211. Kinematic parameters of air molecules

According to kinetic molecular theory, gas is composed of multiple individual particles (molecules, atoms, ions). For the sake of simplicity, these particles can be regarded as absolutely rigid spheres. Observations of the Brownian motion allow suggesting that molecules are perpetually in motion. Large compressibility of gases is an evidence that their molecules are at a big (average) distance from each other. A question arises on estimating kinematic parameters of this phenomenon. They are: mean free path $<\lambda>$ (i.e., the distance that a molecule goes between subsequent collisions with other particles) and mean effective collision area $<\sigma> = \frac{1}{4}\pi<D>^2$ (where D is the maximal distance between the centres of colliding molecules at which their velocities change; it is the effective diameter of a molecule). Mean effective collision area is a quantitative parameter describing the intensity of collisions: the larger the collision area is, the more often collisions occur.

Aim of the work: determination of kinematic parameters of air molecules.

Tasks:

1. Using physical principles which correlate kinematic parameters with the viscosity of air.
2. Getting acquainted with the Poiseuille method of measuring the viscosity of a gas.
3. Measuring viscosity.
4. Estimating the mean free path, frequency, and effective area of collisions for air molecules.

Obviously, the mean frequency of collisions $<\omega>$ is proportional to the collision cross-section. If the concentration of molecules is n and the mean speed of the thermal motion of the molecules is $<u>$, then

$$<\omega> = \sqrt{2} <\sigma> <n> <u>. \qquad (1)$$

Additional factor allows for the fact that the mean relative speed of the molecules (with respect to each other) is $\sqrt{2}$ times larger that their mean speed $<u>$.

The mean free path $<\lambda>$ can be derived as the average distance at which a molecule moves per time unit (this value is actually the mean speed) divided by the number of collisions occurring during the same time:

$$\langle\lambda\rangle=\frac{\langle u\rangle}{\langle\omega\rangle}=\frac{1}{\sqrt{2}\langle\sigma\rangle\langle n\rangle}=\frac{2\sqrt{2}}{\pi\langle D\rangle^2 n} \quad . \qquad (2)$$

Thus, simple mechanistic concepts allows correlating microscopic parameter of gas with each other.

On the other hand, the correlation between the dynamical viscosity η and microscopic parameter can be found provided that the Maxwell's distribution of velocities takes place. In this case we can write down:

$$\eta = \tfrac{1}{3} \rho \langle \lambda \rangle \langle u \rangle, \qquad (3)$$

where ρ is the density of the gas.

In the equilibrium state the Maxwell's distribution is true, which predicts that the mean speed of molecules is

$$\langle u \rangle = \sqrt{\frac{8RT}{\pi\mu}} \ , \qquad (4)$$

where R is the gas constant and μ is the molecular weight of the gas.

Thus, we can express the mean free path as a function of macroscopic parameters:

$$\langle \lambda \rangle = \frac{3\eta}{\rho \langle u \rangle} = \frac{3\eta}{\rho} \sqrt{\frac{\pi\mu}{8RT}} \ . \qquad (5)$$

Thus, knowing the molecular weight of air and having measured its viscosity and temperature, we can calculate the free path <λ> using Eq. (5), and then estimate the mean collision area from Eq. (2).

Experimental setup

The idea of the experiment is based on the Poiseuille formula for the gas flow rate Q which passes as a laminar flow through a cylindrical tube of the radius r and length l driven by a constant pressure difference Δp at the ends of the tube:

$$. \ Q = \frac{\pi r^4}{8\eta l} \Delta p \qquad (6)$$

Thus, to determine the viscosity of gas, its volume flow rate through a capillary of a known length and inner radius should be measured together with the pressure drop, and then

$$\eta = \frac{\pi r^4}{8l} \frac{\Delta p}{Q} \ . \qquad (7).$$

Experiment is carried out on a setup drawn in figure 1. Capillary K is attached to the vessel Γ and a water manometer M. Water is released through a regulated valve B1 in order to create the needed pressure difference Δp on the ends of the capillary (the valve B2 should be closed). Created underpressure is monitored by the manometer M ($\Delta p = \rho_w g \Delta h$, where ρ_w is the density of water at the ambient temperature, g is the acceleration of gravity, and Δh is the difference of the water levels in the manometer). As a result, air will enter the system through the capillary and come into the vessel Γ. The volume of water flowing per time unit is determined by a gauge glass and a stopwatch; this value is equal to the volume of air passing through the tube K at the same time. Valve B2 with a funnel is used to return water from the gauge glass back to the vessel before a new measurement.

Algorithm of measurements

1. Put an auxiliary vessel E under the valve B1 and open the valve slowly to create the maximal possible pressure difference Δh in the manometer. Wait until the flow stabilizes and the reading from the manometer stop changing quickly; measure Δh_1. Do not touch the valve now and replace the vessel E by the measuring cup C. Measure the duration of flowing of a certain portion of water V and the pressure difference Δh_2 which will be shown by the manometer at the end of this process. Close the valve B1. Pour the water from the measuring cap and the auxiliary vessel back to the funnel inserted to the valve B2. Water can be returned from the funnel to the vessel Γ after the funnel is filled to half of its height.
2. Calculate the average pressure drop at the capillary's end during the process:
$$\Delta p = \rho_w g \frac{\Delta h_1 + \Delta h_2}{2} \quad .$$
3. Measure the temperature of water (or the room temperature) to find its density.
4. Calculate the flow rate of air as $Q = V/t$.
5. Repeat the measurements and calculations described in steps 1-2 with several (8 to 10) smaller values of Δp, so that finally you can plot the dependency $Q(\Delta p)$. The value of Q should correspond to the vertical axis; Δp, to the horizontal axis.
6. Analyze the obtained dependency $Q(\Delta p)$. Does it agree with the Poiseuille formula (6)? At the limit of small Δp's, the dependency should be linear, and $Q(0) = 0$. Otherwise, measurements should be repeated more accurately.
7. Using your experimental data and Eq. (7), calculate the average viscosity of air. The sizes of the capillary (l and r) are written on the apparatus.
8. Calculate the free path of air molecules using Eq. (5).
9. Measure atmospheric pressure with the aid of the barometer (if no barometer is present, take the value of mean atmospheric pressure).
10. Calculate the mean effective collision area from Eq. (2). Concentration (number of molecules per unit volume) can be found as $\langle n \rangle = p/(kT)$.
11. Estimate the diameter of an air molecule and the frequency of collisions between them.

Notes

1. Consider again step 1 in the experimental procedure. The volume V is set according to the desired accuracy of measurements. Let the accuracy level should be 10%. Time is measured manually using a stopwatch, and one cannot make it more exact than the human reaction time (~0.3 s) allows. This means, that the given relative error can be achieved if the duration of the process exceeds 3 s. Thus, you should pour water into the cup for 3 s or more.
2. Molecules are travelling at different speeds and thus it is difficult to determine the speed of each of the molecules. Instead, we can determine the fraction F of molecules that have the speed v given by the following equation:

$$f(v)=\sqrt{\left(\frac{m}{2\pi k T}\right)^3} 4\pi v^2 \exp\left(\frac{-mv^2}{2kT}\right) \quad .$$

   It is called Maxwell's distribution.

Questions

1. Correlation between microscopic and macroscopic parameters of gases. The basic equation of kinetic molecular theory.
2. How the phenomenon of viscous flow in gases is explained within the context of kinetic molecular theory?
3. How the viscosity of a gas depends on temperature and pressure?
4. How the free path and frequency of collisions depend on temperature and pressure?
5. Physical meaning of the effective collision cross-section.
6. How much is the average speed of thermal motion of air molecules at the room temperature? What is the frequency of collisions in these conditions?
7. Why the reading Δh from the manometer decreases as the liquid flows out of the vessel Г?