Integral Equation Method in the Theory of Dielectric Waveguides

E. M. Karchevskiy
Kazan State University, Russia

Abstract— The eigenvalue problems for generalized natural modes of dielectric waveguides are formulated as problems for the set of time-harmonic Maxwell equations with partial radiation conditions (Sveshnikov radiation conditions) at infinity in the cross-sectional plane. All of the known natural-mode solutions (i.e., guided modes, leaky modes, and complex modes) satisfy the partial radiation conditions at infinity. The generalized eigenvalues of these problems are the complex propagation constants β on a logarithmic Reimann surface. The partial radiation conditions in these problems are connected with the fact that wavenumbers may be complex. For real wavenumbrs on the principal ("proper") sheet of this Reimann surface, one can reduce the partial radiation conditions to either the Sommerfeld radiation condition or to the condition of exponential decay. The partial radiation conditions may be considered as a generalization of the Sommerfeld radiation condition and can be applied for complex wavenumbers. These conditions may also be considered as the continuation of the Sommerfeld radiation condition from a part of the real axis of the complex parameter to the appropriate logarithmic Reimann surface.

The generalized natural modes of an inhomogeneous dielectric waveguide without a sharp boundary and a step-index dielectric waveguide with smooth boundary of cross-section are considered. The original problems by integral equation method are reduced to nonlinear spectral problems with Fredholm integral operators. Theorems on spectrum localization are proved, and then it is proved that the sets of all eigenvalues of the original problems can only be some sets of isolated points on the Reimann surface, ant it also proved that each eigenvalue β depends continuously on the frequency and refraction index and can appear and disappear only at the boundary of the Reimann surface. The Galerkin methods for numerical calculations of the generalized natural modes are proposed, and the convergence of the methods is proved. Some results of numerical experiments are discussed.