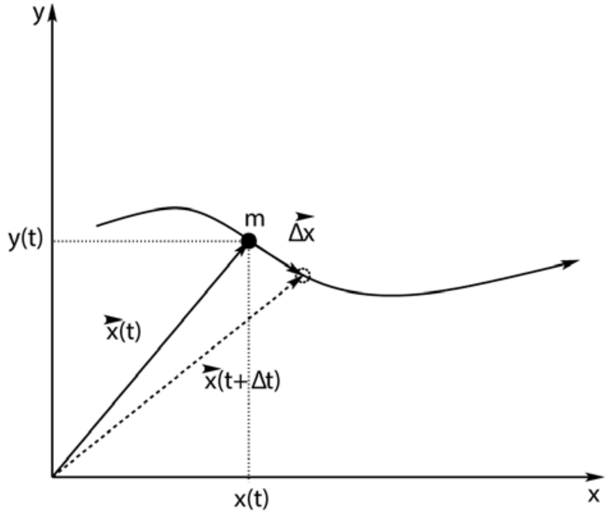
Lecture 1. Newton's Laws

Coordinates

Physics is the study of *dynamics*. Dynamics is the description of the actual forces of nature that, we believe, underlie the causal structure of the Universe and are responsible for its *evolution in time*. We are about to embark upon the intensive study of a simple description of nature that introduces the concept of a *force*, due to Isaac Newton. A force is considered to be the *causal agent* that produces the effect of *acceleration* in any massive object, altering its dynamic state of motion.



a) meters – the SI units of length
b) seconds – the SI units of time
c) kilograms – the SI units of mass

Coordinatized visualization of the motion of a particle of mass m along a trajectory $\vec{x}(t)$. Note that in a short time Δt the particle's position changes from $\vec{x}(t)$ to $\vec{x}(t+\Delta t)$.

$$\vec{x}(t)=x(t)\hat{x}+y(t)\hat{y}$$

Lecture 1. Newton's Laws

Velocity

The *average* velocity of the particle is by definition the vector change in its position $\Delta\vec{x}$ in some time Δt divided by that time:

$$\vec{v}_{av} = \frac{\Delta\vec{x}}{\Delta t}$$

Sometimes average velocity is useful, but often, even usually, it is not. It can be a rather poor measure for how fast a particle is actually moving at any given time, especially if averaged over times that are long enough for interesting changes in the motion to occur.

The *instantaneous* velocity vector is the time-derivative of the position vector:

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{x}(t + \Delta t) - \vec{x}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Speed is defined to be the *magnitude* of the velocity vector:

$$v(t) = |\vec{v}(t)|$$

Acceleration

To see how the velocity changes in time, we will need to consider the acceleration of a particle, or the rate at which the velocity changes. As before, we can easily define an *average* **acceleration** over a possibly long time interval Δt as:

$$\vec{a}_{av} = \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}}{dt}$$

The acceleration that really matters is (again) the limit of the average over very *short* times; the time derivative of the velocity. This limit is thus defined to be the *instantaneous* acceleration:

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

Lecture 1. Newton's Laws

Newton's Laws

a) Law of Inertia: Objects at rest or in uniform motion (at a constant velocity) in an *inertial reference frame* remain so unless acted upon by an unbalanced (net, total) force. We can write this algebraically as:

$$\vec{F} = \sum_i \vec{F}_i = 0 = m\vec{a} = m\frac{d\vec{v}}{dt} \Rightarrow \vec{v} = constant\ vector$$

b) Law of Dynamics: The total force applied to an object is directly proportional to its acceleration in an *inertial reference frame*. The constant of proportionality is called the **mass** of the object. We write this algebraically as:

$$\vec{F} = \sum_i \vec{F}_i = m\vec{a} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

where we introduce the *momentum* of a particle, $\vec{p} = m\vec{v}$.

c) Law of Reaction: If object A exerts a force $\vec{F}_{AB}$ on object B *along a line connecting the two objects*, then object B exerts an equal and opposite reaction force of $\vec{F}_{AB} = -\vec{F}_{BA}$ on object A. We write this algebraically as:

$$\vec{F}_{ij} = -\vec{F}_{ji}\ (or) \sum_{i,j} \vec{F}_{ij} = 0$$

where i and j are arbitrary particle labels. The latter form will be useful to us later; it means that the sum of all *internal* forces between particles in any closed system of particles cancels!

Forces

Classical dynamics at this level, in a nutshell, is very simple. Find the total force on an object. Use Newton’s second law to obtain its acceleration (as a differential equation of motion). Solve the equation of motion by direct integration or otherwise for the position and velocity.

The next most important problem is: how do we evaluate the total force?

There are *fundamental* forces – *elementary* forces that we call “laws of nature” because the forces themselves aren’t caused by some other force, they are themselves the actual causes of dynamical action in the visible Universe.

The Forces of Nature (strongest to weakest):

a) Strong Nuclear (bound together the quarks, protons and neutrons)

b) Electromagnetic (combines the positive nucleus with electrons)

c) Weak Nuclear (acts at very short range. This force can cause e.g. neutrons to give off an electron and turn into a proton)

d) Gravity

Lecture 1. Newton's Laws

Force Rules

a) **Gravity** (near the surface of the earth):

$$F_g = mg, \qquad g \approx 9{,}81 \frac{meter}{second^2} \approx 10 \frac{meter}{second^2}$$

b) **The Spring** (Hooke's Law) in one dimension:

$$F_x = -k\Delta x$$

c) **The Normal Force:**

$$F_\perp = N$$

d) **Tension** in an Acme (massless, unstretchable, unbreakable) string:

$$F_S = T$$

e) **Static Friction:**

$$f_S \leq \mu_s N$$

f) **Kinetic Friction:**

$$f_k = \mu_k N$$

g) **Fluid Forces, Pressure:** A fluid in contact with a solid surface (or anything else) in general exerts a force on that surface that is related to the pressure of the fluid:

$$F_P = PA$$

h) **Drag Forces:**

$$F_d = -bv^n$$

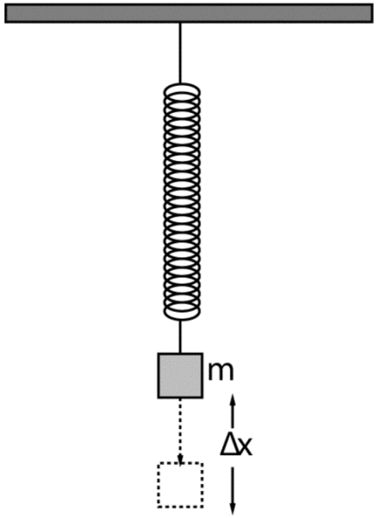
Force Balance – Static Equilibrium

If all of the forces acting on an object balance:

$$\vec{F}_{tot} = \sum_i \vec{F}_i = m\vec{a} = 0$$

Example: Spring and Mass in *Static Force Equilibrium*

Suppose we have a mass *m* hanging on a spring with spring constant *k* such that the spring is stretched out some distance Δ*x* from its unstretched length.



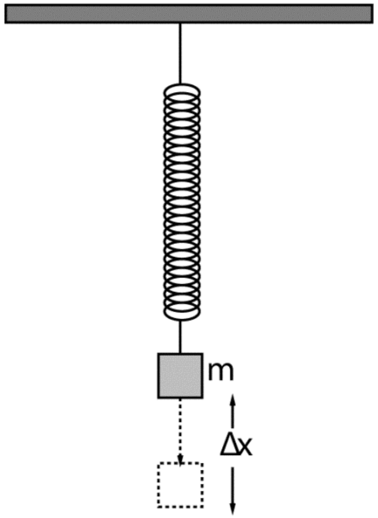
A mass *m* hangs on a spring with spring constant *k*. We would like to compute the amount Δ*x* by which the string is stretched when the mass is at rest in static force equilibrium.

$$\sum F_x = -k(x - x_0) - mg = ma_x$$

or (with $\Delta x = x - x_0$, so that Δ*x* is negative as shown)

$$a_x = -\frac{k}{m}\Delta x - g$$

Force Balance – Static Equilibrium



In static equilibrium, $a_x = 0$ (and hence, $F_x = 0$) and we can solve for Δx:

$$a_x = -\frac{k}{m}\Delta x - g = 0$$

$$\frac{k}{m}\Delta x = g$$

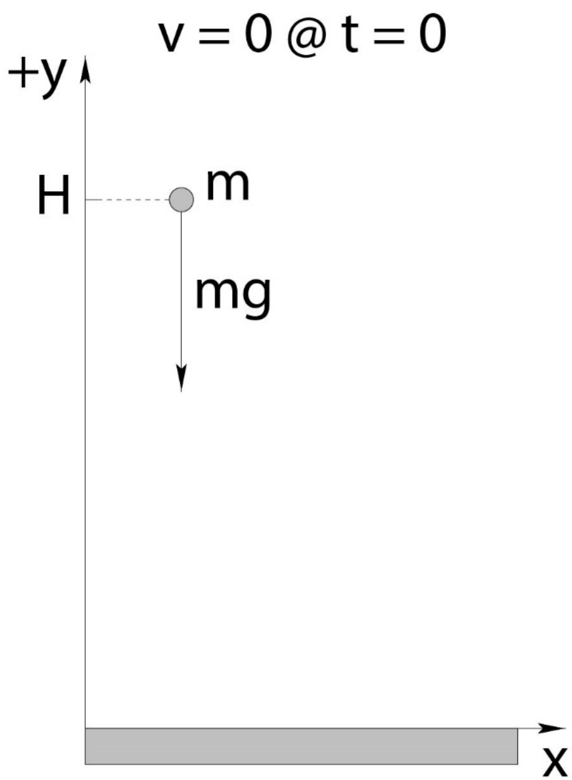
$$\Delta x = \frac{mg}{k}$$

Simple Motion in One Dimension

A mass m at rest is dropped from a height H above the ground at time $t = 0$; what happens to the mass as a function of time?

1. You must select a coordinate system to use to describe what happens.
2. You must write Newton's Second Law in the coordinate system for all masses, being sure to include all forces or force rules that contribute to its motion.
3. You must solve Newton's Second Law to find the accelerations of all the masses (equations called the equations of motion of the system).
4. You must solve the equations of motion to find the trajectories of the masses, their positions as a function of time, as well as their velocities as a function of time if desired.
5. Finally, armed with these trajectories, you must answer all the questions the problem poses using algebra and reason

Example: A Mass Falling from Height H



Draw in all of the forces that act on the mass as proportionate vector arrows in the direction of the force.

$$\vec{F} = -mg\hat{y}$$

or if you prefer, you can write the dimension-labelled scalar equation for the magnitude of the force in the y-direction:

$$F_y = -mg$$

$$F_y = -mg = ma_y$$

$$ma_y = -mg$$

$$a_y = -g$$

$$\frac{d^2y}{dt^2} = \frac{dv_y}{dt} = -g$$

where g = 10 m/second^2

Example: A Mass Falling from Height H

The last line (the algebraic expression for the acceleration) is called the equation of motion for the system

$\frac{dv_y}{dt} = -g$ Next, multiply both sides by *dt* to get:

$dv_y = -gdt$ Then integrate both sides:

$\int dv_y = -\int gdt$ doing the indefinite integrals to get:

$$v_y(t) = -g \cdot t + C$$

The final C is the constant of integration of the indefinite integrals. We have to evaluate it using the given (usually initial) conditions. In this case we know that:

$$v_y(0) = -g \cdot 0 + C = C = 0$$

Thus:

$$v_y(t) = -gt$$

We now know the velocity of the dropped ball as a function of time!

Example: A Mass Falling from Height H

However, the solution to the dynamical problem is the trajectory function, *y(t)*. To find it, we repeat the same process, but now use the definition for v_y in terms of y:

$$\frac{dy}{dt} = v_y(t) = -gt \text{ Multiply both sides by } dt \text{ to get:}$$

$$dy = -gt\,dt \text{ Next, integrate both sides:}$$

$$\int dy = -\int gt\,dt \text{ to get:}$$

$$y(t) = -\frac{1}{2}gt^2 + D$$

The final D is again the constant of integration of the indefinite integrals. We again have to evaluate it using the given (initial) conditions in the problem. In this case we know that:

$$y(0) = -\frac{1}{2}g0^2 + D = D = H$$

because we dropped it from an initial height $y(0) = H$. Thus:

$$y(t) = -\frac{1}{2}gt^2 + H$$

and we know everything there is to know about the motion!

Example: A Mass Falling from Height H

Finally, we have to answer any questions that the problem might ask! Here are a couple of common questions you can now answer using the solutions you just obtained:

a) How long will it take for the ball to reach the ground?

b) How fast is it going when it reaches the ground?

To answer the first one, we use a bit of algebra. "The ground" is (recall) $y = 0$ and it will reach there at some specific time (the time we want to solve for) t_g.

We write the condition that it is at the ground at time t_g :

$$y(t_g) = -\frac{1}{2}gt^2 + H = 0$$

If we rearrange this and solve for t_g we get:

$$t_g = \pm\sqrt{\frac{2H}{g}}$$

Example: A Mass Falling from Height H

To find the speed at which it hits the ground, one can just take our correct (future) time and plug it into v_y! That is:

$$v_g = v_y(t_g) = -g t_g = -g\sqrt{\frac{2H}{g}} = -\sqrt{2gH}$$

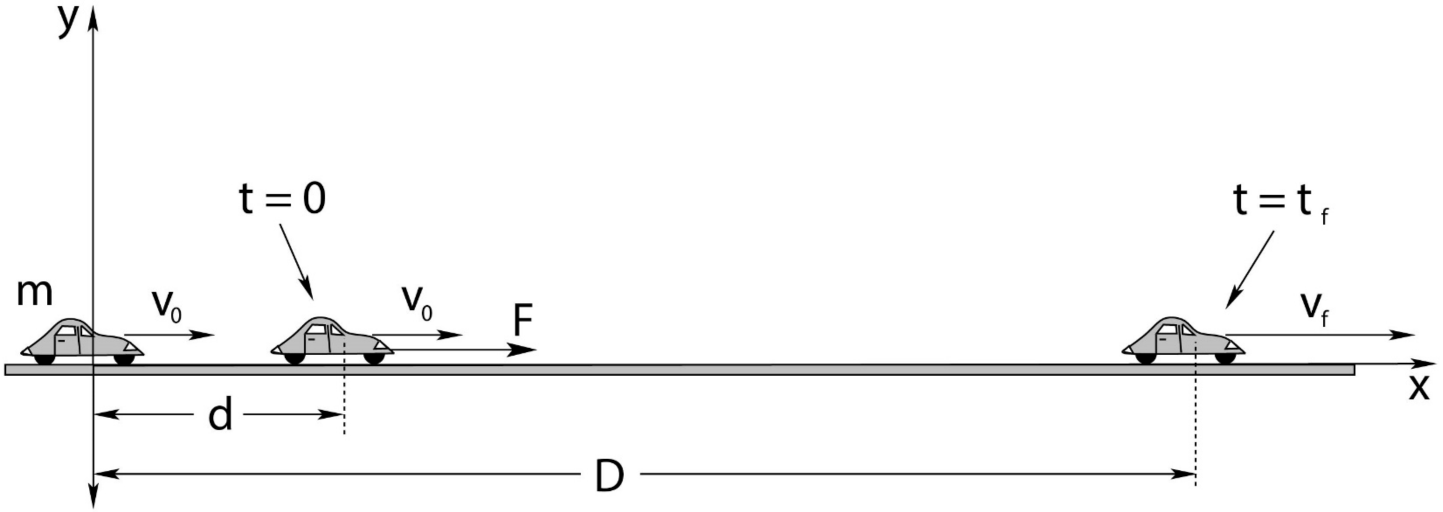
Note well that it is going down (in the negative y direction) when it hits the ground.

Example: A Constant Force in One Dimension

A car of mass m is travelling at a constant speed v_0 as it enters a long, nearly straight merge lane. A distance d from the entrance, the driver presses the accelerator and the engine exerts a constant force of magnitude F on the car.

a) How long does it take the car to reach a final velocity $v_f > v_0$?

b) How far (from the entrance) does it travel in that time?



Example: A Constant Force in One Dimension

We will write Newton's Second Law and solve for the acceleration (obtaining an equation of motion). Then we will integrate twice to find first $v_x(t)$ and then $x(t)$.

$$F = ma_x$$

$$a_x = \frac{F}{m} = a_0 \quad \text{(a constant)}$$

$$\frac{dv_x}{dt} = a_0$$

Next, multiply through by dt and integrate both sides:

$$v_x(t) = \int dv_x = \int a_0 dt = a_0 t + V = \frac{F}{m} t + V$$

V is a constant of integration that we will evaluate below.

Note that if $a_0 = F/m$ was not a constant (say that $F(t)$ is a function of time) then we would have to do the integral:

$$v_x(t) = \int \frac{F(t)}{m} dt = \frac{1}{m} \int F(t) dt = ???$$

Example: A Constant Force in One Dimension

At time $t = 0$, the velocity of the car in the x-direction is v_0, so $V = v_0$ and:

$$v_x(t) = a_0 t + v_0 = \frac{dx}{dt}$$

We multiply this equation by dt on both sides, integrate, and get:

$$x(t) = \int dx = \int (a_0 t + v_0)\, dt = \frac{1}{2} a_0 t^2 + v_0 t + x_0$$

where x_0 is the constant of integration. We note that at time $t = 0$, $x(0) = d$, so x_0 = d. Thus:

$$x(t) = \frac{1}{2} a_0 t^2 + v_0 t + d$$

$$v_x(t) = a_0 t + v_0$$

$$x(t) = \frac{1}{2} a_0 t^2 + v_0 t + x_0$$

Motion in Two Dimensions

The idea of motion in two or more dimensions is very simple. Force is a vector, and so is acceleration. Newton’s Second Law is a recipe for taking the total force and converting it into a differential equation of motion:

$$\vec{a} = \frac{d^2 r}{dt^2} = \frac{\vec{F}_{tot}}{m}$$

If we write the equation of motion out in components:

$$a_x = \frac{d^2 x}{dt^2} = \frac{F_{tot,x}}{m}$$

$$a_y = \frac{d^2 y}{dt^2} = \frac{F_{tot,y}}{m}$$

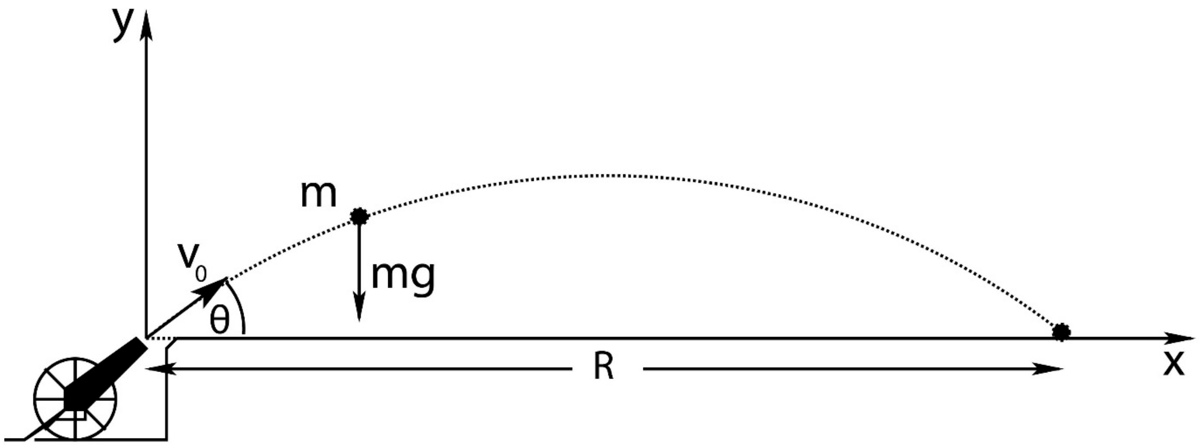
$$a_z = \frac{d^2 z}{dt^2} = \frac{F_{tot,z}}{m}$$

we will often reduce the complexity of the problem from a “three dimensional problem” to three “one dimensional problems”.

Select a coordinate system in which one of the coordinate axes is aligned with the total force.

Lecture 1. Newton's Laws

Example: Trajectory of a Cannonball



An idealized cannon, neglecting the drag force of the air. Let x be the horizontal direction and y be the vertical direction, as shown. Note well that $\vec{F}_g = -mg\vec{y}$ points along one of the coordinate directions while F_x = (F_z =) 0 in this coordinate frame.

A cannon fires a cannonball of mass m at an initial speed v_0 at an angle θ with respect to the ground as shown in figure. Find:

a) The time the cannonball is in the air.

b) The range of the cannonball.

Example: Trajectory of a Cannonball

Newton's Second Law for both coordinate directions:

$$F_x = ma_x = 0$$

$$F_y = ma_y = m\frac{d^2y}{dt^2} = -mg$$

We divide each of these equations by *m* to obtain two equations of motion, one for *x* and the other for *y*:

$$a_x = 0$$

$$a_y = -g$$

We solve them independently. In *x*:

$$a_x = \frac{dv_x}{dt} = 0$$

The derivative of any constant is zero, so the *x*-component of the velocity does not change in time. We find the initial (and hence constant) component using trigonometry:

$$v_x(t) = v_{0x} = v_0 \cos\theta$$

Example: Trajectory of a Cannonball

We then write this in terms of derivatives and solve it:

$$v_x(t) = \frac{dx}{dt} = v_0 \cos(\theta)$$

$$dx = v_0 \cos(\theta)\, dt$$

$$\int dx = v_0 \cos(\theta) \int dt$$

$$x(t) = v_0 \cos(\theta)\, t + C$$

We evaluate C (the constant of integration) from our knowledge that in the coordinate system we selected, $x(0) = 0$ so that $C = 0$. Thus:

$$x(t) = v_0 \cos(\theta)\, t$$

Example: Trajectory of a Cannonball

The solution in *y* is more or less identical to the solution that we obtained above dropping a ball, except the constants of integration are different:

$$a_y = \frac{dv_y}{dt} = -g$$

$$dv_y = -gdt$$

$$\int dv_y = -\int g\,dt$$

$$v_y(t) = -gt + C'$$

For this problem, we know from trigonometry that:

$$v_y(0) = v_0 \sin(\theta)$$

so that $C' = v_0 \sin(\theta)$ and:

$$v_y(t) = -gt + v_0 \sin(\theta)$$

Example: Trajectory of a Cannonball

We write v_y in terms of the time derivative of y and integrate:

$$\frac{dy}{dt} = v_y(t) = -gt + v_0\sin(\theta)$$

$$dy = (-gt + v_0\sin(\theta))\,dt$$

$$\int dy = \int(-gt + v_0\sin(\theta))\,dt$$

$$y(t) = -\frac{1}{2}gt^2 + v_0\sin(\theta)t + D$$

Again we use $y(0) = 0$ in the coordinate system we selected to set $D = 0$ and get:

$$y(t) = -\frac{1}{2}gt^2 + v_0\sin(\theta)t$$

Example: Trajectory of a Cannonball

Collecting the results from above, our overall solution is thus:

$$x(t) = v_0 \cos(\theta)\, t$$

$$y(t) = -\frac{1}{2} g t^2 + v_0 \sin(\theta) t$$

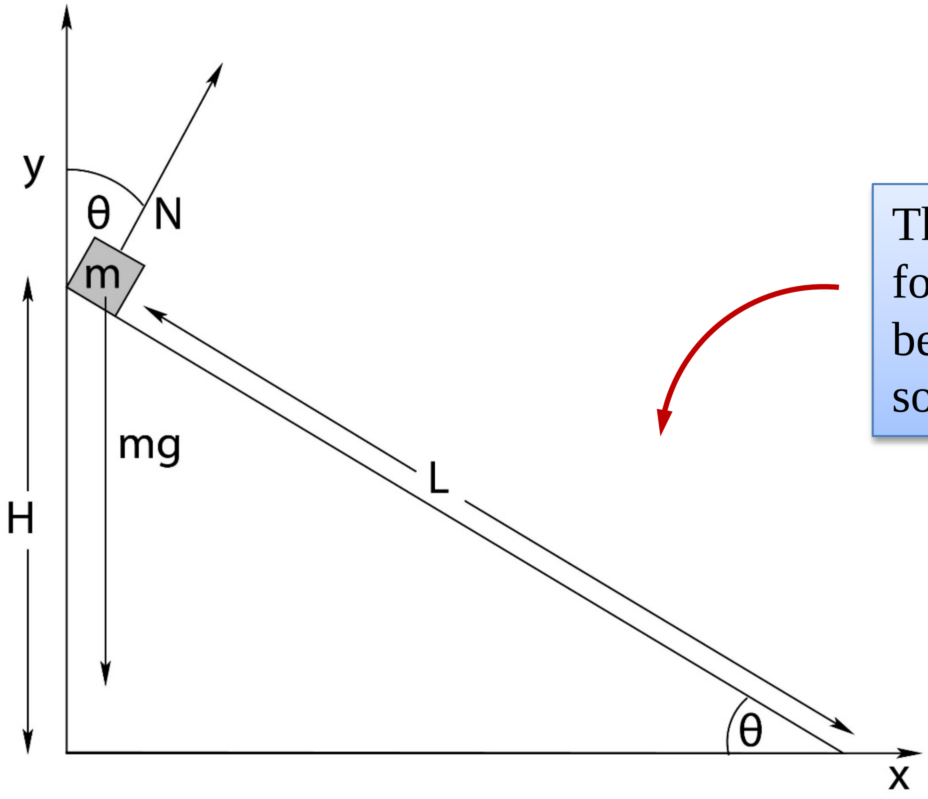
$$v_x(t) = v_{0x} = v_0 \cos\theta$$

$$v_y(t) = -gt + v_0 \sin(\theta)$$

We know exactly where the cannonball is at all times, and we know exactly what its velocity is as well.

The Inclined Plane

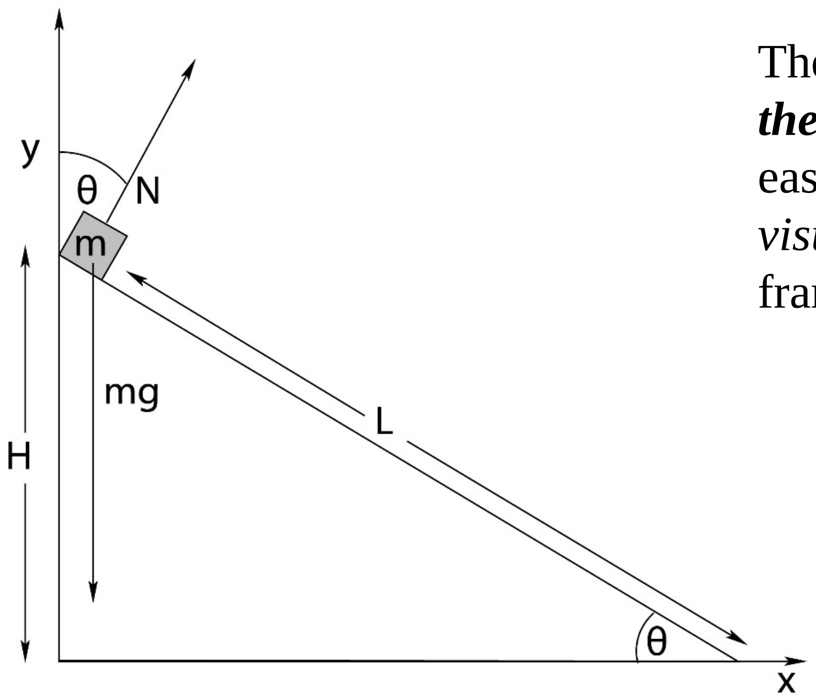
In this problem we will talk about a new force, the *normal* force. Recall from above that the normal force is whatever magnitude it needs to be to prevent an object from moving in to a solid surface, and is always perpendicular (normal) to that surface in direction.



This is the naive/wrong coordinate system to use for the inclined plane problem. The problem can be solved in this coordinate frame, but the solution (as you can see) would be quite difficult.

The Inclined Plane

A block m rests on a plane inclined at an angle of θ with respect to the horizontal. There is no friction, but the plane exerts a normal force on the block that keeps it from falling straight down. At time $t = 0$ it is released (at a height $H = L\sin(\theta)$ above the ground), and we might then be asked any of the "usual" questions – how long does it take to reach the ground, how fast is it going when it gets there and so on.



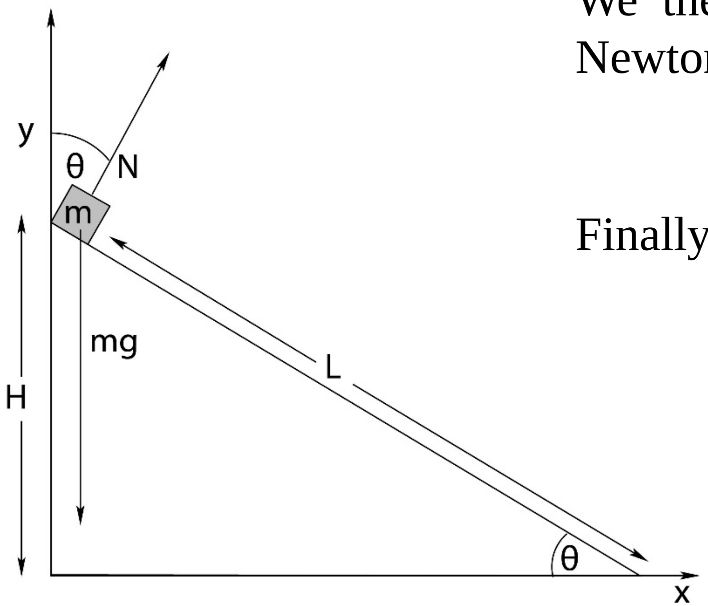
The motion we expect is for the block to ***slide down the incline***, and for us to be able to solve the problem easily we have to use our *intuition* and ability to *visualize* this motion to select the best coordinate frame.

The Inclined Plane

Let's try to decompose these forces in terms of our coordinate system:

$$N_x = N\sin\theta$$
$$N_y = N\cos\theta$$

where $N = |\vec{N}|$ is the (unknown) magnitude of the normal force.



We then add up the total forces in each direction and write Newton's Second Law for each direction's total force :

$$F_x = N\sin\theta = ma_x$$
$$F_y = N\cos\theta - mg = ma_y$$

Finally, we write our equations of motion for each direction:

$$a_x = \frac{N\sin\theta}{m}$$
$$a_y = \frac{N\cos\theta - mg}{m}$$

Unfortunately, we cannot solve these two equations as written yet. That is because we do not know the value of N; it is in fact something we need to solve for!

The Inclined Plane

To solve them we need to add a condition on the solution, expressed as an equation. The condition we need to add is that the motion is down the incline, that is, at all times:

$$\frac{y(t)}{L\cos\theta - x(t)} = \tan\theta$$

That means that:

$$y(t) = \big(L\cos\theta - x(t)\big)\tan\theta$$

$$\frac{dy(t)}{dt} = -\frac{dx(t)}{dt}\tan\theta$$

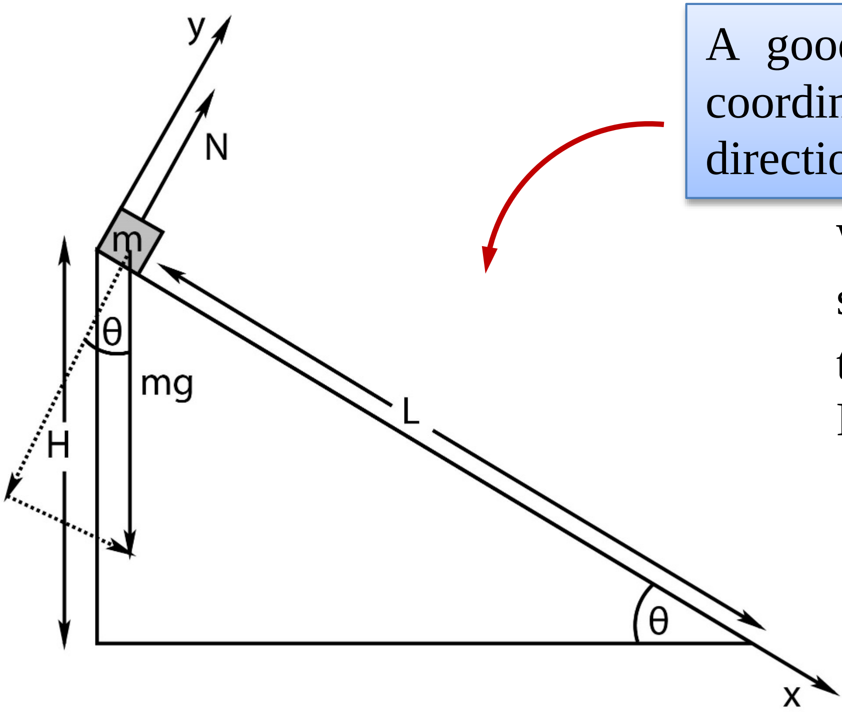
$$\frac{d^2y(t)}{dt^2} = -\frac{d^2x(t)}{dt^2}\tan\theta$$

$$a_y = -a_x\tan\theta$$

We can use this relation to eliminate (say) a_y from the equations above, solve for a_x, then backsubstitute to find a_y.

The solutions we get will be so very complicated (at least compared to choosing a better frame), with both *x* and *y* varying nontrivially with time!

The Inclined Plane



A good choice of coordinate frame has (say) the x-coordinate lined up with the total force and hence direction of motion.

We can decompose the forces in this coordinate system, but now we need to find the components of the gravitational force as $\vec{N} = N\hat{y}$ is easy! Furthermore, we know that $a_y = 0$ and hence $F_y = 0$.

$$F_x = mg\sin\theta = ma_x$$

$$F_y = N - mg\cos\theta = ma_y = 0$$

We can immediately solve the y equation for:

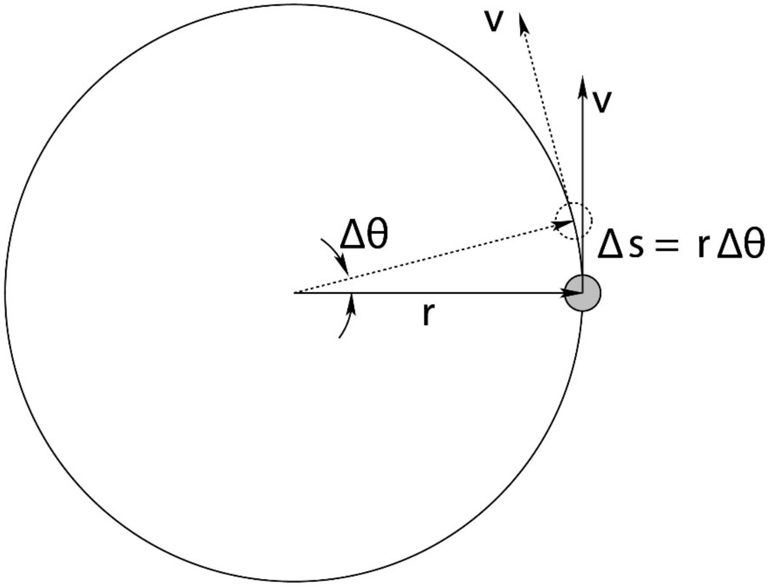
$$N = mg\cos\theta$$

and write the equation of motion for the *x*-direction: $a_x = g\sin\theta$ which is a constant.

From this point on the solution should be familiar – since $v_y(0) = 0$ and $y(0) = 0$, $y(t) = 0$ and we can **ignore** *y* altogether and the problem is now **one dimensional**!

See if you can find how long it takes for the block to reach bottom, and how fast it is going when it gets there. You should find that $v_{bottom} = \sqrt{2gH}$

Circular Motion



A small ball, moving in a circle of radius *r*. We are looking down from above the circle of motion at a particle moving counterclockwise around the circle. At the moment, at least, the particle is moving at a constant speed *v* (so that its *velocity* is always *tangent* to the circle).

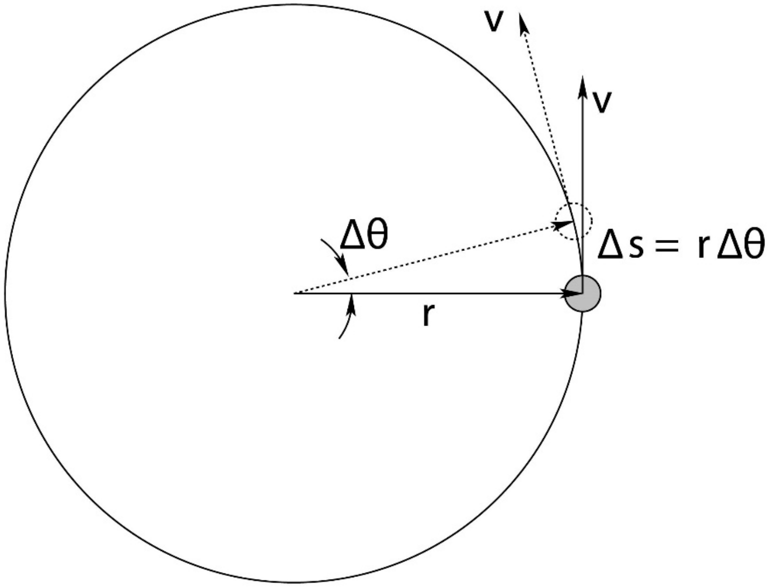
The length of a circular arc is the radius times the angle subtended by the arc we can see that:

$$\Delta s = r\Delta\theta$$

Note Well! In this and all similar equations θ must be measured in **radians**, never degrees

Lecture 1. Newton's Laws

Circular Motion



The average speed v of the particle is thus this distance divided by the time it took to move it:

$$v_{avg} = \frac{\Delta s}{\Delta t} = r\frac{\Delta \theta}{\Delta t}$$

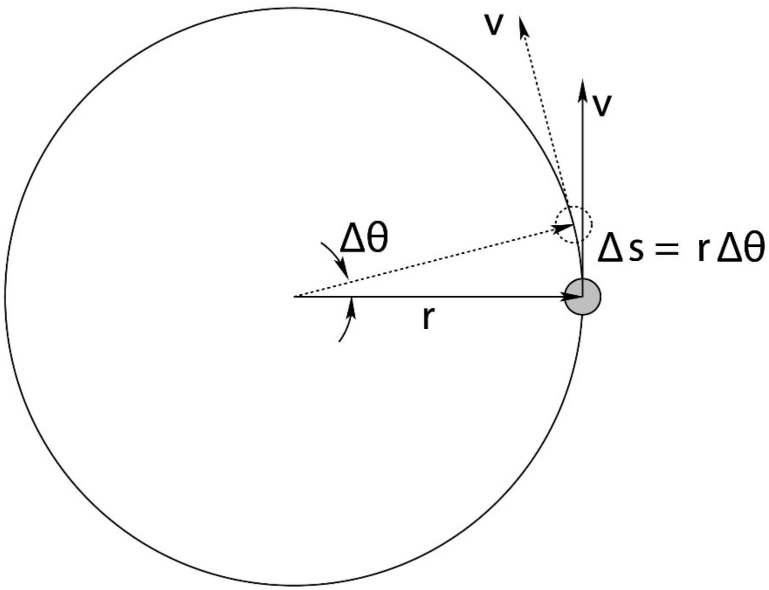
Of course, we really don't want to use average speed (at least for very long) because the speed might be varying, so we take the limit that $\Delta t \rightarrow 0$ and turn everything into derivatives, but it is much easier to draw the pictures and visualize what is going on for a small, finite Δt :

$$v = \lim_{\Delta t \rightarrow 0} r\frac{\Delta \theta}{\Delta t} = r\frac{d\theta}{dt}$$

This speed is directed tangent to the circle of motion (as one can see in the figure) and we will often refer to it as the tangential velocity.

Lecture 1. Newton's Laws

Circular Motion



$$v_t = r\frac{d\theta}{dt}$$

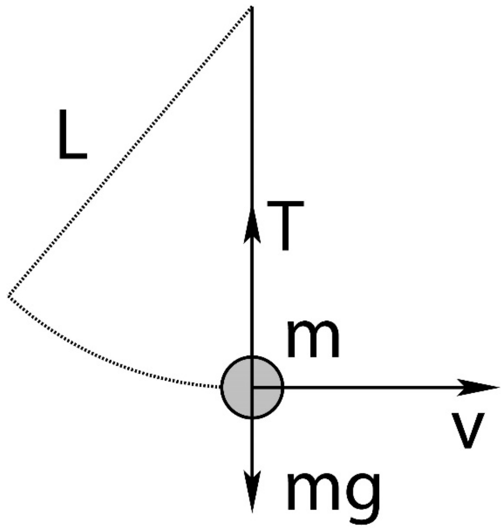
In this equation, we see that the speed of the particle at any instant is the radius times the rate that the angle is being swept out by the particle per unit time. This latter quantity is a very useful one for describing circular motion, or rotating systems in general.

We define it to be the ***angular velocity***:

$$\omega = \frac{d\theta}{dt}$$

Thus: $v = r\omega$ or $\omega = \frac{v}{r}$

Centripetal Acceleration



A ball of mass *m* swings down in a circular arc of radius *L* suspended by a string, arriving at the bottom with speed *v*. What is the tension in the string?

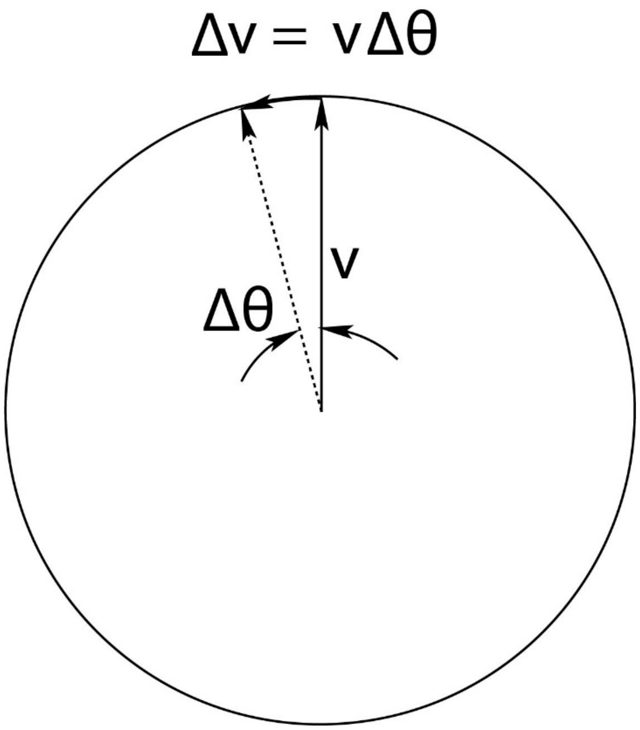
At the bottom of the trajectory, the tension *T* in the string points straight up and the force *mg* points straight down. No other forces act, so we should choose coordinates such that one axis lines up with these two forces. Let’s use +*y* vertically up, aligned with the string. Then:

$$F_y = T - mg = ma_y = m\frac{v^2}{L}$$

$$\text{or } T = mg + m\frac{v^2}{L}$$

The *net* force towards the center of the circle must be algebraically equal to mv^2/r

Example: Ball on a String



The velocity of the particle at t and $t + \Delta t$. Note that over a very short time Δt the speed of the particle is at least approximately constant, but its *direction* varies because it always has to be perpendicular to $\vec{r}$, the vector from the center of the circle to the particle. The velocity swings through the *same angle* $\Delta\theta$ that the particle itself swings through in this (short) time.

In time Δt, then, the magnitude of the *change* in the velocity is:

$$\Delta v = v\Delta\theta$$

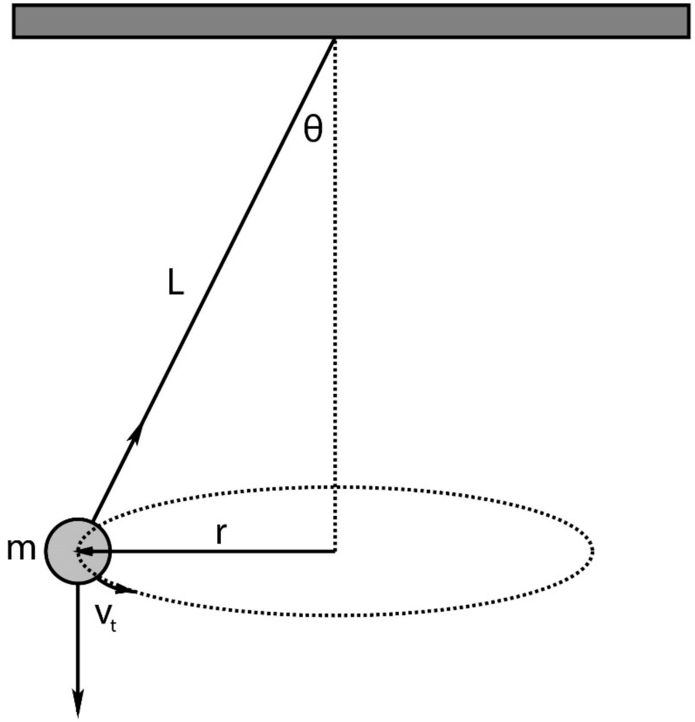
Consequently, the average magnitude of the acceleration is:

$$a_{avg} = \frac{\Delta v}{\Delta t} = v\frac{\Delta\theta}{\Delta t}$$

The instantaneous magnitude of the acceleration is: $a = \lim_{\Delta t \to 0} v\frac{\Delta\theta}{\Delta t} = v\frac{d\theta}{dt} = v\omega = \frac{v^2}{r} = r\omega^2$

If a particle is moving in a circle at instantaneous speed v, **then** its acceleration towards the center of that circle is v^2/r (or $r\omega^2$ if that is easier to use in a given problem).

Example: Tether Ball/Conic Pendulum



Ball on a rope (a tether ball or conical pendulum). The ball sweeps out a right circular cone at an angle θ with the vertical when launched appropriately.

Suppose you hit a tether ball so that it moves in a plane circle at an angle θ at the end of a string of length L. Find T (the tension in the string) and v, the speed of the ball such that this is true.

Note well in this figure that the only "real" forces acting on the ball are gravity and the tension T in the string. Thus in the y-direction we have:

$$\sum F_y = T\cos\theta - mg = 0$$

and in the x-direction (the minus r-direction, as drawn) we have: $\sum F_x = T\sin\theta = ma_r = \frac{mv^2}{r}$

Thus $T = \frac{mg}{\cos\theta}$

$$v^2 = \frac{Tr\sin\theta}{m} \quad \text{or} \quad v = \sqrt{gL\sin\theta\tan\theta}$$

Lecture 1. Newton's Laws

Example: Tangential Acceleration

Sometimes we will want to solve problems where a particle speeds up or slows down while moving in a circle. Obviously, this means that there is a nonzero *tangential acceleration* changing the *magnitude* of the tangential velocity.

Let's write $\vec{F}$ (total) acting on a particle moving in a circle in a coordinate system that rotates along with the particle – *plane polar coordinates*. The tangential direction is the $\vec{\theta}$ direction, so we will get:

$$\vec{F} = F_r \hat{r} + F_\theta \hat{\theta}$$

From this we will get two equations of motion (connecting this, at long last, to the dynamics of two dimensional motion):

$$F_r = -m\frac{v^2}{r}$$

$$F_t = ma_t = m\frac{dv}{dt}$$