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Use of long-term nongravitational force models for fitting astrometric observations of comet Encke

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Abstract

Based on the equations derived in (Usanin et al., 2016) a new solution combining the observations of 30 apparitions of the comet Encke from 1911 to 2010 is obtained. For the first time in the worldwide practice the solution is obtained by using converging differential correction of all 60 observed returns of the comet, however, the deviations are still unsatisfactory. The single solution has allowed to draw some preliminary conclusions. The contributions of planetary and nongravitational perturbations to the change of the elements of the orbit during the entire period of observation are determined. The extrapolation of the solution shows that for the past two thousand years the elements of the orbit orientation could change for a half of turnover, which should be taken into account when identifying the comet and associated meteor showers in ancient records. The predictions made by Z. Sekanina and I. Ferrín about oncoming termination of the comet activity are confirmed.

Keywords: Comets; Celestial mechanics; Nongravitational forces; Marsden's model; 2P/Encke

1. Processing of observations of the comet Encke: dynamical model and weights determination

The data on the comet Encke ground-based optical astrometric observations from 1881 was taken from the "MPCOBS" base of the Minor Planet Center (Marsden et al., 1993). The information on the observations taken from 1786 to 1961 was provided at our request by B.G. Marsden who noticed that he could guarantee neither reliability of the observations nor their conformity with what the observers had originally published (Marsden, personal communication, 2010). The observations used in the present work had been already converted to the equator and equinox of J2000.0 and were represented according to the standard format of the Minor Planet Center by Marsden himself, therefore we could use them during the calculations on a par with the other ones without introducing any additional reductions. The orbital elements obtained earlier (Marsden and Sekanina, 1974) from these observations have not been reconsidered until now and are reproduced in all comet catalogues, therefore, we may certainly be sure that this data is as reliable as possible. From each pair of identical records one was deleted. Thus, the data on 2695 observations from 60 apparitions of the comet Encke from 1786 to 2010 (except 1868) was collected.

A set of computer codes were created for observations processing according to the derived equations (Usanin et al., 2016). Theoretical positions for the comet were calculated by numerical integration of the equations of motion with relativistic term and nongravitational effects in two options: with model of significant non-volatile mass without impeding the sublimation and the low-mass crust

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that reduces effective area linearly in thickness – without simplification. To integrate the equations of motion the Everhart method with nodes of 35th order and constant step of 1 day was used, while for solving equations of nongravitational effects the Radau quadratures were applied. Gravitational attractions of the Sun, major planets, the Moon, Ceres, Pallas, Vesta, Pluto, and Charon were taken into account. Initial positions in the J2000.0 ecliptic coordinate system and masses of the gravitating bodies were taken from the JPL NASA "HORIZONS" service (Giorgini et al., 1996), but the origin of coordinates was shifted to their barycenter according to the relativistic formula. The differences between TDB and UT time scales, diurnal parallax (position of an observer influenced by precession and nutation of the Earth's axis of rotation), planetary aberration (light-time), and differential relativistic deflection of light caused by the gravity of the Sun were taken into account when comparing theoretical and observed positions of the comet. The motion parameters of the comet and their root-mean-square deviations were determined by the weighted least squares method using differential correction with damping. At all stages of the development the software was checked by solving control examples. The error in closure after integrating the motion equations of the comet Encke from 2011 to 1785 and inversely was 6.9 km. The total integrating error could be a few tens of km, which is acceptable for the considered problem.

Since over the period of 225 years during which the comet Encke was observed the accuracy of observations dramatically increased, it is incorrect to combine significant number of the comet apparitions without determining observations' weights. Usually, the values which are inversely proportional to the squares of a priori root-mean-square deviations σ_{apr} of observations are accepted as weights. It is clear that they should be defined so that they do not depend on the model of nongravitational force, but at the same time according to the sample as representative as possible. Since the nongravitational forces are significant near the perihelion, division into groups containing halves of the number of observations from pairs of consecutive apparitions satisfies these conditions. The first and the last apparitions are included in the corresponding groups completely. Apparently erroneous observations incompatible by 3σ criterion with the other ones are revealed and removed. According to the results of differential correction in 59 groups, 145 observations are eliminated, for the rest 2550 the values of σ_{apr} are determined and presented in Table 1. The values in Table 1, although differ in figures due to distinctions in sets of observations, are consistent with the data on the corresponding time periods from the earlier works (Marsden and Sekanina, 1974; Giorgini et al., 1996). It should be noted that in the modern epoch the accuracy of astrometric observations of comets is much lower than of asteroids, for instance. These errors arising due to the diffuse appearance of comets are usually considered to be random. The question of systematical offset of photocenters relative to centers of mass of comets remains debatable (Chesley et al., 2001).

Table 1
A priori root-mean-square deviations in the groups of observations.

Interval	Observations eliminated	Observations left	σ_{apr}, ″
2007–2010	13	184	1.69
2003–2007	104	589	0.68
2000–2003	15	552	0.72
1997–2000	1	86	1.56
1994–1997	1	130	2.31
1990–1994	1	83	2.16
1987–1990	0	29	1.93
1984–1987	0	33	1.51
1980–1984	0	30	4.61
1977–1980	1	27	5.83
1974–1977	3	19	3.31
1971–1974	0	19	2.97
1967–1971	0	15	3.70
1964–1967	0	9	2.28
1961–1964	1	18	1.07
1957–1961	2	22	1.68
1954–1957	0	13	1.73
1951–1954	0	11	2.26
1947–1951	0	13	3.71
1941–1947	0	8	3.58
1937–1941	0	12	2.34
1934–1937	2	24	2.30
1931–1934	0	18	5.14
1928–1931	0	10	2.43
1924–1928	0	28	3.33
1921–1924	0	27	6.80
1918–1921	0	16	8.78
1914–1918	0	11	4.35
1911–1914	0	6	1.36
1908–1911	0	4	1.40
1905–1908	0	11	1.89
1901–1905	1	21	5.69
1898–1901	0	14	13.0
1895–1898	0	20	3.72
1891–1895	0	28	3.66
1888–1891	0	12	3.17
1885–1888	0	25	3.75
1881–1885	0	29	7.60
1878–1881	0	10	4.32
1875–1878	0	11	3.60
1871–1875	0	14	3.26
1865–1871	0	19	3.12
1862–1865	0	20	2.67
1858–1862	0	14	3.17
1855–1858	0	21	2.75
1852–1855	0	46	3.08
1848–1852	0	34	3.30
1845–1848	0	8	5.75
1842–1845	0	12	6.67
1838–1842	0	17	6.65
1835–1838	0	9	8.76
1832–1835	0	5	5.31
1829–1832	0	16	7.02
1825–1829	0	30	7.45
1822–1825	0	20	8.56
1819–1822	0	11	10.1
1805–1819	0	11	11.5
1795–1805	0	8	28.4
1786–1795	0	8	33.4

2. Combining two periods containing 30 apparitions each

Determination of weights allowed combining a large number of apparitions. Using the derived equations for the model with crust we could not achieve the differential correction convergence. All the further converging solutions were obtained according to the equations for the model with significant nonvolatile mass (Usanin et al., 2016).

It should be noted that in every case we assess errors of the motion parameters according to the standard practice using only diagonal elements of the covariance matrix. It is well known that, actually, all the motion parameters are correlated with one another to some degree, and therefore, their true uncertainty may be several times larger than the formal accuracy (Whipple and Sekanina, 1979).

By means of weighted differential correction and using 2045 observations of the comet Encke from 30 apparitions during the interval 1911–2010 the following motion parameters for November 15, 2011 and their root-mean-square deviations have been determined and are listed in the Table 2: barycentric ecliptic (J2000.0) coordinates and velocity projections, Marsden's parameters, and our two additional non-gravitational parameters (Usanin et al., 2016).

Heliocentric ecliptic orbital elements corresponding to coordinates and velocity are $\omega = 186.5489962°$, $\Omega = 334.5719653°$, $i = 11.7772718°$, $e = 0.848144208$, $a = 2.214307537$ au, $M = 139.2709904°$. Residual deviations of the observed positions (expressed in terms of angular ecliptic coordinates λ and β) from the positions calculated by the model are shown in Fig. 1. The a posteriori root-mean-square deviations is $\sigma_{apo} = 4.66''$, its ratio to a priori root-mean-square deviations is $\sigma_{apo}/\sigma_{apr} = 1.72$.

Similarly, the solution by 505 observations from 30 apparitions during the period 1786–1908 has been obtained, the parameters for March 30, 1910 and their root-mean-square deviations are represented in Table 3. Heliocentric ecliptic orbital elements are: $\omega = 184.627381°$, $\Omega = 335.770477°$, $i = 12.597127°$, $e = 0.84744689$, $a = 2.21608697$ au, $M = 208.388377°$. Residual deviations of the comet's positions are given in Fig. 2. $\sigma_{apo} = 160''$, $\sigma_{apo}/\sigma_{apr} = 10.3$.

Table 2
Motion parameters of the comet Encke for JD 2455880.5 TDB obtained by 2045 observations from 30 apparitions during 1911–2010.

Parameter	Value $\pm$ 1σ
x, au	$+3.56811198 \pm 0.00000047$
y, au	$-1.69289905 \pm 0.00000069$
z, au	$+0.00117817 \pm 0.00000093$
V_x, au/day	$+0.00332508351 \pm 0.00000000135$
V_y, au/day	$+0.00213873817 \pm 0.00000000055$
V_z, au/day	$+0.00070091890 \pm 0.00000000045$
A_1	-0.0025476 ± 0.0000870
A_2	$-0.00033515 \pm 0.00000389$
A_3	-0.009929 ± 0.000866
α, day^{-1}	(1.9235 ± 0.0788) ṡ 10^{-5}
χ	0.03772 ± 0.00156

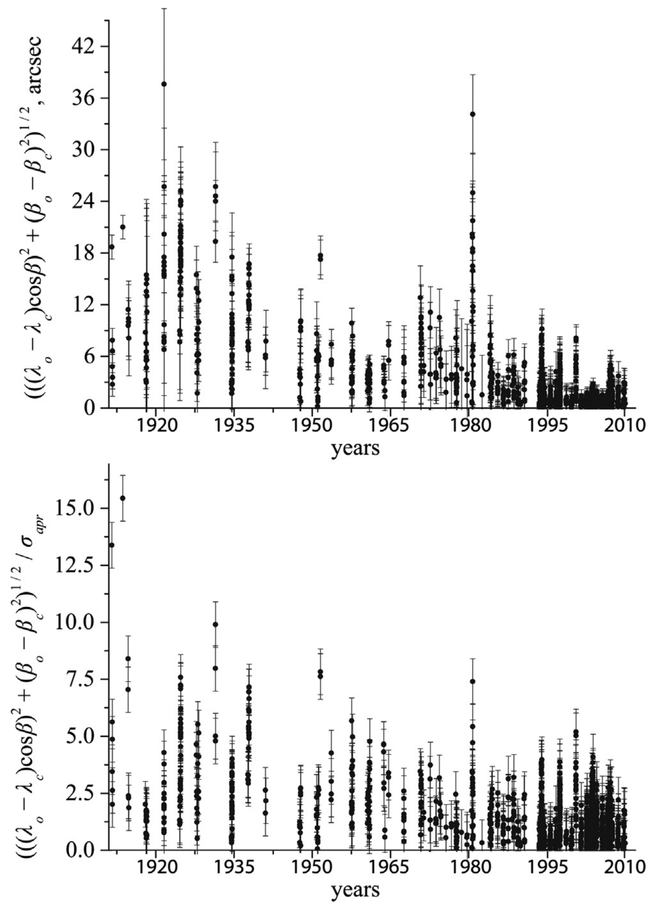

Fig. 1. Residual deviations of the observed positions from the ones calculated according to parameters from Table 2 in arcseconds and units of σ_{apr}.

Table 3
Motion parameters of the comet Encke for JD 2418760.5 TDB obtained by 505 observations from 30 apparitions during 1786–1908.

Parameter	Value $\pm$ 1σ
x, au	$+3.8831664 \pm 0.0000155$
y, au	-1.0662166 ± 0.0000594
z, au	$+0.1387856 \pm 0.0000675$
V_x, au/day	$-0.0005357087 \pm 0.0000000605$
V_y, au/day	$+0.0035715925 \pm 0.0000000287$
V_z, au/day	$+0.0006776533 \pm 0.0000000629$
A_1	$+0.0452 \pm 0.0110$
A_2	-0.014490 ± 0.000114
A_3	$+0.4075 \pm 0.0796$
α, day^{-1}	(5.6305 ± 0.0391) ṡ 10^{-5}
χ	0.45187 ± 0.00207

3. The solution for all 60 apparitions and its consequences

Converging solution was found for 2550 observations of the comet Encke from 60 apparitions during the period 1786–2010 during which it made 68 revolutions around the Sun. Residual root-mean-square deviations has turned out to be fairly large, so this solution cannot be considered a complete theory of motion. It should be noted that its physical meaning is limited – it can be applied for one kind

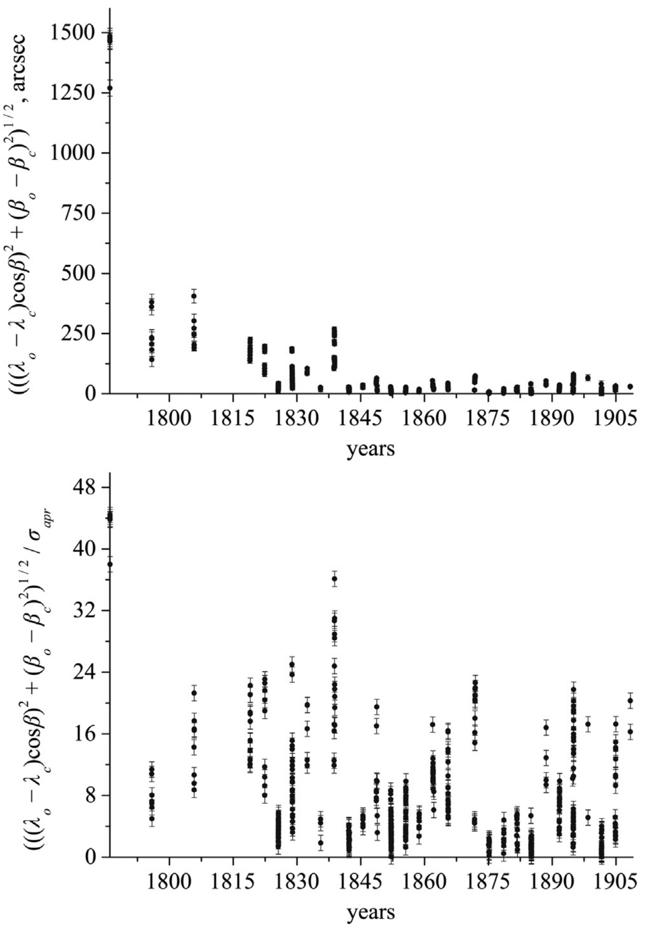

Fig. 2. Residual deviations of the observed positions from the ones calculated according to parameters in Table 3 in arcseconds and units of σ_{apr}.

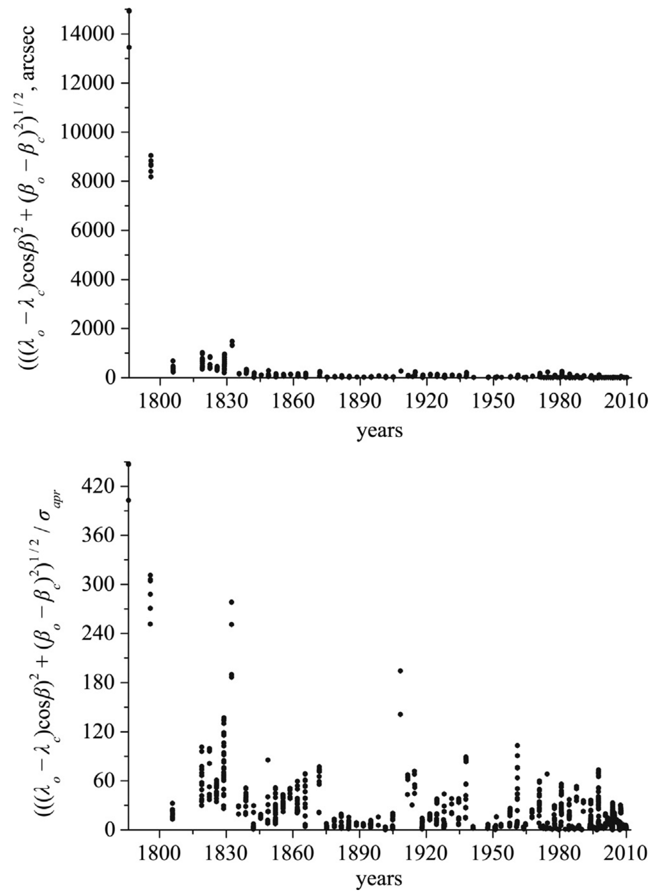

Fig. 3. Residual deviations of the observed positions from the ones calculated according to parameters from Table 4 in arcseconds and units of σ_{apr}.

of problem and cannot be useful for the other ones. The parameters for November 15, 2011 and their root-mean-square deviations are given in Table 4. Heliocentric ecliptic orbital elements are: $\omega = 186.546116°$, $\Omega = 334.573509°$, $i = 11.777901°$, $e = 0.84813537$, $a = 2.21431406$ au, $M = 139.266667°$. Residual deviations of the comet's positions are given in Fig. 3. Root-mean-square deviations is $\sigma_{apo} = 804''$, $\sigma_{apo}/\sigma_{apr} = 35.0$. Calculated in terms of the accuracy for observations taken in 2003–2007, it is equal to $(\sigma_{apo}/\sigma_{apr}) \times \sigma_{apr\ 2003\text{–}2007} = 24.0''$. The value of osculating time of perihelion passage differs from the latest NASA "HORIZONS" model by 0.0211 day. Formal errors of the parameters α and χ have decreased compared with the solution for 30 apparitions. Change of A_2 for this solution is shown in Fig. 4 in comparison with the exponent from Marsden's earlier hypothesis (Marsden, 1969).

Table 4
Motion parameters of the comet Encke for JD 2455880.5 TDB obtained by 2550 observations from 60 apparitions during 1786–2010.

Parameter	Value $\pm$ 1σ
x, au	$+3.5680125 \pm 0.0000067$
y, au	-1.6930206 ± 0.0000107
z, au	$+0.0011242 \pm 0.0000175$
V_x, au/day	$+0.0033254299 \pm 0.0000000147$
V_y, au/day	$+0.0021386169 \pm 0.0000000060$
V_z, au/day	$+0.0007009540 \pm 0.0000000089$
A_1	-0.002910 ± 0.000248
A_2	-0.0002749 ± 0.0000111
A_3	-0.01156 ± 0.00185
α, day^{-1}	(2.8226 ± 0.0107) ṡ 10^{-5}
χ	0.047556 ± 0.000925

Marsden's parameters determined by the intervals with inclusion of modern observations of 30 and 60 apparitions have some common features. In both solutions A_3 is the largest by absolute value and negative. However, as it has been expected from the laws of celestial mechanics, its influence is the least. A_1 has turned out to be negative as well. This fact is quite difficult to explain within the Marsden's model (it is should be taken into account that sublimation is higher on the night side of the nucleus). However, the majority of A_1 values obtained by B.G. Marsden and Z. Sekanina with different sets of observations of the comet Encke are negative and their absolute values are greater than positive ones (Marsden and Sekanina, 1974). Yu.A. Chernetenko found out when taking into account the displacement of the comet Encke photocenter relative to its center of mass the A_1 absolute value

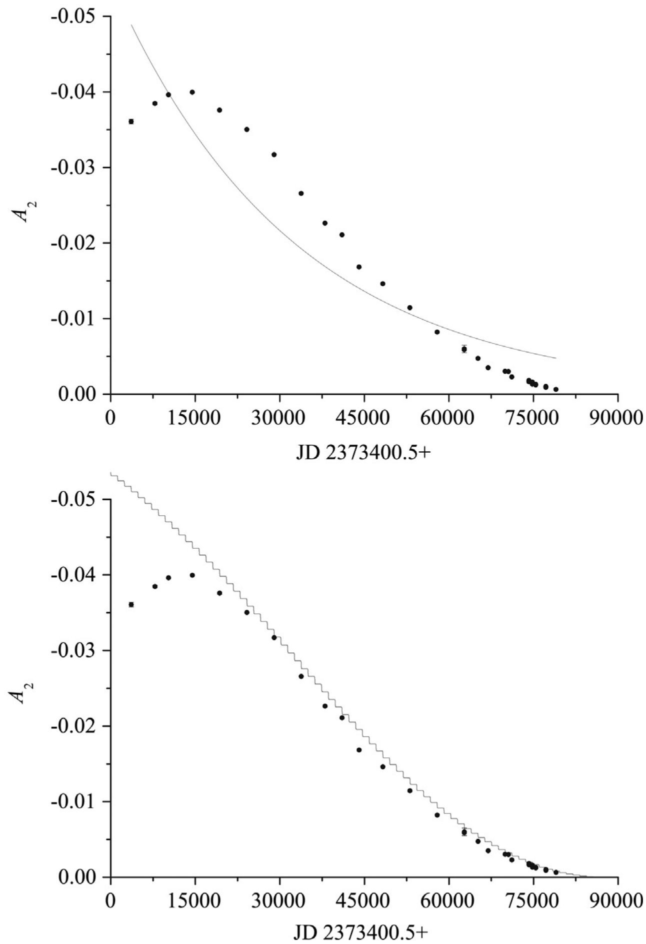

Fig. 4. At the top: approximation of the parameter A_2 change by exponent according to Marsden's earlier hypothesis (calculations made on "Origin 6.1" software, OriginLab Corporation, Marsden's parameters for the epoch JD 2373400.5 are $A_2 = -0.05472$, $B_2 = +0.30905$). At the bottom: the A_2 parameter change obtained by numerical integration of our equations for the model with significant nonvolatile mass and parameters from Table 4.

decreased, but still remained negative (Chernetenko, 1992). Z. Sekanina noted that the theoretically negative A_1 values could be a result of averaging, when the solution was searched according to the standard Marsden's model, but in reality, nongravitational force acts asymmetrically relative to the perihelion due to the presence of several separate sublimation sources on the nucleus (Sekanina, 1993), for instance. Nevertheless, so far there are no known calculations confirming by the method of differential correction any positive values of A_1 for the comet Encke in the asymmetrical model of nongravitational forces for those apparitions in which the standard model definitely gives its negative values. A_2 was found to be the least by absolute value and negative, which coincided with the published data. But its absolute value at the epoch of osculation was greater than it had been expected from the formal solution. It may be explained, since in this model the comet's ablation is taken into account continuously and the fastest ablation is near the perihelion. Consequently, nongravitational accelerations on the descending and ascending branches of the orbit are a bit asymmetric in our model, which leads to a notable resulting acceleration caused by the parameter A_1, which is absent in the standard model. In the present case, the effect of A_1 is opposite to the effect of A_2, so A_2 should be increased by absolute value compared to the standard model. The parameters of the solution for 30 apparitions without inclusion of the modern observations significantly differ. Although the A_2 parameter is still the smallest by absolute value, A_1 is intermediate, and A_3 is the largest, the last two turn out to be positive, but their errors are significant. Because of the opposite A_1 sign in this solution, A_2 is expectedly less than it follows from the formal solution for the same epoch.

Longer-term though less accurate solution has several advantages, like in some tasks average orbital elements of planets have, but does not reflect the accuracy of observations. This solution is more suitable for extrapolation, since the orbits of the comet Encke obtained from less number of observations beyond their periods lead to significant deviations (Marsden, 1969). Only a single solution will allow estimating the contribution of nongravitational perturbations to the total change of the comet's orbital elements, because the transition from one particular solution to another leads to the discontinuities in all parameters, including the orbital elements and the coordinates which cannot be considered nongravitational or gravitational perturbations (they cannot even have a physical meaning because in reality the space coordinates of any physical body only change continuously with time). After the development of more accurate theories the calculations carried out according to this model as an example should be repeated.

Equations of motion with considering gravitational and nongravitational forces and equations for nongravitational parameters with values for the comet Encke taken from Table 4 were integrated up to epoch JD 2373360.5 (December 9, 1785). The calculated time of perihelion passage differs from the one observed in 1786 by 1.36 day (in many models by NASA differences were tens of days). After that this set of parameters was recalculated back to November 15, 2011 in three ways. In the first way mean and true anomalies and time of perihelion passage were recalculated according to Kepler's formulas, the other elements were unchanged ("unperturbed orbit"). In the second way the equations of motion were integrated without nongravitational effects ("orbit with planetary perturbations"). The third way implied integrating equations of motion with both gravitational and nongravitational effects ("orbit with complete perturbations"). The differences between the elements from the second and the first options can be called "planetary perturbations", between the third and the second – "nongravitational perturbations" for the entire observation period (of course, within the accepted model of nongravitational forces). The differences between the third and the first options we consider "complete perturbations" and the sum of absolute values of planetary and nongravitational perturbations will be called "summated perturbations". The corresponding values are given in Table 5. The difference between initial parameters for November 15, 2011 and ones obtained in the third way is an error in closure for the comet Encke during the entire period of its observation characterizing the accuracy of integrator (see above).

Although the comet Encke is known for the presence of strong nongravitational effects and absence of close approaches to planets, it has turned out that nongravitational perturbations significantly prevail over planetary ones only in aphelion distance Q. They are comparable by magnitude in eccentricity e, perihelion distance q, true and mean anomalies v and M, and in the time of perihelion passage T. The values of planetary and nongravitational perturbations in the perihelion distance differ just in hundredths of percent and their signs are opposite resulting in the fact that this element has been almost unchanged since 1785 up to the present time. The planetary perturbations significantly prevail in the major semi-axis of the orbit a and orbital period P. Nongravitational effects contribute only a few percent to perturbations of the orbit orientation elements ω, Ω, i. The least impact they have on the longitude of perihelion π.

For determination of the comet's origin integration of its equations of motion backwards in time is required. The obtained parameters cannot yet allow studying the motion of the comet Encke at large time periods with the required accuracy, but formal extrapolation with considering gravitational and nongravitational perturbations already enables to draw some preliminary conclusions. Based on its results, for the past 2000 years there has been no closer approaches to Jupiter than up to 0.78 au (in the modern time the comet may approach Jupiter up to 0.90 au). Considering that aphelion distance decrease with time is mainly caused by nongravitational acceleration and the fact that in this model it is likely overestimated, the largest value of aphelion distance 4.3 au obtained during the extrapolation should be considered overestimated as well. Thus, this model does not identify causes of the absence of observations of the comet before the date of its discovery in 1786. A hypothesis on inactive state of the comet Encke before the end of 18th century is widespread. But in most observed cases of comets transition from less active state to more active one unrelated to decrease of their perihelion distances the excessive activity lasted no more than 2–3 apparitions, after that comets took their previous shape or even became weaker. Therefore, the claim about the maintenance of acquired activity of the comet Encke at 68 returns to the Sun is unconvincing. Firstly, it can be assumed that the comet has significantly changed the orbit during a very close approach to a terrestrial planet. To determine the circumstances of this approach a highly accurate theory is required, which at the present time is unachievable. Secondly, the comet Encke may be a product of explosive fragmentation of an unknown long-period comet. Because of the comet's small perihelion distance, this assumed explosion most likely might have occurred at small solar elongation and remained unobserved. Then a part of the comet receiving momentum ahead in the direction of motion left the Solar System along a hyperbolic trajectory and the other part which had been thrown back became the comet Encke.

If we still admit that the comet Encke moved within Jupiter's orbit previously, then it is interesting to evaluate the general conditions of its visibility in the past. In this extrapolation, the argument of perihelion has changed by 162° and the longitude of ascending node – by 145° since the beginning of the Common Era. Reliability of this result is supported by the fact that the orbit orientation elements least depend on poorly known nongravitational forces. The dramatic change of these elements should lead to significant shift of the seasons and regions of the sky favorable for observing the comet. Conditions of its orbit approaching the Earth's one are also changing, which must be taken into account in the studies of Taurid meteor complex. The perihelion distance and inclination are stable enough to be useful for possible identification of the comet in ancient records and the meteors generated by it in observational data.

Table 5
Perturbations in heliocentric motion of the comet Encke for the entire period of observation (within the considered model).

Element	Planetary perturbations	Nongravitational perturbations	Complete perturbations	Contribution of nongravitational perturbations to summated, %
ω, °	+4.7055	−0.2772	+4.4283	5.56
Ω, °	−2.9033	+0.2515	−2.6518	7.97
i, °	−2.0472	+0.1140	−1.9332	5.28
e	+0.002315	−0.002462	−0.000147	51.53
a, au	−0.001421	−0.000591	−0.002012	29.38
M, °	−94.5873	+79.6138	−14.9735	45.70
π, °	+1.8022	−0.0258	+1.7765	1.41
q, au	−0.005343	+0.005363	+0.000019	50.09
Q, au	+0.002502	−0.006544	−0.004043	72.34
P, day	−1.1585	−0.4818	−1.6403	29.37
v, °	−19.9686	+17.5319	−2.4367	46.75
T, day	+315.683	−266.562	+49.121	45.78

The extrapolation forward in time has been performed as well and it predicts a complete extinction of the comet in about 2027. Our previous formal solution (Usanin et al., 2016) predicted 2022 as the date of extinction. Back in 1969 Z. Sekanina (Sekanina, 1969) using more simplified formulas and smaller amount of data built several models of the comet Encke nongravitational acceleration change from which he concluded that the complete extinction would take place between 2022 and 2036 (Sekanina, 1969). After that he abandoned these models in favor of precessional ones, some of whose predictions have not been justified by now. The tendency to the extinction of the comet Encke is confirmed by absolutely independent photometric data. In the first 50 years after the discovery it was visible by the naked eye 6 times, in the second 50 years – 3 times, and only 2 times for 50 years before 1988 (Ferrín and Gil, 1988). The course of secular change of the comet's brightness predicts the termination of activity in 2053–2059 (Ferrín, 2008) or even slightly earlier.

4. Conclusions

The problem of developing theories of motion for periodic comets combining all their apparitions is not yet solved and primarily it refers to the comet Encke. The model proposed in this paper explains characteristic features of transversal nongravitational parameter change with time for the comet Encke and allows obtaining a converging solution using astrometric observations of all its apparitions by means of differential correction. But the accuracy of this solution is insufficient. In the future one should extensively and regularly work on the development of new nongravitational forces models and application of the existing ones for constructing single motion theories for all the known periodic comets, and particularly the comet Encke. For each comet an individual model may be required, depending on its physical characteristics considering primarily certain factors in change of nongravitational forces. The discontinuity can only be acceptable in those cases, when it is reliably known about the comet's outburst leading to significant change of momentum. Perhaps even the best theories will not be able to achieve the accuracy of astrometric observations giving some average values of parameters.

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