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Accumulation of a light non-aqueous phase liquid on a flat barrier baffling a descending groundwater flow

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The pioneering solution of Zhukovskii for a steady two-dimensional flow of an ideal heavy fluid with a nonlinear free boundary condition is extended to a Darcian flow of groundwater encumbered by an impermeable barrier. The stoss or/and lee sides of the barrier are covered by a macrovolume of a liquid contaminant. Explicit parametric equations of the sharp interface are obtained by inversion of the hodograph domain. Zhukovskii's gas-finger shape is shown to be a particular case of our new class of free surfaces. For a cap of a light liquid, partially covering the roof, from the given cross-sectional area of the cap, the affixes of the conformal mapping are found as a solution of a system of two nonlinear equations. The horizontal width and vertical height of the cap are determined. If the dimensionless incident velocity is higher than the density contrast, then the interface (cap boundary) cusps at its apex. For a relatively small velocity, the interface spreads to the vertices of the barrier, the apex zone remaining blunt shaped. We depict all the relevant domains and plot the flow nets using computer algebra routines.

Keywords: nonlinear free boundary problem; two-dimensional flow of heavy ideal fluid – seepage; holomorphic functions; hodograph; conformal mappings

1. Historical motivation from Zhukovskii's solution

There are very few analytical solutions to the problem of steady two-dimensional motion of an ideal irrotational heavy fluid with a free surface. This is caused by the necessity to satisfy the Bernoulli equation

$$V^2 + 2gy = \text{const.}, \qquad (1.1a)$$

along an unknown isobaric streamline (free surface). In equation (1.1a), V is the magnitude of the velocity vector $\boldsymbol{V} = V\mathrm{e}^{\mathrm{i}\theta}$, θ is the angle that $\boldsymbol{V}$ makes with the horizontal axis x, $g = 9.8\,\mathrm{m\,s^{-2}}$ and y is the vertical coordinate oriented upwards, against gravity.

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