4\cos^2\phi+\sin^2\phi]}, \qquad (14)$$

and the normal and tangential deforming forces are given by

$$F_N = \frac{\omega^2 a(1-\alpha)[(1-\alpha)^2\cos^2\phi-2\sin^2\phi]}{3[(1-\alpha)^2\cos^2\phi+\sin^2\phi]^{1/2}[(1-\alpha)^4\cos^2\phi+\sin^2\phi]^{1/2}},$$

$$F_k = \frac{\omega^2 a(1-\alpha)[1+2(1-\alpha)^2]\sin\phi\cos\phi}{3[(1-\alpha)^2\cos^2\phi+\sin^2\phi]^{1/2}[(1-\alpha)^4\cos^2\phi+\sin^2\phi]^{1/2}}.$$

A change of the deforming forces causes the change of the polar compression of the ellipsoid of revolution and, consequently, the conjugate change of all its main parameters.

Let us now consider how the radial deforming force on the surface of an ellipsoid will changed with a change of its rotational regime. To do this, we differentiate (14) with respect to ω:

$$\frac{\partial F_r}{\partial \omega} = 2\omega(1-\alpha)F(\phi)\frac{\partial r}{\partial \alpha} - f$$

where $F(\phi) = \left\{1 + \frac{\alpha}{3(1-2\alpha)}\left[\frac{(1-\alpha)^2\cos^2\phi-2\sin^2\phi}{(1-\alpha)^2\cos^3\phi-\sin^2\phi}\right]\right\}\cos^2(B-\phi)$,

$$f = \frac{4\omega a\alpha(1-\alpha)^3\{(1-\alpha)^2[2+(1-\alpha)^2]-[1-(1-\alpha)^2\sin^2\phi]\}\sin^2\phi\cos^2\phi}{3(1-2\alpha)[(1-\alpha)^2\cos^2\phi+\sin^2\phi]^{1/2}[(1-\alpha)^4\cos^2\phi+\sin^2\phi]^2},$$

$$\frac{\partial r}{\partial \alpha} = \frac{a}{3}\left\{\frac{(1-\alpha)^2\cos^2\phi-2\sin^2\phi}{[(1-\alpha)^2\cos^2\phi+\sin^2\phi]^{3/2}}\right\} \qquad (15)$$

where $r$ is the radius vector of a point lying on the surface of an ellipsoid of revolution.

From formula (15), which expresses a change of the radius vector with a change of the polar compression of the ellipsoid, one can see that at $\phi = 35°21'18.5''$ $\partial r / \partial \alpha = 0$, that is, in the zone of this latitude the radial displacements do not occur with a conjugate change of the figure,. The maximum radial displacements are observed at the poles ($\phi = 90°$) and equator ($\phi = 0°$). In the terminology of Stovas (1957), the zone $\phi = \pm(30-40)°$ is the zone of "critical parallels".

The value of function $F(\phi)$ is within $0.997750 < F(\phi) < 1.002225$. The maximum value of function $f$ is 160 g·cm·s$^{-1}$ or 0.7% of $\partial F_r / \partial \omega$. Neglecting the value of $f$ and equating $F(\phi) = 1$, with a sufficient accuracy, we obtain

$$\frac{\partial r}{\partial \alpha} = \frac{1}{2\omega(1-\alpha)} \frac{\partial F_r}{\partial \omega}, \tag{16}$$

whence it follows that a change of the radial deforming force with a change of the angular velocity causes a conjugate change of the radius vector of the ellipsoid and, therefore, a conjugate deformation of all its main parameters.

The displacement $u_N$ of a point on the surface of the ellipsoid along the normal $N$ when changing the polar compression $\alpha$ is represented by the equation

$$u_N = \frac{a}{3} \frac{(1-\alpha)^2 - 2\,\mathrm{tg}^2\,\phi}{[(1-\alpha)^2 + \mathrm{tg}^2\,\phi]^{1/2}[(1-\alpha)^4 + \mathrm{tg}^2\,\phi]^{1/2}} \Delta\alpha \tag{17}$$

where $\Delta\alpha$ is the change of polar compression. It follows from (17) that the greatest vertical displacement will be observed at the poles ($\phi = 90°$) and equator ($\phi = 0°$), that we have already seen from analysis of (15).

If now to use the obtained relations (16) and (17) and perform elementary calculations, taking at $\phi = 0°$ that $u_N$ = 6 m or $\delta r$ = 6 m [as indicated in the Introduction, that is sufficient to change the moment of inertia of the Earth corresponding to (2)], then one can obtain the change of polar compression, which should occur at real fluctuations of the day duration $\Delta\alpha \sim 3 \cdot 10^{-6}$ or, as a percentage, $\Delta\alpha \sim 0.09\ \%$ of the average compression of the Earth obtained by geodetic methods, as well as using satellites, which is 1/298.25 ($\pm 0.02$), according to the MGGSM / MAC data.

We believe that such a change of compression is real, although calculations which use some theoretical models [see, for example, the Lallemand model described by Melchior (1976)] give a significantly smaller result. We think that these theoretical models do not in any way take into account either the redistribution of masses in the Earth's body, and the related density change in the subcrustal layer (see Introduction), and the heterogeneity of the Earth (in any case, the extremely complicated law of density distribution in the body of planet), and, finally, the difference in the compressions of the two hemi-spheres of the Earth (Melchior, 1975) and also the ellipticity of the equator.

## 5 Discussion and Conclusion

We summarize the results as follows. It was established in the paper that since the angular velocity of the Earth's rotation changes abruptly and continuously, increasing and decreasing on a general tidal background of its damping, the signs of the displacement vectors change passing through zero and, therefore, continuous changes of angular velocity of rotation of the plastic-elastic Earth should cause continuous conjugate deformation of the crustal layer, redistribution of masses in the subcrustal layer and the associated change of density, and also, as a consequence of all these phenomena, the polar pulsation, when the Earth polar diameter increases and decreases.

We have clarified the mechanism of the occurrence of deformations of the planet's body under the action of a time-varying deforming (centrifugal) force, we have written strain and stress tensors, and based on rheological equations we have derived equilibrium equations and also calculated the modulus of variation of polar compression and radial displacements with real fluctuations of the angular velocity of the Earth's rotation. The calculated values gave quite real, in our opinion, changes of compression and radial displacements of the Earth's crust and its underlying shells. The opposite was also shown: the observed fluctuations of the amplitude of polar compression, leading to corresponding changes of the moment of inertia of the Earth, are consistent with real fluctuations of the day duration.

Our next work will be devoted to considering the possible consequences of the deformation of the Earth's figure based on the approaches proposed by Belashov (1978, 1985).

**Acknowledgments**. The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University. Data were not used, nor created for this research.

**References**

– Belashov, V.Yu. (1978). Geofizicheskie prichiny i sledstviya neravnomernogo vrashcheniya Zemli [Geophysical Causes and Effects of Non-Uniform Rotation of the Earth]. Leningrad: LVIMU [in Russian].
– Belashov, V.Yu. (1984). O vliyanii magnitosfernoi vozmushchennosti na rotatsionnyi rezhim Zemli: Preprint [On the Infuence of Magnetosphere Disturbance on the Rotational Regime of the Earth: Preprint]. Magadan: SVKNII DVNTs Akad. Nauk SSSR [in Russian].
– Belashov, V.Yu. (1985). *Dlinnoperiodnye nutatsionno-pretsessionnye dvizheniya mgnovennogo polusa vrashcheniya Zemli: Preprint* [Long-Periodical Nutation-Precession Movements of the Instant Pole of Rotation of the Earth: Preprint]. Magadan: SVKNII DVNTs AN SSSR [in Russian].
– Belashov, V.Yu. (1987). Deformation of Figure of the Earth Associated with Changes of Velocity of its Rotation. *Proc. NEISRI FESC ASc. USSR* (pp. 12-20). Magadan: NEISRI FESC ASc. USSR [in Russian].
– Belashov, V.Yu., Nasyrov, I.A. & Gordeev R.S. (2018). On the Problem of the Infuence of the Magnetosphere Disturbance on the Rotational Regime of the Earth. *Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki*, *160*(4), 617–630.
– Melchior, P. (1971). *Physique et Dynamique Planétaires, Vol. 1*. Bruxelles: Vander-éditeur.
– Melchior, P. (1972). *Physique et Dynamique Planétaires*, *Vol. 3*. Bruxelles: Vander-éditeur.
– Pariisky, N.N. (1948). Inconstancy of the rotation of the Earth and its strain. *Tr. Soveshch. Metod. Izuch. Dvizheniya Deform. Zemnoi Kory* (pp. 157–174). Moscow: Geodezizdat [in Russian].
– Stovas, M.V. (1957). Non-uniformity of rotation of the Earth, as the planetary-geomorphological and geotectonic factor. *Geol. Zh. Akad. Nauk Ukr. SSR*, *17*(3), 58–69.
– Stovas, M.V. (1961). Opyt matematicheskogo analiza tektonicheskih processov, vyzyvaemyh izmeneniyami figury Zemli [Experience of Mathematical Analysis of Tectonic Processes caused by changes of the figure of the Earth]. *Autoref. of DSci. Diss.* Leningrad: Leningr. Gornyi Inst. [in Russian].