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# Numerical Simulation of Viscous Liquid Displacement in Hele-Shaw Cell

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**Abstract.** The paper studies the influence of initial shape of liquid interface in Hele-Shaw cell on the development of Saffman–Taylor instability. The impact of perturbation parameters of interface is investigated using computational fluid dynamics methods. The results of the research include concentration outlines, the dependence of the viscous fingering growth rate on the period and the amplitude of the initial boundary perturbation. The value of the perturbation period at which the growth rate of viscous fingering is maximal is determined. At large values of the perturbation period, the growth rate is significantly reduced. The increase in the amplitude of the perturbation leads to an increase in the velocity of viscous fingerings.

## INTRODUCTION

Geometrically similar forms of evolution of the originally smooth interface of two different liquids, differing in density, viscosity, compressibility, received different names in different sections of mechanics. Thus, the behavior of a heavy liquid layer over a light one in the gravity field is called "Rayleigh-Taylor instability". The evolution of the shape of the interface during the passage of the shock wave is called "Richtmyer-Meshkov instability". The present work considers the complicated shape of the interface between fluids with different viscosities, one of which is used to displace the other one in a certain volume. This complication is referred to as the "Saffman-Taylor instability" [1]. This instability is also known as unstable "viscous fingering" because the difference in viscosity between the displacing liquids and the displaced ones is a key factor which causes displacement. The instability is indicated by formation of folds on the interface which are fingers that grow at different rates. The instability was first reported by Hill in 1952 [2]. A lot of research has been carried out since then.

The main reason for great attention to the phenomenon is that the instability plays a huge role in low efficiency of oil extraction process. Thus, the ratio of oil extraction is about 30% in Russia. One of the explanations of this is the fact that 90% of gained oil in Russia is extracted through flooding technology [3-4]. That means, oil extraction is implemented by displacing it with a layer of water with a much lower viscosity than the one of oil.

Saffman-Taylor instability can be observed also in different kinds of application in natural environment and industrial processes, such as hydrology, filtration and chemical processes in the layers. The paper [5] considers instability phenomenon in the flow of liquid in porous medium in the context of tenacity and variability. It refers to important classes of processes such as acid dissolution of porous medium (acid treatment of bottom-hole zone), coal gasification, pore blockage as a result of colmatant deposition, catalyst deactivation and others [6-11]. The paper [12] describes a number of experiments dedicated to phases interface formation in different samples at different forms of perturbation. Heterogeneity of material samples is shown to make a great contribution into the observed variety of flow figures. The critical review [13] studies a modern idea of formation, transfer and fusion of drops into flows in microchannels as well as methods of overcoming a liquid film between them.

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The movement of air under the earth's surface is of great importance in environmental and technical areas, such as environmental remediation, water seepage and groundwater recharge, mine and tunnel ventilation, and gas exchange between soil and atmosphere [14]. The paper [15] presents a modern state of knowledge of mechanisms, analytical and numerical models, as well as ecological and engineering application related to air flow in nature.

A great number of papers is dedicated to simulating Saffman-Taylor instability. Thus, Fourier's spectral method was first used as a basic plot for numerical simulation by Tan and Homsy in 1988 [2]. In 1992 simulated viscous fingering by expanding Hartley's spectral method in three dimensions [2] Zimmerman and Homsy. They showed that 2D simulation is enough to describe essential signs of non-linear viscous fingering. The displacement of liquid in Hele-Shaw cell was simulated with methods of random wandering in the review [16] and it was shown that (a) fingers are linearly stable even at high rates, (b) fingers are non-linearly unstable in relation to noises and external perturbation and (c) results of Saffman-Taylor equations do not correspond to experiments.

In every case of instability occurrence, a dynamical process is supposed to take place, which starts when some dimensionless parameter gets above a critical value. The ideology of "stability of motion", exported to fluid mechanics from solid dynamics, implicitly assumes the existence of stationary states, whose transition conditions are determined and can be characterized by a numerical criterion with the universal properties preserved in a wide range of physical parameters of the process. When considering the Saffman-Taylor instability, if one fluid is displaced by another more viscous one, then the interface will be stable for any perturbations [17]. If the liquid is displaced by a less viscous liquid, the interface will have a critical stability, depending on the characteristics of the perturbation. Theoretical studies [18] show that such instability can be controlled by external influence. One of the methods of external instability control is the effect of elastic vibrations on the process. An indirect confirmation of it is reduction of the water content of the extracted oil under acoustic influence on the formation [19]. The problems of instability control by external influence are still poorly investigated.

In this paper we simulate viscous elastic fingers in Hele-Shaw cell at different parameters of wave action. In our case wave action is equivalent to perturbations of interface. Means of numerical simulation of motion of liquid are used for this purpose.

## CALCULATIONS

Numerical simulation was implemented in the software FlowVision at academic license. Three-dimensional space of Hele-Shaw cell was used as a computational domain (Fig. 1) with the following parameters: height is $h = 176$ mm, length is $l = 400$ mm and width is $d = 0.81$ mm. External influence was set by changing the interface of liquids along the $y$-axis with the equation:

$$x = A \cdot \sin\left(\frac{2\pi N}{h} y\right), \qquad (1)$$

where $A$ is amplitude of perturbations, $N$ is the number of periods of perturbations on the width of the Hele-Shaw cell.

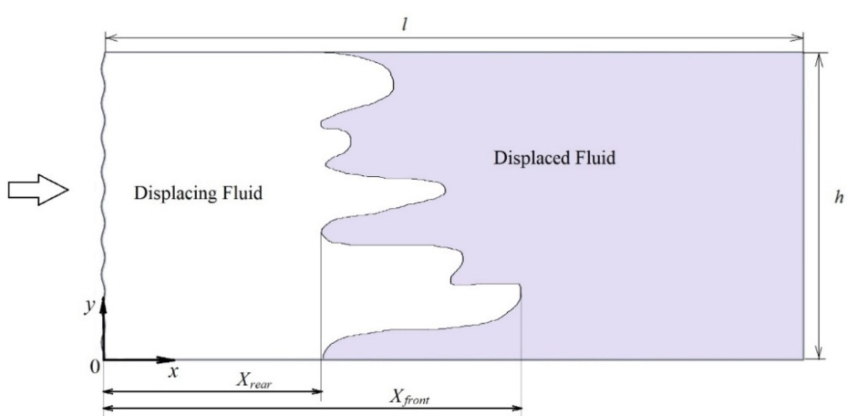


**FIGURE 1.** The schematic geometry of the problem

Laminar flow model was used as a mathematical model in the software FlowVision. The displaced liquid was mineral oil with viscosity $\mu_0$=133 mPa·s. Pure water was used as a displacing liquid. The choice of these two liquids is due to the fact that the following experimental research on physical model of Hele-Shaw cell was planned with these exact liquids, as well as to the fact that the cell is geometrically sized like a real one. The boundary condition at the inlet was the water flow rate $u_x$=3mm/s, at the outlet it was free outlet and at the remaining boundaries it was the wall with adhesion.

The paper investigates the influence of initial perturbation parameters on the length of the area where viscous fingers are observed. Measuring length of the longest viscous finger seems to be a useful tool for determining degree of instability of fingers. The interface of liquids can be visualized according to the concentration of displacing liquid in computational domain.

## RESULTS AND DISCUSSION

Figures of concentration distribution of liquid are obtained for different modes and parameters as a result of conducted numerical experiment. Profiles of concentration for non-perturbed initial interface (a) and perturbed initial interface of liquids with a number of periods of perturbations $N$=6 and $N$=15 are depicted in Fig. 2 as an example. Amplitude of perturbation was the same and equaled $A=0.5\cdot h$ in this case. The blue color shows 100% concentration of viscous oil in figures. The red color shows 100% concentration of water which displaces the oil. The motion of liquid is carried out from left to right. It can be seen that viscous fingers are formed and they grow over the time.

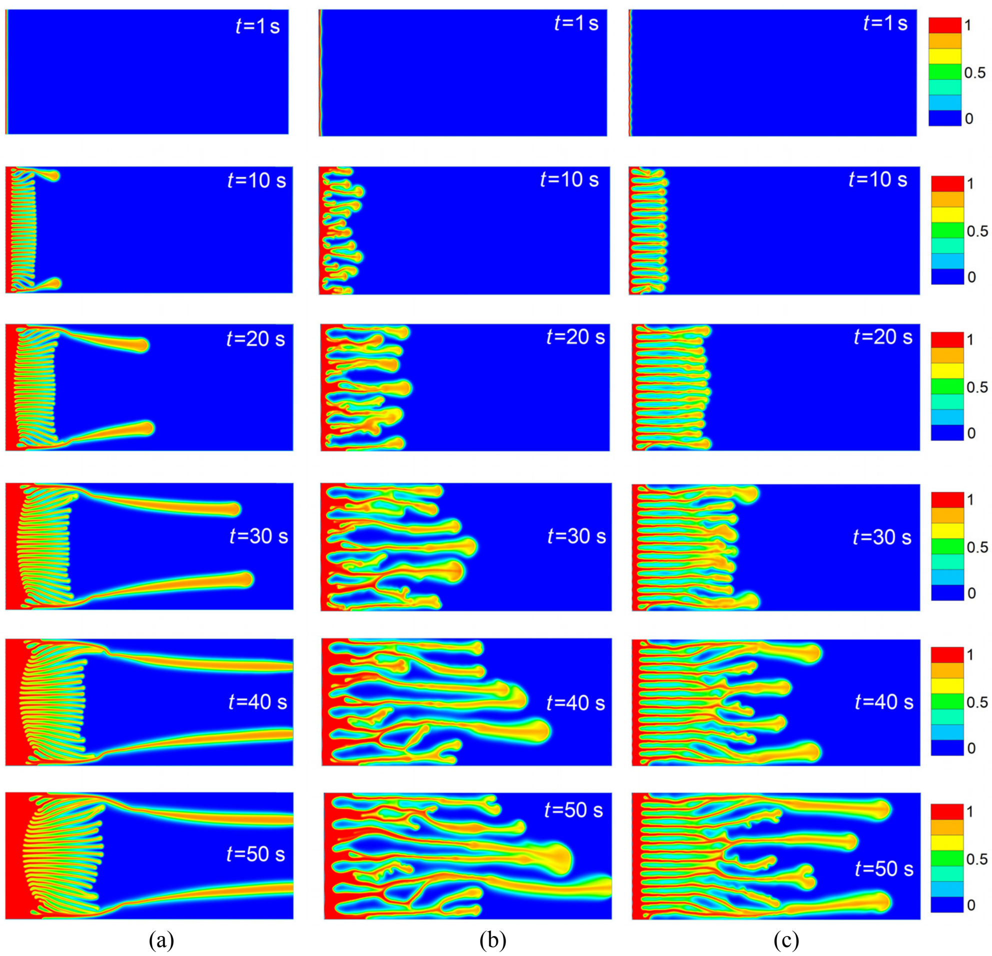


**FIGURE 2.** Concentration iso-contours at without perturbations (a); $A=0.5\cdot h$, $N$=6 (b); $A=0.5\cdot h$, $N$=15 (c)

Two evident viscous fingers are selected from gained folds for the case of non-perturbed initial interface (Fig. 2a) which grow significantly faster than others. When the fluid flows in Hele-Shaw cell, due to viscosity, its velocity will be less near the boundaries with the solid (in Fig. 2a it is at the top and the bottom) than in the middle of the cell at the first moments of time. This will cause the interface to curve. Surface tension forces result in additional pressure which causes growth of fingers. It explains the growth of fingers at the top and the bottom of Hele-Shaw cell. The repeatability of this result was high for different computational grids. The optimal value of initial number of computational cells was approximately 105 in terms of the computational experiment time. Local decomposition of computational cells was carried out near the interface. The calculation process also included automatic compaction of the computational grid in the area of large concentration gradients. The imposition of periodic perturbations of the initial interface leads to a change in the nature of the formation of viscous fingers. At number of periods of perturbations of $N$=6 there is a complex irregular pattern (Fig. 2b). There are processes of merging fingers, resulting in the formation of big fingers. And the thickness of these fingers varies in length, there is local thickening. The front of the fingers is mushroom-shaped.

The increase of number of periods of perturbations of initial interface leads to the more regular formation process of viscous fingers (Fig. 2c). The growth of viscous fingers took place at about the same rate up to approximately 1/3 of length of Hele-Shaw cell. Then the merging processes of the viscous fingers begin and several strong ones are formed, the growth rate of which increases. The remaining small fingers begin the processes of branching and formation of appendages on the short fingers. Similar growth pattern of viscous fingers is observed by many researchers. For example, in the paper [19] it is also shown that the growth rate of viscous fingers is determined by the wave number having an optimal value. In our case, this corresponds to $N$=6.

Similar patterns occur at a higher value of the perturbation amplitude. Figure 3 shows contours of the flow displacements for different periods and the same amplitude $A=1\cdot h$. When the number of periods of perturbations is small (Fig. 3a), the nature of the formation of viscous fingers has similar features to the unperturbed liquid interface.

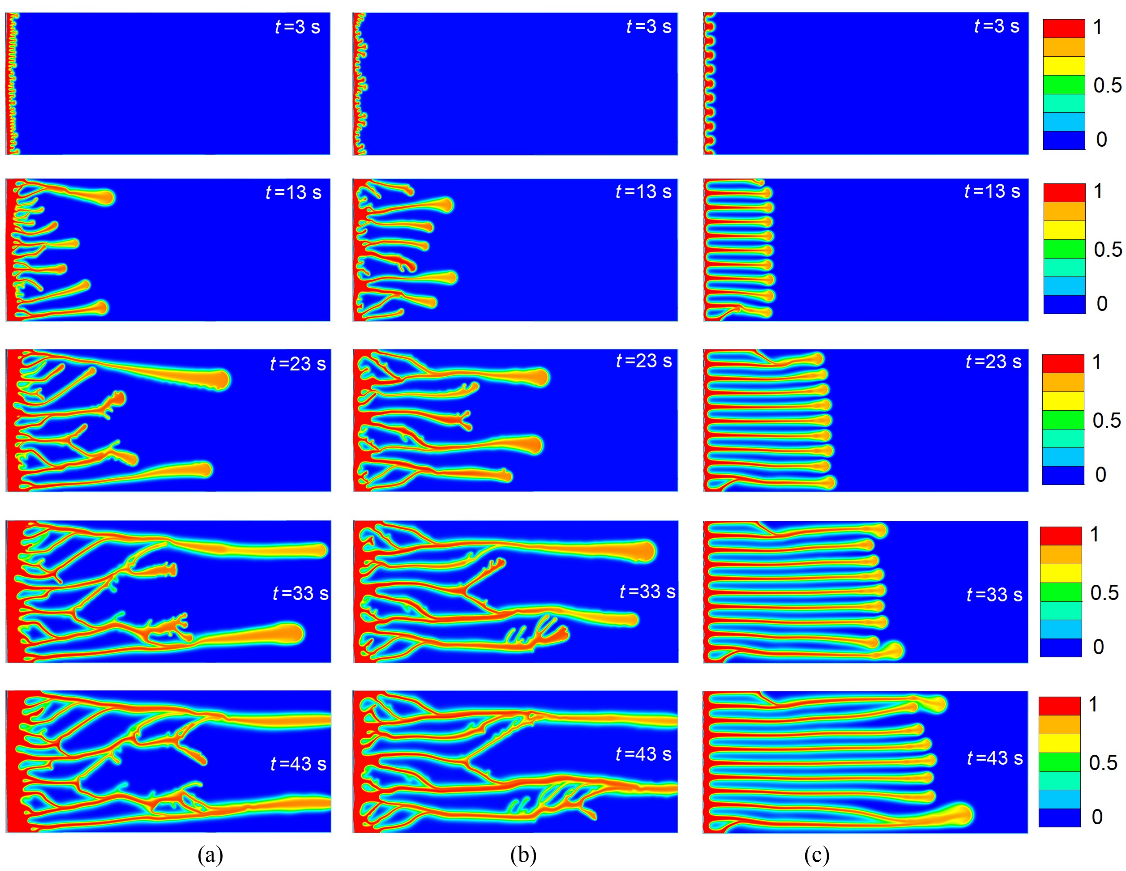


**FIGURE 3.** Concentration iso-contours at $N$=1 (a); $N$=4 (b); $N$=10 (c)

Namely, the two "strong" viscous fingers are formed near the geometrical boundaries of Hele-Shaw cell (in the picture at the top and bottom of the cell). In the process of growth these fingers capture the ones that grow near them more slowly. This result is consistent with the data of many researchers, for example [1,11,20]. Increasing the period of perturbation of the liquid interface leads to a greater number of growing viscous fingers (Fig. 3b). Some of them eventually merge to form a few strong fingers. An even greater increase in the period of the interface perturbation leads to some stabilization along with an increase in the amplitude (Fig. 3c). In particular, when $N$=10 nine viscous fingers are formed with approximately the same growth rate. Only when the length of the viscous fingers is greater than the width of Hele-Shaw cell, the selection of two strong fingers begins due to the merging of neighboring fingers with them.

The following treatment of gained patterns of concentration at different moments of time is aimed to study the influence that perturbation parameters of interface of liquid has on the growth rate of viscous fingers. According to Fig. 1 the coordinates of the leading edge of the longest viscous finger $X_{front}$ and the position of the back of the interface $X_{rear}$ were measured for each moment of time. Considering that the liquid was supplied to the inlet at the moment $t$=0, the average growth rate of the biggest finger can be calculated using the formula:

$$U_{vf} = \frac{X_{front}}{t}, \quad (2)$$

where $t$ is the time when initial interface is at the distance $X_{front}$.

For analysis of obtained data, we will consider them as dimensionless parameter of rate $U_{vf}/u_x$. Figure 4 shows dependences of relative growth rate of viscous fingers on time at the number of periods of perturbations $N$=6. The first stage is up to t=10 seconds. The increase in the growth rate of viscous fingers is basically the same for both unperturbed interface and for different amplitudes of interface perturbation. Over the time the growth rate of viscous fingers in Hele-Shaw cell increases on the unperturbed interface reaching the maximum of four times the supply rate of the liquid into the cell. A small amplitude of interface perturbation leads to a relative stabilization of growth rate of viscous fingers. The maximum growth rate of viscous fingers is reduced for this period by 19-31% depending on the period of perturbation.

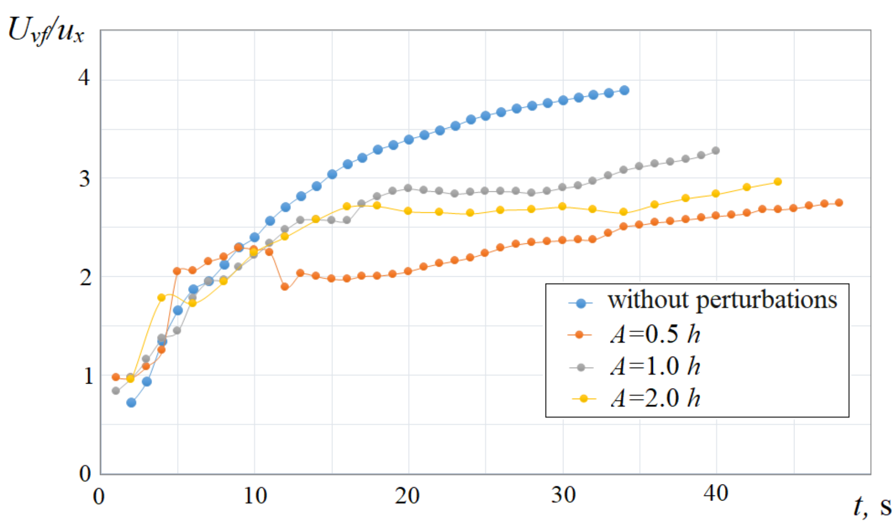


**FIGURE 4.** The change in the rate of growth of viscous fingers over time at $N$=6

Figure 5 shows maximum values of growth rate of viscous fingers for various periods and amplitudes of interface perturbation of liquid. The figure indicates that the growth rate of viscous fingers tends to reduce when the period of perturbation increases. A frequency dependence is found at some amplitudes of interface perturbation, at A = 4·h in our case, i.e. at $N$=4 the maximum growth rate of viscous fingers is close to the growth rate for unperturbed interface. The increase or decrease in perturbation period leads to the reduction of maximum growth rate of viscous fingers.

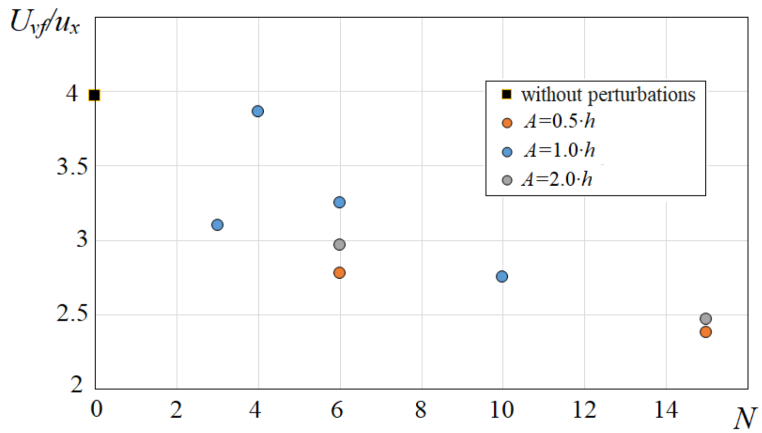


**FIGURE 5.** The change in the rate of growth of viscous fingers over time at $T$=6

## CONCLUSION

The dependence of growth rate of "viscous fingers" is studied during displacement of water with oil in Hele-Shaw cell based on the series of conducted numerical experiments. It is determined that characteristics of initial perturbation have an impact mainly on formation process of viscous fingers and almost do not influence their further development. It is detected that at first stages of the experiment the growth rate of viscous fingers rises sharply and after reaching $t=0.1\cdot l/u_x$ the change in rate is small. The relative growth rate of "viscous fingers" changes from 2 to 4 in the studied range of variable parameters. Obtained new results can be used for simulation of various modes of acoustic influence on porous media to increase oil recovery.

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