122. Determination of the moment of inertia of a flywheel using oscillations

Tasks to solve

- Getting acquainted with the method of measuring the moment of inertia of a body using oscillations.
- Learning the Steiner's theorem.
- Determining the moment of inertia of a flywheel.

Instruments

Flywheel on a mounting	1
Additional body	1
Timer	1
Calipers	1
Balances	1

Description of the experiment

The experimental setup (Fig. 1) is a massive flywheel which can rotate with a small friction around a horizontal axis. The axis passes through the centre of mass of the wheel, and therefore the wheel is in the state of indifferent equilibrium. If an additional body is fastened on the rim of the wheel, then a position of stable equilibrium appears. If the system is disturbed from this equilibrium positon by rotating at an angle α_m and then released, the wheel will begin to oscillate with a period T. If the angle α_m is small, the oscillation can be regarded as harmonic oscillations following the formula $\alpha = \alpha_m \sin(\omega_0 t)$.

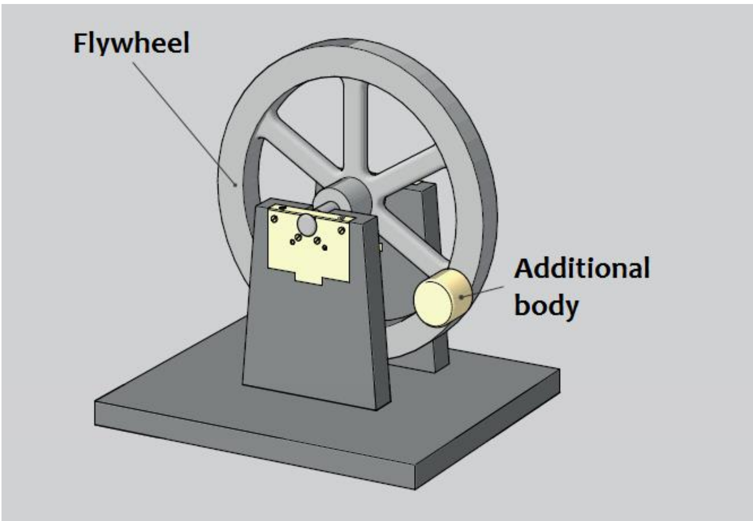

Figure 1.

When the disturbed system passes through the equilibrium position, its angular speed attains the maximal value of $\alpha_m \cdot \omega_0$, and hence the maximal kinetic energy is

$$E_m = \frac{I\alpha_m^2\omega_0^2}{2}.$$

The moment of inertia of the system I includes the moment of inertia I_w of the flywheel itself and of the additional body I_b.

On the other hand, the potential energy of the system is $E = mgh$, where m is the mass of the additional body and h is the height by which it is raised from the equilibrium position. Geometrical consideration in Fig. 2 shows that

$$h = d - d\cos\alpha = 2d\sin^2\frac{\alpha}{2},$$

where d is the distance from the centre of the wheel to the centre of mass of the additional body.

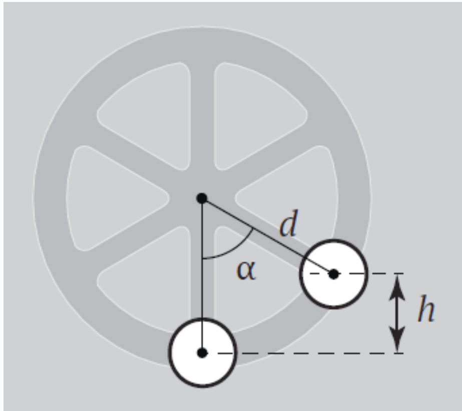

Figure 2.

In the case of small-amplitude oscillations (only such oscillations can be regarded as harmonic in our case) we can use the approximation sinα ~ α[1]. If friction is negligible we can assume that the maximal values of the potential and kinetic energies are equal (based on the law of conservation of energy). If ω_0 is rewritten using the value of the period, we get the following expression for the moment of inertia of the flywheel:

$$I_w = I - I_b = mgd\frac{T^2}{4\pi^2} - I_b. \qquad (1)$$

The values on the right side of Eq. (1) can be measured directly (g is a known constant), and the value of I_b can be found using the Steiner's theorem:

$$I_b = I_0 + md^2. \qquad (2)$$

Here I_0 is the moment of inertia of the additional body with respect to the axis passing through the body's centre of mass and parallel to the system's (wheel's) axis of rotation. In this work the body of cylindrical shape is used which has the moment of inertia

$$I_0 = mR^2/2, \qquad (3)$$

where R is the radius of the cylinder.

Algorithm of measurements

1. Unscrew the additional body and measure its mass.
2. Find the diameter $2R$ of the body and the distance d (Fig. 2).
3. Using Eqs. (3) and (2), calculate the moments of inertia of the additional body with respect to its symmetry axis (I_0) and to the wheel's rotation axis (I_b).
4. Attach the body to the rim of the wheel.
5. Deflect the wheel by a little angle and release it. The wheel will oscillate.
6. Determine time t corresponding to as large as possible number of oscillations (N). Find the average period of one oscillation: $T = t/N$.
7. Repeat steps 5–6 several times (at least 5; the amplitude angles α_m will inevitably be slightly different) and find the mean period value $\langle T \rangle$.
8. Calculate the moment of inertia of the wheel using Eq. (1). Estimate the inaccuracy of the experiment.

[1] Angles should be expressed in radians (which are in fact a dimensionless measure).

Questions

1. Write down the equation of motion of the flywheel with the plummet (body on the rim), estimate at which angles the solution of this equation can be assumed to be a harmonic function within the level of accuracy of the instruments in this work.
2. Derive the formula (1).
3. How the friction force influences the accuracy of measurements? Which features of the experimental setup allow neglecting the friction force?
4. How the moment of inertia and mass of the additional body influence the accuracy of measurements? Which requirements should meet the body?
5. Compare the method described here with other methods of measuring the moment of inertia that you know.
6. Plot approximate dependences of the angular coordinate, angular speed, angular acceleration of the wheel with the body on time.