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Nonlinear dynamics of the 3D Alfvén waves in plasma of ionosphere and magnetosphere ☆

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ABSTRACT

The nonlinear dynamics of the 3D solitary Alfvén waves propagating nearly parallel to the external magnetic field in plasma of ionosphere and magnetosphere, which are described by the model of the 3-DNLS equation, is studied analytically and numerically. Under the assumption of negligible dissipative effects the analytical estimates and the sufficient conditions for the stability of 3D solutions of the 3-DNLS equation are obtained, based on the transformational properties of the system's Hamiltonian for the whole range of the equation coefficients. On the basis of asymptotic analysis the solutions asymptotics are presented. To study the evolution of the 3D Alfvén solitary waves including propagation of the Alfvén waves' beams in a magnetized plasma the equation are integrated numerically using the simulation codes specially developed. The results show that the 3-DNLS equation in non-dissipative case can have the stable 3D solutions in form of the 3D Alfvén solitons, and also on a level with them the 3D solutions collapsing or dispersing with time. In terms of the self-focusing phenomenon the results obtained can be interpreted as the formation of the stationary Alfvén wave beam propagating nearly parallel to magnetic field, or Alfvén wave beam spreading, or the self-focusing of the Alfvén wave beam. The influence of the dissipation in the medium on structure and character of evolution of 3D Alfvén waves is studied.

1. Introduction. Basic equations

We study stability and dynamics of the multidimensional soliton-like Alfvén structures forming on the low-frequency branch of oscillations in the ionospheric and magnetospheric plasma which are described by equation (Belashov, 2011)

$$\partial_t u + \mathrm{A}(t, u)\ \ u = f, \qquad f = \kappa \int_{-\infty}^{x} \Delta_\perp u dx, \qquad \Delta_\perp = \partial_y^2 + \partial_z^2 \tag{1}$$

when for

$$\mathrm{A}(t, u) = 3s\ \ |p|^2 u^2 \partial_x - \partial_x^2 (i\lambda + \nu) \tag{2}$$

it falls into 3D derivative nonlinear Schrödinger (3-DNLS) equation class. Derivation of Eq. (1) with differential operator (2) was presented in detail in (Belashov and Vladimirov, 2005) with use of the same approach and conditions as in (Petviashvili and Pokhotelov, 1992; Pokhotelov et al. 1996a; 1996b). In the case when $\beta \equiv 4\pi nT/B^2 > 1$ the 3-DNLS Eqs. (1), (2) describes dynamics of the finite-amplitude Alfvén waves propagating nearly parallel to homogeneous magnetic field **B** for $u = h = (B_y + iB_z)/2B|1 - \beta|$, $h = B_\perp/B_0$ where $p = (1 + ie)$, and e is the "eccentricity" of the polarization ellipse of the Alfvén wave (Belashov, 1997), $\nu = \frac{\rho_0}{2\rho}(c_\infty^2 - c_0^2)\tau \int_0^\infty \xi\varphi(\xi) d\xi$ defines the logarithmic damping rate, and it is the characteristic rate of the relaxation damping of the "sound" wave (Belashov and Vladimirov, 2005). Here ρ is perturbed plasma density $\left(\lim_{|x|\to\infty} \rho = \rho_0\right)$, c_∞ and c_0 are the velocities of the high and low-frequency "sound" mode (the last one coincides with $c_0 = (T_e/m_i)^{1/2}$) and $\varphi(t, \tau)$ is the function defining the relaxation process. The upper and lower signs of $\lambda = \pm\ 1$ correspond to the right and left circularly polarized wave, respectively; the sign of nonlinearity is accounted by the factor $s = \mathrm{sgn}\ \ (1 - p) = \pm 1$ in the nonlinear term; and $\kappa = -\ r_A/2$, $r_A = v_A/\omega_{0i}$.

Eqs. (1), (2) are not completely integrable. Therefore, to study the stability of multidimensional solitons we use the method developed in (Belashov, 1999) and investigated the Hamiltonian bounding with its deformation conserving momentum by solving

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the corresponding variation problem. In analytical study of this set we use also asymptotic analysis of its multidimensional solutions. To study evolution of 3D solitons including propagation of the Alfven waves' beams in a magnetized plasma the equations were being integrated numerically using the simulation codes specially developed and described in detail in (Belashov and Vladimirov 2005).

2. Stability of 3D solutions of the 3-DNLS equation

To study the stability of multidimensional solutions of Eqs. (1), (2) with $\nu = 0$ we use the same approach as in (Belashov, 999) similar to one in (Belashov, 2014a) used for the BK equation. We rewrite 3-DNLS Eqs. (1), (2) by performing the formal change $u \to h$ into the Hamiltonian form

$$\partial_t h = \partial_x (\delta \mathrm{H}/\delta h) \tag{3}$$

where $\delta\mathrm{H}/\delta h$ is a variational derivative, with the Hamiltonian (Belashov, 2014b)

$$\mathrm{H} = \int_{-\infty}^{\infty} \left[\tfrac{1}{2} |h|^4 + \lambda s h h^* \partial_x \varphi + \tfrac{1}{2}\kappa \left(\nabla_\perp \partial_x w\right)^2 \right] d\,\mathbf{r},$$
$$\partial_x^2 w = h, \qquad \varphi = \arg(h) \tag{4}$$

which has a sense of energy of the system, and solve the variation problem, $\delta\,(\mathrm{H}+\nu P_x)=0$, where $P_x = \frac{1}{2}\int |h|^2 d\,\mathbf{r}$ is the momentum projection onto the x axis, ν is the Lagrange's factor, that illustrates the fact that all finite solutions of Eq. (3) are the stationary points of the Hamiltonian for fixed P_x. Conforming with Lyapunov's theorem, the stationary points of a dynamical system realizing maximum or minimum of H are absolutely stable; if the extremum is local then the locally stable solutions are possible. The unstable states correspond to monotonous dependence of H on its variables, i.e. to the case when the stationary point is a saddle point. Thus, it is needed to prove the Hamiltonian's boundedness (from below) for fixed P_x. Consider the scale transformation $h\ (x, \mathbf{r}_\perp) \to \zeta^{-1/2}\eta^{-1} h\ (x/\zeta,\ \mathbf{r}_\perp/\eta)\ (\zeta,\ \eta\ \in \mathbb{C})$ conserving P_x, in complex vector space $\mathbb{C}$. The Hamiltonian as a function of ζ, η is given by

$$\mathrm{H}(\zeta, \eta) = a\zeta^{-1}\eta^{-2} + b\zeta^{-1} + c\zeta^2\eta^{-2} \tag{5}$$

where

$a = (1/2)\int |h|^4 d\,\mathbf{r}$, $\quad b = \lambda s \int h h^* \partial_x \varphi\ d\,\mathbf{r}$, $\quad c = (\sigma/2)\int (\nabla_\perp \partial_x w)^2 d\,\mathbf{r}$.

The necessary conditions for the existence of the extremum, $\partial_\zeta \mathrm{H} = 0$, $\partial_\eta \mathrm{H} = 0$, immediately allows us to obtain the extremum's coordinates

$$\zeta = -a/c, \qquad \eta = \left\{-(a/b)\left[1 + (a^2/c^2)\right]\right\} \tag{6}$$

where $b < 0$ if $\eta \in \mathrm{R} \subset \mathrm{C}$ because $a > 0$, $c > 0$ by definition, and $b > 0$ if $\eta \in \mathrm{C}$. The sufficient conditions for the existence of the local minimum of H at the point (ζ_i, η_i) are given by (Belashov, 2014a)

$$\begin{vmatrix} \partial_\zeta{}^2\mathrm{H}\ (\zeta_i, \eta_j) & \partial_{\zeta\eta}{}^2\mathrm{H}\ (\zeta_i, \eta_j) \\ \partial_{\eta\zeta}^2\mathrm{H}\ (\zeta_i, \eta_j) & \partial_\eta{}^2\mathrm{H}\ (\zeta_i, \eta_j) \end{vmatrix} > 0, \quad \partial_\zeta{}^2\mathrm{H}\ (\zeta_i, \eta_j) > 0 \tag{7}$$

and we therefore obtain for $b < 0$

$$a/c < d = \left(2\sqrt{2}\right)^{-1}\sqrt{13 + \sqrt{185}} \tag{8}$$

Thus it follows from (5)–(8) that the Hamiltonian H of (3) is limited from below, i.e.

$$\mathrm{H} > -3\,b\,d/(1 + 2d^2) \tag{9}$$

where $b < 0$ if condition (8) holds. In this case the 3D solutions of 3-DNLS equation are stable. The solutions are unstable in the opposite case, $a\,c^{-1} \geq d$, $b < 0$. Condition $b < 0$ corresponds to the right circularly polarized wave with $\beta = 4\pi nT/B^2 > 1$, i.e. when $\lambda = 1$, $s = -1$ in Eqs. (1), (2), and to the left circularly polarized wave when $\lambda = -1$, $s = 1$. It is necessary to note that the sign change $\lambda = 1 \to -1$, $s = -1 \to 1$ is equivalent to the change $t \to -t$, $\kappa \to -\kappa$ and for negative κ the Hamiltonian becomes negative in the area "occupied" by the 3D wave weakly limited in the $\mathrm{k}_\perp$-direction; in this case condition (9) is not satisfied. The change of the sign of b to positive [when $\lambda = 1$, $s = 1$ or $\lambda = -1$, $s = -1$ in Eqs. (1), (2)] is equivalent to the analytical extension of solution from real values of y, z to the pure imaginary ones: $y \to -iy$, $z \to -iz$ and, therefore, equivalent to the change of sign of κ in the basic equations. In this case instead of inequality (9) the opposite inequality will take place. From the physical point of view this means that if the opposite inequality is satisfied, the right polarized wave with the positive nonlinearity and the left polarized wave with the negative nonlinearity are stable. Note that in the particular case $\kappa = 0$ in Eqs. (1), (2) (1D approximation), instead of inequality (9) and the opposite one, it is easy to obtain the conditions H >0 and H <0, respectively, that is completely in agreement with the results obtained in (Dawson and Fontán, 1988a) for the 1-DNLS equation.

Thus the analysis of the transformation properties of the Hamiltonian of the 3-DNLS equation allows us to determine the ranges of the respective coefficients as well as H which has the sense of the energy of the system, corresponding to the stable and unstable 3D solutions. So, we have proved the possibility of existence in the 3-DNLS model of absolutely stable 3D solutions.

Now, following (Belashov and Vladimirov, 2005) consider the character of the asymptotics of the axially-symmetric solitary pulse solution of the 3-DNLS equation when $\Delta_\perp = \partial_\rho^2 + (1/\rho)\,\partial_\rho$. In this case Eqs. (1), (2) can be written in the form of the set

$$\partial_\eta \left[\partial_t h + s\partial_\eta\left(|h|^2 h\right) - i\lambda\partial_\eta^2 h - \nu\ \partial_\eta^2 h \right] = \kappa\ \left(\partial_\eta^2 + (1/\rho)\,\partial_\eta\right) h\,,$$
$$\partial_\zeta \left[\partial_t h + s\partial_\zeta\left(|h|^2 h\right) - i\lambda\partial_\zeta^2 h - \nu\ \partial_\zeta^2 h \right] = \kappa\ \left(\partial_\zeta^2 + (1/\rho)\,\partial_\zeta\right) h$$

written in thereference frame with the axes $\eta = x + \rho$, $\zeta = \rho - x$ rotated through an angle $\pi/4$ relative to the axes x and ρ. Further obvious transformations give us the set

$$\partial_t h + s\partial_{\eta'}\left(|h|^2 h\right) - i\lambda\partial_{\eta'}^2 h - \nu\ \partial_{\eta'}^2 h = 0\,,$$
$$\partial_t h + s\partial_{\zeta'}\left(|h|^2 h\right) - i\lambda\partial_{\zeta'}^2 h - \nu\ \partial_{\zeta'}^2 h = 0 \tag{10}$$

written in the coordinates $\eta' = \eta + \kappa t$, $\zeta' = \zeta + \kappa t$, i.e. in the frame moving along the corresponding axis with the velocity $-\kappa$. So, we can conduct the analysis for only one equation of the set (10) and then, fulfilling the inverse change of the variables, extend the results to the 3D axially-symmetric solutions $h\ (x,\ \rho,\ t)$ of the 3-DNLS equation (1), (2) with $u \equiv h$ and $\Delta_\perp = \partial_\rho^2 + (1/\rho)\,\partial_\rho$.

As it is known from Dawson and Fontán (1988a), an exact solution of the 1D DNLS equation is given by

$$h\ (x,\ t) = (A/2)^{1/2}\left[\exp\left(-Ax\right) + i\exp\left(Ax\right)\right]\exp\left(-iA^2 t\right)\cosh^{-2}(2Ax) \tag{11}$$

where A is the amplitude of the wave [see (Belashov and Vladimirov, 2005) for detail). Now we can apply the inverse change of the variables, $x = (\eta + \zeta)/2$, $\rho = (\eta - \zeta)/2$, and extending solution (11) to the 3D case (1), (2) with $\Delta_\perp = \partial_\rho^2 + (1/\rho)\,\partial_\rho$ write at once for $\nu = 0$

$$h\ (x,\ \rho,\ t) = (A/2)^{1/2}\left[\exp\left(-A\chi\right) + i\exp\left(A\chi\right)\right]\exp\left(-iA^2 t\right)\cosh^{-2}(2A\chi)$$

where $\chi = (x \pm \rho + (\kappa - V)\ t)$, and V is the velocity of the wave propagation relative to the coordinate axis x or ρ for the first or the second equations of set (10), respectively.

The dependence of the form of the solution on dissipation in the system as well as the dynamic characteristics of the solution for $\nu > 0$ will be considered in the next section in detail.

Note, that our analytical results are well confirmed by the results of our numerical experiments on study of structure and stability of multidimensional solitons in the model of the 3-DNLS equations (Belashov and Vladimirov, 2005; Belashov, 2014b). So, we have obtained that for a single solitons, on a level with wave spreading and wave collapse (in other terminology, self-contraction), the formation of multidimensional 3D solitons can be observed. Let us now consider the nonlinear effects for 3D soliton structures propagating in a magnetized plasma.

3. Nonlinear effects for the Alfvén waves propagating along magnetic field

For the numerical investigation, we consider the 3-DNLS equation in the integral-differential form (1), (2) and integrate it in the axially-symmetric geometry when $\Delta_\perp = \partial_\rho^2 + (1/\rho)\ \partial_\rho,\quad \rho^2 = y^2 + z^2$. The initial conditions are taken in the form of the axially-symmetric solitary pulses of two types:

1. Soliton-like axially symmetric pulse

$$h\ (x,\ \ \rho,\ \ 0) = h_0(x)\exp\left[\ i\varphi(x) - \rho^2/l_\rho^2\right] \tag{12}$$

with $h_0(x) = 2\sqrt{2}\delta\ \sin\ \vartheta\left[\cosh(4\delta^2\ \sin\ \vartheta x) + \cos\ \vartheta\right]^{-1/2}$ and $\varphi(x) = -\ 2s\delta^2\ \cos\ \vartheta x - (3s/4)\int_{-\infty}^{x} h_0^2(x)dx$ where $0 < \vartheta < \pi$;

2. Modulated plane wave:

$$h\ (x,\ \ \rho,\ \ 0) = H_0\ \exp\ \left(2\pi ix/\lambda - x^2/l_x^2 - \rho^2/l_\rho^2\right) \tag{13}$$

where λ is the wavelength, H_0 is the amplitude, and l_x and l_ρ are the characteristic scales of the Gaussian envelop modulation in the x and ρ-directions. Note that for $\rho=0$, the initial conditions (12) and (13) are equivalent to those used for the numerical simulation of the evolution of the 1D Alfvén wave in (Dawson and Fontán, 1988a,b).

To investigate the structure and evolution of the 3D pulses, we have done a number of simulation runs for both signs of the integral parameter b and various initial values of the Hamiltonian by defining various initial values for the pulse amplitude and the widths l_x and l_ρ. Thus, for non-dissipative case, when $\nu = 0$ in Eqs. (1), (2), we have obtained the following results.

1. For $\lambda=1$, $s=-1$, large $\kappa>0$, and the initial pulse weakly limited in the transverse ρ-direction when the stability condition (9) is satisfied, the evolution for large t results in formation of the stable 3D (axially-symmetric) solution (Fig. 1).
2. At the opposite signs of λ and s [that is equivalent to change $t \to -t$, $\kappa \to -\kappa$ in Eqs. (1), (2)] the Hamiltonian (4) of the 3-DNLS equation becomes negative, and, as it follows from the results of numerical experiments, a 3D Alfvén wave spreads with evolution (Fig. 2).
3. At $\lambda = 1,\quad s = -1$ for small $\kappa > 0$ and initial pulse rather strong limited in the ρ-direction the conditions of the existence of the local minimum of H (7) are not satisfy, and in the numerical experiments one can observe development of the 3D collapsing solutions of the 3-DNLS (Figs. 3 and 4). Note, that this effect is typical for all nonlinear systems where the Hamiltonian is unlimited for fixed first integrals (in this case, for the momentum P_x) and the quadratic terms in the expression for H [the first and third terms in expression (4)] are positively defined. For example, the same effects have been observed in the systems describing the evolution of the FMS waves (Belashov, 2014a) and Langmuir waves (Zakharov, 1975) waves in a plasma.

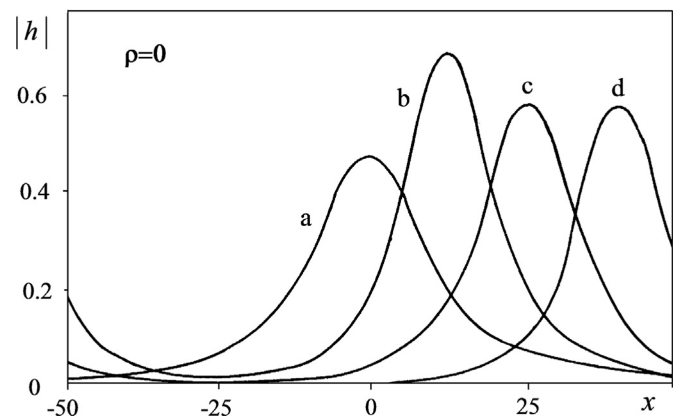

Fig. 1. Evolution of a 3D right circularly polarized nonlinear pulse (12) for $\lambda=1$, $s=-1$, $\kappa=1$; $H>-3bd/(1+2d^2)>0$: (a) $t=0$, (b) $t=25$, (c) $t=50$, (d) $t=75$.

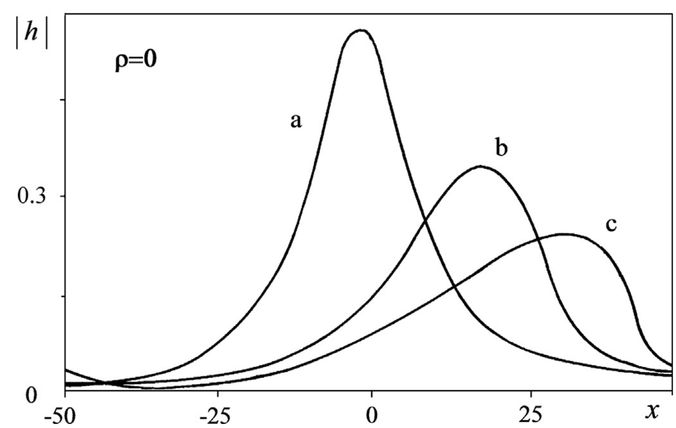

Fig. 2. Evolution of a 3D right circularly polarized nonlinear pulse (13) for $\lambda=-1$, $s=1$, $\kappa=1$; $H>0$: (a) $t=0$, (b) $t=50$, (c) $t=100$.

The series of the numerical experiments being carried out for $b>0$ when $\lambda = 1,\quad s = 1$ and $\lambda = -1, s = -1$ in Eqs. (1), (2) with $\nu=0$ showed that for these conditions in all cases (for different initial values of the Hamiltonian and the parameters $l_x,\quad l_\rho$) the initial 3D axially symmetric pulses spread with evolution. That is rather obvious so far as with such conditions for coefficients the inequality $H < -\ 3\,b\,d/\left(1 + 2d^2\right)$ (see Section 2) does not satisfy and, therefore, the 3D solutions of the 3-DNLS equation are unstable.

But, if we will fulfill the transform $h' = -sh^*$ in the 3-DNLS equation, i.e. consider left circularly polarized waves, that the signs in the expression for the Hamiltonian (4) change to opposite and for all cases considered above we will observe a mirror opposite picture. Thus, a case $\lambda = -1,\quad s = -1$ for big values $\kappa > 0$, and also the cases $\lambda = 1,\quad s = 1$ and $\lambda = -1,\quad s = -1$ for small values $\kappa > 0$ correspond the cases (1), (2) and (3), accordingly, with the opposite signs of the Hamiltonian. An example of the dynamics of a 3D left circularly polarized pulse is shown in Fig. 5.

Summing the above up, conclude that the 3-DNLS Eqs. (1), (2) with $\nu=0$ can have the stable 3D solutions in form of the 3D Alfvén solitons, and also on a level with them the 3D solutions

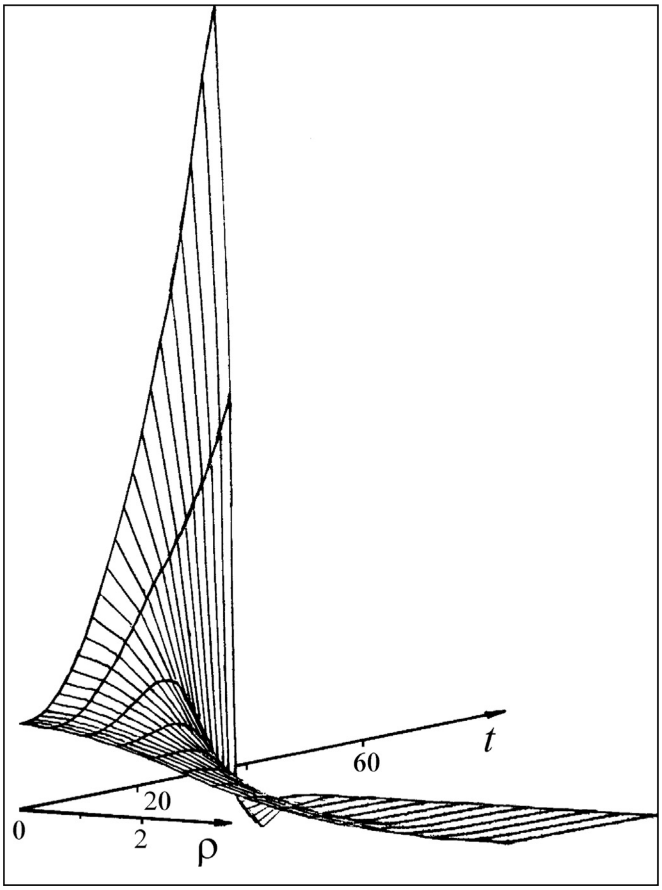

Fig. 3. Dynamics of a 3D right circularly polarized nonlinear pulse (13) (cross-section by the ρ-plane in the point $h_{\max}$) for $\lambda=1$, $s=-1$, $\kappa=0.2$; $0<\mathrm{H}<-3bd/(1+2d^2)$.

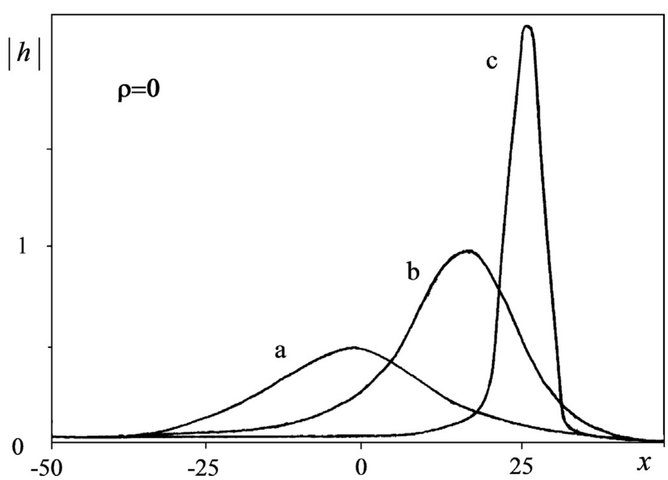

Fig. 4. Evolution of a 3D right circularly polarized nonlinear pulse (12): (a) $t=0$, (b) $t=25$, (c) $t=30$; the equation coefficients and Hare the same as in Fig. 3.

collapsing or dispersing with time. The form of solution is defined by signs of the equation coefficients λ and s, and also by the form of initial condition. The results obtained above can be also interpreted in terms of the self-focusing phenomenon. So, formal change $x \leftrightarrow t$ enables us to do a transition from the Cauchy problem (1), (2) and (12) or (1), (2) and (13) to the boundary-value problem describing the propagation of the 3D Alfvén wave beam localized in the ρ-plane along the x axis from boundary $x=0$. In this case the results obtained above can be interpreted as: (1) the formation of the stationary Alfvén wave beam propagating along

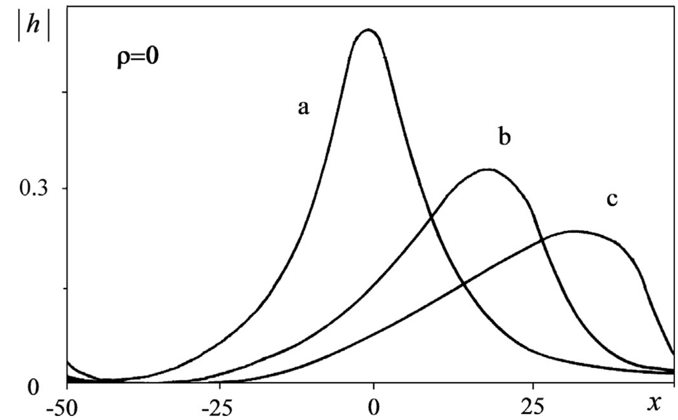

Fig. 5. Evolution of a 3D nonlinear left circularly polarized pulse (13) for $\lambda=s=\kappa=1$; $\mathrm{H}>0$: (a) $t=0$, (b) $t=50$, (c) $t=100$.

the x axis; (2) Alfvén wave beam spreading; and (3) the self-focusing of the Alfvén wave beam. It is interesting to note that we observe here the dynamics of the Alfvén wave beam propagating in plasma with $\beta>0$ at near-to-zero angles with respect to the external magnetic field **B,** which is qualitatively similar to the dynamics of the FMS wave beam propagating in plasma with dispersion coefficient $\gamma_1 \ll 0$ [see (Belashov, 2014a)] at angle close to $\pi/2$ with respect to the external magnetic field (Manin and Petviashvili, 1983).

The presence of dissipation in a plasma can be caused by many reasons. For example, in (Nariyuki et al., 2012) it is shown that the dissipation processes of circularly polarized parent Alfvén waves in solar wind plasmascan be observed due to the presence of the beam induced obliquely propagating waves, such as kinetic Alfvén waves (KAW) as a result of the nonlinear wave–wave coupling. Similar effects can be observed at nonlinear interaction of KAW and FMS wave for intermediate β-plasma when $\beta \ll 1$ (Modi and Sharma, 2013). But in context of our problem we consider here the case when the presence of dissipation in the system [$\nu>0$ in Eqs. (1), (2)] is caused by the relaxation processes of the viscous type in a medium (the particular physical reason of the energy dissipation depends on the type of the medium). In this case the dissipation changes the character of the evolution of the 3D nonlinear pulse, and in evolution of the Alfvén wave the exponential decrease of its amplitude with time is observed

$$|h|^2 = (1+e^2)\ \hbar^2(t) = (1+e^2)\ \hbar^2(0)\ \exp\ (\Gamma\ t),$$

where $h=(1+ie)\ \hbar$. In this case, the damping rate is of the same order of magnitude as in the BK model (Belashov and Vladimirov, 2005) (we have obtained in our numerical simulations the averaged value $\Gamma \sim -3.1$). Moreover, similar to the BK model, some steepening of the pulse's front takes place, and the back slope of the pulse decreases. The proof of that behavior is, in particular, in the different character of the change of the integrals of motion, $P=(1+e^2)\ \int \hbar^2 d\,\mathrm{r}$ and

$$\mathrm{H} = \int_{-\infty}^{\infty} \left[\tfrac{1}{2}\left(1+e^2\right)^2 \hbar^4 + \lambda s\ \left(1-e^2\right)\ \hbar^2 \partial_x \varphi + \tfrac{1}{2}\sigma\left(\nabla_\perp \partial_x w\right)^2 \right] d\mathrm{r},$$

$$\partial_x^2 w = (1+ie)\ \hbar, \qquad \varphi = \arg\ [(1+ie)\ \hbar],$$

in the regions behind and in front of the main maximum. Indeed, in all cases P and |H| decrease faster in front of the pulse. For various values of the coefficients in the 3-DNLS equation, the character of the evolution is the following:

1. For $\lambda=1$, $s=-1$, and relatively large $\kappa>0$, the initial pulse

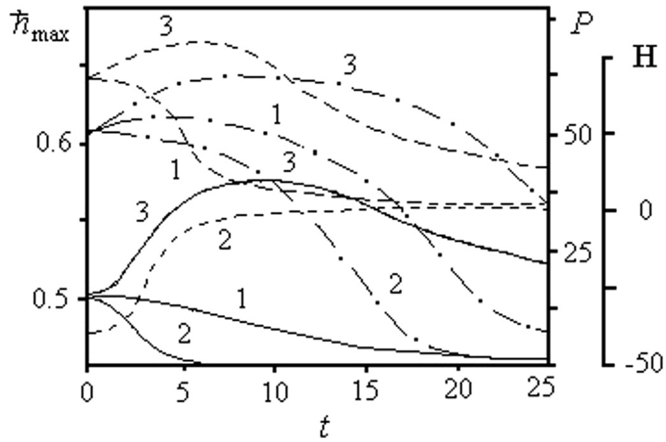

Fig. 6. Change of the amplitude of a 3D axially symmetric Alfvén wave (solid lines), P (chain lines) and H (dashed lines) of the 3-DNLS equation with $\nu=1$: (1) $\lambda=1$, $s=-1$, $\kappa=1.5$; (2) $\lambda=-1$, $s=1$, $\kappa=1.5$; (3) $\lambda=1$, $s=-1$, $\kappa=0.1$.

is weakly limited in the direction perpendicular to its propagation, and loses its energy with evolution (H → 0 with $t\to\infty$). In this case, the amplitude of the pulse decreases with time (as we noted above, exponentially, with the rate proportional to ν) and, as a result, the solitary wave disperses. Recall here, that in the case of $\nu=0$, the evolution after initial "sub-focusing" of the pulse leads to the formation of the stable 3D Alfvén soliton (see above).

2. For $\lambda=-1$, $s=1$, when the Hamiltonian becomes negative and the Alfvén wave pulse for $\nu=0$ spreads with its evolution, the presence of the dissipation accelerates this process significantly (for $\nu\sim1$ we have obtained in the simulations averaged $\Gamma\sim-3.4$). The effect of the steepening of thefront of the pulse takes place as well in this case.
3. For $\lambda=1$, $s=-1$, relatively small $\kappa>0$, and the initial pulse strongly limited in the transverse ρ-direction, when development of the wave collapse is observed in the simulations for $\nu=0$, the presence of the dissipation can rapidly delay or (for large $\nu>0$) even stop this process. In this case, the role of the dissipation in the 3-DNLS model is different from that in the model of the 3D BK equation: it is now the decisive factor in the stopping of the wave collapse.

Fig. 6 shows the change with time of the amplitude and the integrals P and H (averaged throughout the region of numerical integration) for the three cases described above and $\nu=1$.

4. Conclusion

We have considered analytically and numerically the nonlinear dynamics of the 3D solitary nonlinear Alfvén waves propagating nearly parallel to the external homogeneous magnetic field in a plasma on the basis of model of the 3-DNLS equation. For non-dissipative case we have obtained the analytical estimates and the sufficient conditions for the stability of 3D solutions of the 3-DNLS equation and proved that the equation can have the stable 3D solutions in form of the 3D Alfvén solitons, and also on a level with them the 3D solutions collapsing or dispersing with time. The asymptotics of the solitary solutions were studied. Our numerical experiments showed that in terms of the self-focusing phenomenon one can observe as a result of the evolution the formation of the stationary Alfvén wave beam propagating along the external magnetic field as well as Alfvén wave beam spreading or the self-focusing of the Alfvén wave beam. The influence of the dissipation of viscous type in a plasma on structure and character of evolution of 3D Alfvén waves was studied. In our research we have not taken into account the possible effects of magnetic field inhomogeneity and non-stationarity which can take place in the Earth's ionosphere and magnetosphere. So, inhomogeneity can result, for example, in soliton acceleration (Popel, et al., 1995) and other phenomena associated with infringement of balance between nonlinear and dispersive effects, for example, in destruction of the soliton, jast as it take place for the FMS waves propagating in the ionosphere and magnetosphere (Belashov and Vladimirov, 2005; Belashov, 2014a). In our model the effect of the inhomogeneous magnetic field can be accounted if we assume that $B_0=f(\mathrm{r})$ and, hence, β, κ and v_A are the function of $\mathbf{r}$ in (1)–(4). To study the effects of non-stationarity of magnetic field we can assume that $B_0=f(t)$ or, by analogy with the Kadomtsev–Petviashviv (KP) equation (Belashov, 1995), introduce into the 3-DNLS equation the term which describes the wave fluctuations of the magnetic field in time. Thus, we can expect the effects of formation of short-wave structures with soliton destruction and development of turbulence of wave field, being similar to that for the KP model (Belashov and Vladimirov, 2005). However the generalizations of the 3-DNLS model mentioned above leave beyond our research here.

The results obtained are very important for the best understanding of physics of nonlinear wave processes in plasma of ionosphere and magnetosphere and can be rather useful at interpretation of the results of experimental studies in laboratory and space experiments on excitation, evolution and interaction dynamics of the Alfvén solitons as well as the self-influence effects (the wave collapse and the wave self-focusing of the wave beams).

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