J_ID: WRCR Customer A_ID: WRCR23056 Cadmus Art: WRCR23056 Ed. Ref. No.: 2017WR021391 Date: 10-January-18 Stage: Page: 1

AGU PUBLICATIONS

Water Resources Research

TECHNICAL REPORTS: METHODS

10.1002/2017WR021391

Analytical Solution for Interface Flow to a Sink With an Upconed Saline Water Lens: Strack's Regimes Revisited

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Key Points:
- Finite-length interface between moving fresh and static saline water is obtained for 2-D and 3-D flow to sink
- Two inflexion points on the interface merge into a cusp at the tip of the interface when critical regime is attained
- Seemingly freshwater aquifers may unexpectedly derive saline/contaminated water from clandestine DNAPL lenses

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Citation:
Kacimov, A. R., & Obnosov, Y. V. (2018). Analytical solution for interface flow to a sink with an upconed saline water lens: Strack's regimes revisited. *Water Resources Research*, *54*. https://doi.org/10.1002/2017WR021391

Received 26 JUN 2017
Accepted 23 DEC 2017
Accepted article online 4 JAN 2018

Abstract A study is made of a steady, two-dimensional groundwater flow with a horizontal well (drain), which pumps out freshwater from an aquifer sandwiched between a horizontal bedrock and ponded soil surface, and containing a lens-shaped static volume of a heavier saline water (DNAPL-dense nonaqueous phase liquid) as a free surface. For flow toward a line sink, an explicit analytical solution is obtained by a conformal mapping of the hexagon in the complex potential plane onto a reference plane and the Keldysh-Sedov integral representation of a mixed boundary-value problem for a complex physical coordinate. The interface is found as a function of the pumping rate, the well locus, the ratio of liquid densities, and the hydraulic heads at the soil surface and in the well. The shape with two inflexion points and fronts varies from a small-thickness bedrock-spread pancake to a critical curvilinear triangle, which cusps toward the sink. The problem is mathematically solvable in a relatively narrow band of geometric and hydraulic parameters. A similar analytic solution for a static heavy bubble confined by a closed-curve interface (no contact with the bedrock) is outlined as an illustration of the method to solve a mixed boundary-value problem.

Plain Language Summary Since the Ghyben-Herzberg-Bear-Dagan-Strack-Polubarinova-Kochina works, standard conceptualizations of sharp interface flows assume a heavy saline fluid of infinite extent as in seawater intrusion problems. Then, if the pumping rate exceeds a critical value, upcresting, and breakthrough of saline water take place. Pumping should be either reduced or stopped for the interface to recede/flatten (hydrodynamic de-escalation). We examine a different situation: undesired DNAPL makes a finite-size lens, with no contact with the sea. If the pumping rate is small, DNAPLS spreads as a flat "pancake." At high rates, DNAPL becomes taller and slender and eventually invades the sink. We tracked how criticality-cusping-breakthrough is attained. We warn well operators that a sudden rise of salinity may emerge in seemingly benign hydrogeological situations of shallow aquifers with no seawater intrusion or deep saline water massifs. For instance, small undetected saline volumes close to the bedrock of pumped aquifers may occur due to clandestine illegal disposal (injection) of a reject brine from desalination plants or due to natural dissolution of salt domes. If a critical regime is attained then the "undesired" lens can be rapidly sucked from the aquifer by a surge of high-rate pumping (hydrodynamic escalation).