Mathematical and Numerical Analysis of Dielectric Waveguides by the Integral Equation Method

E. Karchevskiy[1] and Y. Shestopalov[2]

[1]Kazan Federal University, Russia
[2]Karlstad University, Sweden

Abstract— Many different numerical techniques are applied for computing eigenmodes of dielectric waveguides [1, 2]; namely, finite-element, finite-difference, beam propagation, and spline collocation methods and multidomain spectral approach. Often the authors concentrate on the algorithm's aspects and physical interpretation of the numerical results rather than on fundamental mathematical aspects including the existence, properties, and distribution of the spectra on the complex plane of the spectral parameter. In this study, we propose a new approach to mathematical and numerical analysis of the dielectric waveguides based on the methods of spectral theory of operator-valued functions [3, 4] and integral equations (IEs) [4–6]. The eigenvalue problems for the determination of natural modes (surface, leaky, and complex eigenmodes) of inhomogeneous optical waveguides and step-index optical waveguides with the smooth cross-sectional boundary are formulated [3–5] for the time-harmonic Maxwell equations with partial radiation conditions at infinity in the cross-sectional plane. The initial problems are reduced with the aid of the IE method (using appropriate Green functions) to nonlinear spectral problems with Fredholm integral operators. Theorems on the spectrum localization are proved. It is shown that the sets of all eigenvalues of the initial problems may consist of isolated points on the Reimann surface of the spectral parameter (longitudinal wavenumber) and each eigenvalue depends continuously on the frequency and permittivity and can appear or disappear only at the boundary of the Reimann surface.

The initial problems for surface waves are reduced to linear eigenvalue problems for integral operators with real-valued symmetric weakly singular kernels. The existence, localization, and dependence of the spectrum on parameters are investigated. The collocation and Galerkin methods for the calculation of natural modes are proposed, the convergence of the methods is proved, and some results of numerical experiments are discussed.

REFERENCES

1. Horikis, T. P., "Dielectric waveguides of arbitrary cross sectional shape," *Applied Mathematical Modeling*, Vol. 37, No. 7, 5080–5091, 2013.
2. Xiao, J. and X. Sun, "Full-vectorial mode solver for anisotropic optical waveguides using multidomain spectral collocation method," *Optics Communications*, Vol. 283, No. 14, 2835–2840, 2010.
3. Shestopalov, V. P. and Yu. V. Shestopalov, *Spectral Theory and Excitation of Open Structures*, Peter Peregrinus Ltd., 1996.
4. Chernokozhin, E., Y. Smirnov, and Y. Shestopalov, *Logarithmic Integral Equations in Electromagnetics*, VSP Int. Science Publishers, Utrecht, Boston, Köln, Tokyo, 2000.
5. Kartchevski, E. M., A. I. Nosich, and G. W. Hanson, "Mathematical analysis of the generalized natural modes of an inhomogeneous optical fiber," *SIAM J. Appl. Math.*, Vol. 65, No. 6, 2033–2048, 2005.
6. Dautov, R. Z. and E. M. Karchevskiy, *Integral Equations Method and Exact Nonlocal Boundary Conditions in the Theory of Dielectric Waveguides*, Kazan State University, Kazan, 2009.