Features of exact calculation of EM fields of current carrying conductors by use of modified method of mirror images in view of electrical and magnetic properties of real media

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In references [1-4] we have solved the problem of exact calculation of the field of refraction of a spherical electromagnetic wave on flat boundary of section of media with due account of the electrical and magnetic properties and also conductivity of real physical media by use of the updating of the mirror images method developed by us. At this, we have obtained the exact solutions for all spectrum of frequencies of the EM field. Now, we represent the results of application of this technique for the problems when the field is admissible to consider as stationary one [5], and we can assume, that $\omega \to 0$. At $\omega = 0$ the field of elementary electrical vibrator is defined by expressions (see [5] and fig. 1):

$$H_\varphi = -\frac{I_э l \sin\Theta}{4\pi r^2}, \quad E_r = \frac{I_э l \cos\Theta}{\gamma_э 2\pi r^3}, \quad E_\Theta = \frac{I_э l \sin\Theta}{\gamma_э 4\pi r^3}. \tag{1}$$

Scalar potential and strength of an electric field of a charge have form:

$$\varphi = q/4\pi\varepsilon_a r, \quad E_r = q/4\pi\varepsilon_a r^2. \tag{2}$$

For case shown in fig. 1 the field is raised by a horizontally polarized wave. Using the modified method of mirror images consider now geometry of calculated fields and currents.

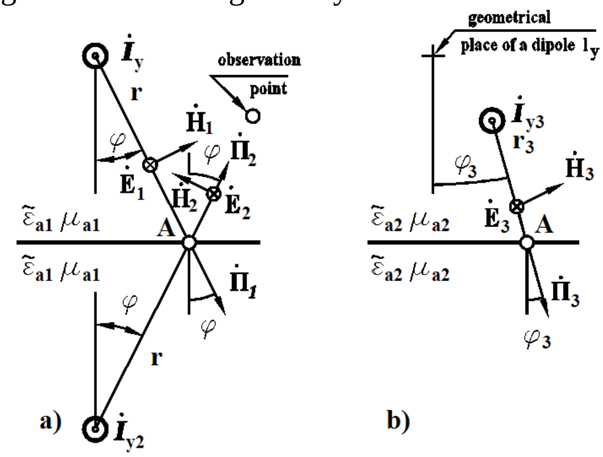

Fig. 1. Horizontally polarized wave:
a) top to a floor-space, b) bottom to a floor-space

For spherical horizontally polarized wave excited by the element of an alternating current (see fig. 1) using the results of [1-3] we obtain the following dependences between currents:

$$\dot I_{y2} = \frac{W_2\cos\varphi - W_1\cos\varphi_3}{W_2\cos\varphi + W_1\cos\varphi_3}\dot I_{y1}, \quad \dot I_{y3} = \frac{2W_1\cos\varphi}{W_2\cos\varphi + W_1\cos\varphi_3}\frac{\gamma_1^2}{\gamma_2^2}\dot I_{y1}, \tag{3}$$

and geometrical expressions:

$$r_1/r_3 = \gamma_2/\gamma_1, \quad \sin\varphi/\sin\varphi_3 = \gamma_2/\gamma_1 \tag{4}$$

where $W_1 = \sqrt{\mu_{a1}/\tilde\varepsilon_{a1}}$ and $W_2 = \sqrt{\mu_{a2}/\tilde\varepsilon_{a2}}$ in expressions (3) are the wave resistances of media.

At $\omega = 0$ these expressions take form

$$I_{y2} = G_3^{-1}[G_2\cos\varphi - G_1\cos\varphi_3]I_{y1}, \quad I_{y3} = G_3^{-1}2G_1\cos\varphi\frac{\gamma_{э1}\mu_{a1}}{\gamma_{э2}\mu_{a2}}I_{y1};$$
$$G_1 = \sqrt{\mu_{a1}/\gamma_{э1}}, \; G_2 = \sqrt{\mu_{a2}/\gamma_{э2}}, \; G_3 = G_2\cos\varphi + G_1\cos\varphi_3; \tag{5}$$

$$r_1/r_3 = \sqrt{\gamma_{э2}\mu_{a2}/\gamma_{э1}\mu_{a1}}, \quad \sin\varphi/\sin\varphi_3 = \sqrt{\gamma_{э2}\mu_{a2}/\gamma_{э1}\mu_{a1}}. \tag{6}$$

The calculation algorithm is the following:

1) field of the element of current is calculated using expressions (1), (2);

2) values of I_2, I_3, r_3 and φ_3 for all points on the boundary of section of media are defined using formulas (5), (6);

3) field intensity in any point of space is calculated using extrapolation of a solution obtained for boundary of section of media following to procedure of a method of mirror images.

The pictures for electrical and magnetic fields obtained as a result of calculation are shown in figs. 2 and 3.

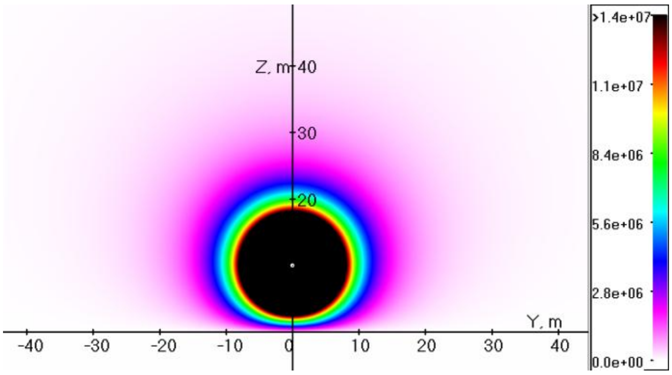

Fig. 2. Electrical field |**E**|. Top to a floor-space – air. Current is at height 10 m. Current intensity I_1= 1A, l=0,1 m

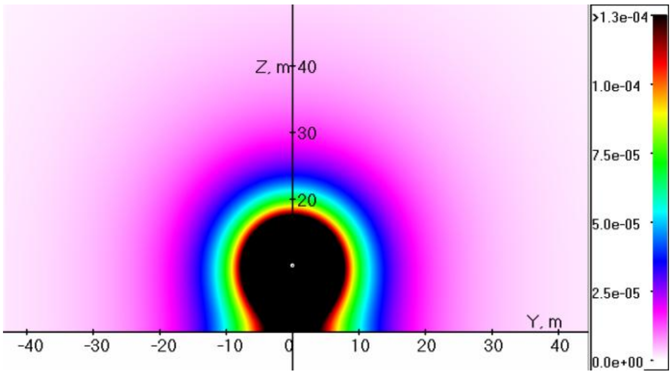

Fig. 3. Magnetic field |**H**|

It is necessary to pay attention to that, with direct using of procedure described, the strength of an electric field calculated is abnormally high (see fig. 2). It is connected by that the equations for Hertz dipole were written for ideal calculation scheme. Let us analyze known formula [6] $\dot E = (i\omega\tilde\varepsilon_a)^{-1}\,\mathrm{rot}\,\dot{\mathbf{H}}$. It is obvious, that $i\omega\tilde\varepsilon_a \dot E = \mathrm{rot}\,\dot{\mathbf{H}}$, that is identical $j = \mathrm{rot}\,\dot{\mathbf{H}}$ where j is the current density.

In case of when the current proceeds on a wire it is necessary to take parameters of a material of a wire, instead of environments. According to that, dependences (1) will take the form of

$$\dot E_r = \frac{\dot I_э e^{-i\gamma r} l\cos\Theta}{i\omega\tilde\varepsilon_{\text{апр}} 2\pi r^3}(1+i\gamma r), \quad \dot E_\Theta = \frac{\dot I_э e^{-i\gamma r} l\sin\Theta}{i\omega\tilde\varepsilon_{\text{апр}} 4\pi r^3}\left(1+i\gamma r-\gamma^2 r^2\right)$$

or

$$\dot E_r = \frac{\dot E_\tau e^{-i\gamma r} V\cos\Theta}{2\pi r^3}(1+i\gamma r), \qquad \dot E_\Theta = \frac{\dot E_\tau e^{-i\gamma r} V\sin\Theta}{4\pi r^3}\left(1+i\gamma r-\gamma^2 r^2\right),$$

where $\dot E_r = \dot U$ is voltage; V is the conductor volume. At $\omega = 0$ we obtain

$$E_r = \left(2\pi r^3\right)^{-1} E_\tau V\cos\Theta = \left(\gamma_{\text{эпр}} 2\pi r^3\right)^{-1} I_э l\cos\Theta, \quad E_\Theta = \left(4\pi r^3\right)^{-1} E_\tau V\sin\Theta = \left(\gamma_{\text{эпр}} 2\pi r^3\right)^{-1} I_э l\sin\Theta,$$

that quite corresponds to classical representations.

Let us consider now how it will affect final dependences.

Dependences between electrical and magnetic components, and also between currents take form: $\dot{W}_{\Theta} = \dfrac{\dot{E}_{\Theta}}{\dot{H}_{\varphi}} = -\dfrac{1+i\gamma r-\gamma^2 r^2}{i\omega\tilde{\varepsilon}_{a\,np} r(1+i\gamma r)}$, and

$$\dot{I}_{y2} = \frac{\gamma_2\cos\varphi-\gamma_1\cos\varphi_3}{\gamma_2\cos\varphi+\gamma_1\cos\varphi_3}\dot{I}_{y1}, \qquad \dot{I}_{y3} = \frac{2\gamma_1\cos\varphi}{\gamma_2\cos\varphi+\gamma_1\cos\varphi_3}\frac{\gamma_1^2}{\gamma_2^2}\dot{I}_{y1}. \tag{7}$$

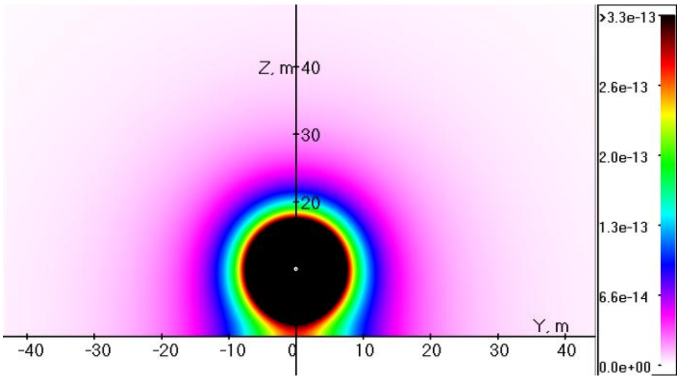

Fig. 4. Electrical field |**E**|. Top to a floor-space – air. Bottom to a floor-space – the ground. Current is at height 10 m. Current intensity I_1= 1A, l=0,1 m

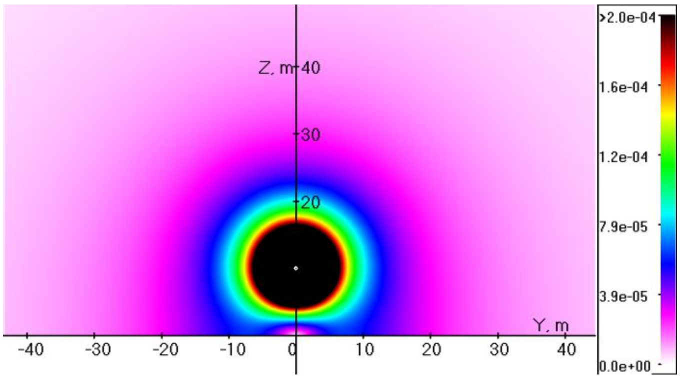

Fig. 5. Magnetic field |**H**|

As consequence, dependences between currents for stationary electromagnetic field become following:

$$I_{y2} = \frac{\sqrt{\gamma_{э2}\mu_{a2}}\cos\varphi-\sqrt{\gamma_{э1}\mu_{a1}}\cos\varphi_3}{\sqrt{\gamma_{э2}\mu_{a2}}\cos\varphi+\sqrt{\gamma_{э1}\mu_{a1}}\cos\varphi_3}I_{y1}, \quad I_{y3} = \frac{2\sqrt{\gamma_{э1}\mu_{a1}}\cos\varphi}{\sqrt{\gamma_{э2}\mu_{a2}}\cos\varphi+\sqrt{\gamma_{э1}\mu_{a1}}\cos\varphi_3}\frac{\gamma_{э1}\mu_{a1}}{\gamma_{э2}\mu_{a2}}I_{y1}.$$

The dependences between radius-vectors and angles will not change. As a result, instead of the pictures of fields represented in figs. 2 and 3, we have obtained the results shown in figs. 4 and 5.

In conclusion let us note the following.

1. At comparison of figs. 2 and 4 it is evident, that strength of an electric field differs in 10^{20} times. It is explained that in the first case the current flows in a dielectric (air), and in the second case it flows in a conductor.
2. At the analysis of figs. 3 and 5 it is visible, that character of a field corresponds to field calculated for a case when the segment of a wire is located above the ferromagnetic plane. In our case the plane is not ferromagnetic, but only spending. This effect is explained that in an offered method we consider not only magnetic properties of media, but also spending ones.
3. It is necessary to calculate the field of a conductor with alternating current under the formulas considering the material of conductor, namely: (1), (2), (5), fictitious currents under (7) using geometrical expressions (4).

References

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piece of a conductor with an alternating current above a semiconductive plane", Proc. Institute of Higher Education. Problems of Energy, 2009, № 7-8, pp. 82-93.
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