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RESEARCH ARTICLE

Self-force on a static charge in the long throat of a wormhle

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Abstract We consider the self-force on a static charge in the long throat of a wormhole. The example of the self-force calculation in the specific profile of the wormhole throat are given. We demonstrate that the self-force is repulsive in the considered case.

Keywords Self-force · Wormhole · Long throat

1 Introduction

The motion of an electric charged test particle on the background of a curved spacetime and/or a spacetime with nontrivial topological structure is determined by the coupling between the charge and the field that this charge induces. The origin of this self-force is associated with nonlocal structure of the field, the source of which is the particle. The formal expressions for the self-force have been derived [1–5] for various types of fields. A number of authors analyzed this phenomenon in the black hole spacetimes [6–52]. In the spacetimes with nontrivial topological structure the investigations of this effect have the additional interesting features [53–55]. The self-force for an charged particle at rest in the static wormhole spacetimes was analyzed in detail in Refs. [56–60]. It was shown that for an electrical charge the self-force is attractive for the considered profiles of the wormhole throat. In the present paper we analyze the self-force for an electric particle at rest in the long throat of a wormhole. The term "long throat" means that the radius of the throat is much smaller than its length. We demonstrate that the self-force is repulsive. This is in contrast to the supposition of Ref. [56] that the

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solution of Maxwell equations. The self-force acting on the charge q is determined by the retarded solution of Eq. (1)

$$A^{\mu}(x) = \int G^{\mu}{}_{\tilde{\nu}}(x, \tilde{x}) J^{\tilde{\nu}}(\tilde{x}) \sqrt{-\tilde{g}}\, d^4\tilde{x}, \tag{48}$$

where $J^{\tilde{\nu}}(\tilde{x})$ is proportional to q and $G^{\mu}{}_{\tilde{\nu}}(x, \tilde{x})$ is the retarded Green's function which satisfy

$$g^{\nu\lambda}(x) G^{\mu}{}_{\tilde{\nu}}(x, \tilde{x})_{;\nu\lambda} - G^{\nu}{}_{\tilde{\nu}}(x, \tilde{x}) R^{\mu}_{\nu}(x) = -4\pi g^{\mu}{}_{\tilde{\nu}}(x, \tilde{x}) \frac{\delta^{(4)}(x - \tilde{x})}{\sqrt{-g}}, \tag{49}$$

$g^{\mu}{}_{\tilde{\nu}}(x, \tilde{x})$ is the bivector of parallel transport of a vector at x to one at $\tilde{x}$. A^t_{hom} does not depend on the charge q. Therefore A^t_{hom} is not the retarded solution and we must consider the field A^t_{hom} as external for the charge q.

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