

# Parametrically Excited Microelectromechanical System in Navigation Problems

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## Abstract

*The analysis of the operation of a microelectromechanical system under conditions of parametric modulation of the static stiffness of the rotor in the coherent mode of excitation of its oscillations has carried out. The corresponding mathematical model have constructed, numerical experiments have carried out. Based on this results a method of extending the functional capabilities of navigation devices has proposed.*

## 1. Introduction

Microelectromechanical systems (MEMS) are miniature, reliable and inexpensive devices that have become in demand in almost all industries and in the consumer market. MEMS sensors are usually equipped with integrated signal processing electronics and do not have moving parts. This determines their high reliability and the ability to provide stable readings in sufficiently harsh environmental conditions (temperature drops, impacts, humidity, vibration, electromagnetic and high frequency interference). As the technologies of their production and quality improve, MEMS became more widely used in the aerospace, energy and other high-tech industries. One of the areas where MEMS devices are used is inertial navigation. Application of MEMS as accelerometers and gyroscopes allows implement methods of inertial navigation at a new level, when miniature sensors serve as sources of data on the parameters of the movement of aircraft, vehicles and ships. As noted in [1], several companies have achieved success in creating tactical accuracy sensors. These companies are: Honeywell (USA), AIS (USA), LITEF (Germany), Gladiator technologies (USA), L-3 Com (USA), ADI (USA), Sensoror (Norway), Thales (France). But publications devoted to the extension of the measuring range are practically absent (among the available publications devoted to

the issues of increasing the stability of characteristics, we can mention the articles of foreign researchers M.S. Weinberg and A.Shkel).

In the present paper, we consider a method for increasing the accuracy and preserving the resonance tuning of the device, realized on the extreme self-adjusting control system, in which perturbation in the form of a periodically changing gyroscopic moment acting on the MMG, and the control search signal are separating, but act on the system simultaneously.

## 2. Modulation of static stiffness of the suspension

The specificity of the work of MMG allows, based on the use of circuitry solutions, without changing the design of the mechanical circuit, to increase the sensitivity of the device to the measured angular velocity by its parametric excitation [2].

The greatest sensitivity of the micromechanical gyroscope (MMG) provided under the conditions of the maximum amplitude of the primary oscillations, i.e. under the conditions of realization of the resonant tuning mode. However, operation of MEMS in conditions of a significant range of ambient temperatures, reaching 100° C or more, results a deviation of its own parameters and the frequency of the primary-oscillation excitation generator, as well as aging of the sensitive element material, that disturbs the resonant tuning.

The method under consideration based on the modulation of the static stiffness of the MMG's suspension. The modulation effected by a change in a small range of the amplitude of the alternating current applied to the additional winding of the moment sensor with a frequency  $\omega_m$  that ensures the parametric excitation of the MMG.

We solve two problems at the same time: firstly, we find an unknown extremum of the oscillation amplitude of the sensitive element, which depends on the frequency of the secondary oscillations, and

secondly, this frequency is stabilizing with the necessary accuracy with respect to the extremum value.

Figure 1 illustrates the principle of parametric modulation of the gyroscope. In this case, such energy-intensive parameters as the static stiffness of the suspension and the equatorial moments of inertia can be modulate. As a result its deviation, determined by the torque sensor of the compensating measurement, loop by means of an external alternating current source, that generates a search signal, affects the sensing element, and its reaction creates an information signal about the detuning of the device.

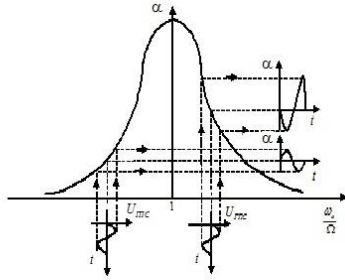


Fig. 1. The MMG reaction to the search signal in its detuning

### 3. Construction of a mathematical model

To obtain the MEMS mathematical model, we use the variational principle of Ostrogradskii-Hamilton (see for example [3]).

Let us choose the angles  $\alpha$ ,  $\beta$  of rotation of the rotor around the torsions and the angle  $\gamma$  of rotation around the anchor fixed to the thrust bearing as the generalized coordinates, and the moments of damping and the moments of elasticity of the torsions as generalized forces.

Differential equations of motion for the case of a constant angular velocity of rotation of the base, taking into account the modulation of the static stiffness along the sensitivity axis and the generalized forces, after the factorization procedure [4] and the subsequent linearization with the help of the Jacobi matrix will have the form:

$$\begin{aligned} A\ddot{\alpha} + \mu_\alpha\dot{\alpha} + (k_\alpha + \Delta k \sin(\omega_m t - \phi))\alpha &= -(C + A - B)\dot{\gamma}\Omega_{y0}, \\ B\ddot{\beta} + \mu_\beta\dot{\beta} + k_\beta\beta &= (C + B - A)\dot{\gamma}\Omega_{x0} - (A + B - C)\dot{\alpha}\Omega_{z0}, \\ C\ddot{\gamma} + \mu_\gamma\dot{\gamma} + k_\gamma\gamma &= (C + A - B)\dot{\alpha}\Omega_{y0} - (C + B - A)\dot{\beta}\Omega_{z0} + M_0 \sin \Omega t, \end{aligned} \quad (1)$$

where  $A$ ,  $B$ ,  $C$  – the main moments of inertia of the MMG rotor relative to the axes of the coordinate system associated with the rotor;  $\mu_\alpha$ ,  $\mu_\beta$ ,  $\mu_\gamma$  – the coefficients of viscous friction in the corresponding

coordinates;  $k_\alpha$ ,  $k_\beta$ ,  $k_\gamma$  – the stiffness of the elastic suspension elements relative to the axes of the secondary and primary gyroscope oscillations;  $\Omega_{x0}$ ,  $\Omega_{y0}$ ,  $\Omega_{z0}$  – the projection of the portable angular velocities of the motion of the gyroscope base;

$M_0$  – the amplitude of the torque developed by the drive relative to the axis of the primary oscillations;  $\Omega$  – the frequency of excitation of primary vibrations;  $\Delta k$  – the increment of stiffness of elastic suspension elements;  $\phi$  – the initial phase of oscillation.

### 4. Analysis of model's equations

Taking into account the fact that

$$k_\beta \gg k_\alpha, \quad M_0 \gg (C + A - B)\dot{\alpha}\Omega_{y0},$$

we assume  $\beta = \dot{\beta} = 0$ , i.e. in the first approximation, do not consider the second equation for the gyroscope. Then the simplified equations will take the form:

$$\begin{aligned} A\ddot{\alpha} + \mu_\alpha\dot{\alpha} + k_\alpha(1 + m \sin(\omega_m t - \phi))\alpha &= -(C + A - B)\dot{\gamma}\Omega_{y0}, \\ C\dot{\gamma} + \mu_\gamma\dot{\gamma} + k_\gamma\gamma &= M_0 \sin \Omega t, \end{aligned} \quad (2)$$

where  $m = \frac{\Delta k}{k_\alpha}$  – the modulation factor.

The solution of the second equation of system (2) has the form:

$$\gamma(t) = \frac{M_0(1 - e^{-a_\gamma t}) \cos \Omega t}{2a_\gamma C \Omega}, \quad (3)$$

where  $a_\gamma = \frac{\mu_\gamma}{2C}$ .

In the steady-state mode, expression (3) can be written as:

$$\gamma(t) = \gamma_0 \cos \Omega t, \quad (4)$$

where  $\gamma_0 = \frac{M_0}{\mu_\gamma \Omega}$ .

For the angular velocity of the oscillations of the anchor relative to the axis of the primary oscillations, we obtain:

$$\dot{\gamma}(t) = -\gamma_0 \sin \Omega t. \quad (5)$$

Substituting expression (5) into the first equation of system (2), we will have:

$$\begin{aligned} \ddot{\alpha} + 2a_\alpha\dot{\alpha} + \omega_0^2(1 + m \sin(\omega_m t - \phi))\alpha &= K \Omega_{y0} \sin \Omega t, \\ &= K \Omega_{y0} \sin \Omega t, \end{aligned} \quad (6)$$

where  $a_\alpha = \frac{\mu_\alpha}{2A}$ ,  $\omega_0 = \sqrt{\frac{k_\alpha}{A}}$ ,  $K = \frac{(C+B-A)\gamma_0\Omega}{A}$ .

A feature of the differential equation (6) is the presence of a term associated with a periodic change in the positional moment

$$\omega_0^2 m \sin(\omega_m t - \phi)\alpha.$$

The presence of such a periodically changing energy-consuming parameter MMG, as static rigidity, which is part of the corresponding inhomogeneous differential equation, provides favorable conditions for parametric excitation of the mechanical contour of the gyroscope under consideration at the modulation frequency  $\omega_m = 2\Omega$  [5].

## 5. The results obtained

The Figure 2 shows the graphs of the solution of equation (6) carried out with the help of the Maple 9 mathematical package at  $\omega_m = 2\Omega$  in the coherent mode of parametric excitation for various detuning values caused by instability of its natural oscillation frequency.

An analysis of the results shows that the solution of equation (6) depends essentially on the value of the modulation index  $m$  and the damping coefficient  $a_\alpha$ . The critical value of the modulation index can be found both by numerical experiments and analytically [5].

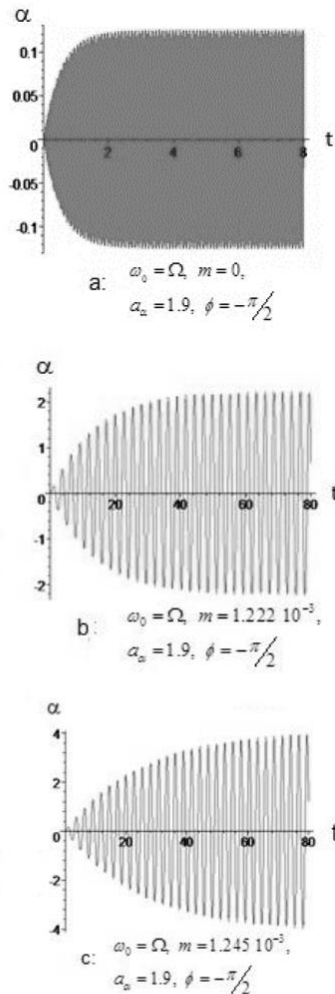


Fig. 2. Graphs of the solution for different values of the modulation coefficient

From the graphs in Figure 2 it follows, that the modulation of the static stiffness of the MMG provides an increase of its sensitivity in a steady state (the differentiating mode of the gyroscope) by several tens of times in comparison with a device in which parametric excitation is absent, which significantly increases the magnitude of its transmission coefficient. This is due to the fact, that modulation of static stiffness substantially reduces the amount of viscous friction. This increases the duration of the linear portion of the increase in the amplitude of the oscillations, which corresponds to a significant increase in the device time constant and, accordingly, to a narrowing of its bandwidth.

It should note that in the coherent excitation regime, an increase in the sensitivity of the MMG is critical to the magnitude of the phase shift between the periodically changing static rigidity of the torsion suspension and the external gyroscopic moment created by the portable angular velocity of rotation of the base of the device.

In Figure 3 shows plots reflecting the response to the device search signal generated by the torque sensor at different values of the detuning  $\delta\omega$ .

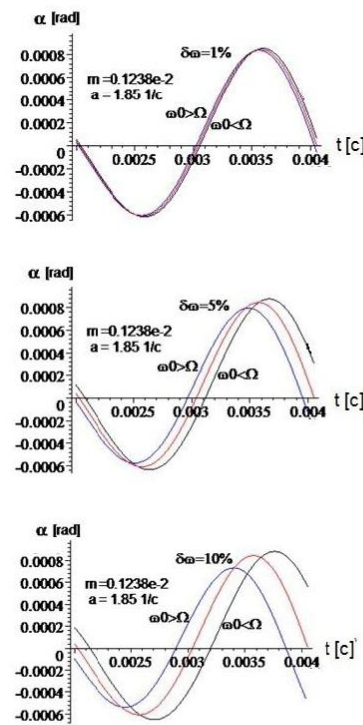


Fig. 3. The dependence of the phase of secondary oscillations on the magnitude and direction of the detuning

In this case, its phase comparison with the original search signal makes it possible, with the help of a phase detector, to extract information about the amount of detuning and to generate control of the master oscillator of the primary oscillations in the direction of its compensation. The results obtained are in good agreement with the experimental data.

## 6. Conclusion

The innovation of the results is as follows:

1) A new method for extending the functional capabilities of the MMG proposed, based on its parametric excitation;

2) A mathematical model of MMG constructed in the conditions of its parametric excitation;

3) Numerical calculations have shown that the presented mathematical model allows not only to illustrate the process of operation of the gyroscope in this mode, but also to find conditions that significantly increase the sensitivity and the quality of operation of MMG in a wide range of operating temperatures.

4) The proposed method makes it possible to perform measurements with greater accuracy than in the known analogs [6]. That extends the functional capabilities of the MMG as inertial navigation

devices.

## 7. References

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