212. Measuring Poisson constant and isochoric heat capacity of air

Poisson constant (also called adiabatic index) is the ratio of the isobaric heat capacity (at constant pressure, C_p) to the isochoric heat capacity (at constant volume, C_V):

$$\gamma = \frac{C_p}{C_V}. \qquad (1)$$

This coefficient appears in the relation describing an adiabatic process:

$$pV^{\gamma} = \text{const}. \qquad (2)$$

Since the Mayer's relation is fulfilled for ideal gases:

$$C_p - C_V = R, \qquad (3)$$

then measurement of γ allows finding the isochoric heat capacity as

$$C_V = \frac{R}{\gamma - 1}. \qquad (4)$$

Having measured γ and using the conclusions of the kinetic molecular theory, we can obtain the information on the number *i* of excited degrees of freedom at the given temperature, because

$$\gamma = \frac{C_p}{C_V} = \frac{i+2}{i}, \qquad (5)$$

and thus make a suggestion about the number of atoms comprising air molecules.

Aim of the work: measuring Poisson constant and isochoric heat capacity of air

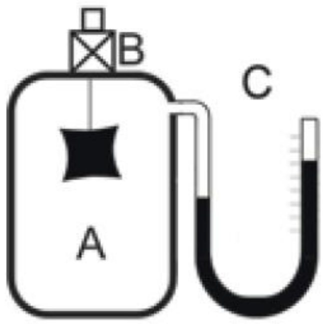
Tasks:

1. Getting acquainted with calculation of the Poisson constant in the slope of the kinetic molecular theory;
2. Getting acquainted with theoretical basis of the Clement–Desorm method of measuring the Poisson coefficient;
3. Measuring the adiabatic index;
4. Estimating the number of degrees of freedom for air molecules;
5. Measuring isochoric heat capacity of air.

Experimental setup and the idea of the Clement–Desorm method

In what follows we will use the knowledge that air at the room temperature shows properties of an ideal gas.

The setup of Celemt–Desorm is shown on the picture. Glass vessel A (with a small bag with silica gel inside to keep air dry) can communicate with the atmosphere through the valve B. Pressure gauge C allows measuring the pressure difference between the vessel and the atmosphere. Initially, when the valve B is open, air in the vessel exists at the atmospheric pressure p_0 and the room temperature T_0. If a small amount of air is pumped quickly into the



vessel A and the valve B is closed immediately after that, the pressure and temperature in the vessel will increase. Since there is a heat flow through the walls of the vessel, the temperature inside the vessel will become equal to the ambient (room) temperature in a while, and the pressure will decrease somewhat to a new value

$$p_1 = p_0 + h_1, \qquad (6)$$

where h_1 is the reading from the manometer (pressure gauge). Let us call this state of the gas with $T_1 = T_0$ and pressure p_1 *the first state*.
If the valve is opened quickly, air in the vessel A will experience adiabatic expansion, the pressure will equal with the atmospheric pressure, and the temperature will fall to a new value T_2. This will be *the second state* with parameters T_2 and $p_2 = p_0$.
If the valve is closed immediately after that (when the pressure reaches the value p_2), a process will begin in which the temperature inside will grow and finally achieves T_0, and the pressure will gain a new equilibrium value p_3. This will be *the third state* with parameters $T_3 = T_0$ and

$$p_3 = p_0 + h_3, \qquad (7)$$

where h_3 is the reading from the manometer in the third state of the gas.
The laws of the ideal gas are formulated for a fixed amount of the gas. Therefore, we will consider an imaginary volume of the gas never leaving the vessel in the following discussion. For this volume we can write down the relations (8) and (9), taking into account that the transition from state 1 to state 2 is the adiabatic process, and the transition 2 → 3 is the isochoric process:

$$\frac{p_1^{\gamma-1}}{T_1^{\gamma}} = \frac{p_0^{\gamma-1}}{T_2^{\gamma}} \qquad (8)$$

and

$$\frac{p_3}{T_1} = \frac{p_0}{T_2}. \qquad (9)$$

Combining Eqs. (6) and (8), we obtain:

$$\left(\frac{p_0 + h_1}{p_0}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)^{\gamma} \text{ or } \left(1 + \frac{h_1}{p_0}\right)^{\gamma-1} = \left(1 + \frac{T_1 - T_2}{T_2}\right)^{\gamma}.$$

If we can assume that $h_1 / p_0 << 1$ and $(T_1 - T_2)/T_2 << 1$, then both sides of the equation above can be presented as infinite series, and two first members of the series will give good enough approximation:

$$1 + \gamma \frac{T_1 - T_2}{T_2} = 1 + (\gamma - 1)\frac{h_1}{p_0} \text{ and hence } p_0 \frac{T_1 - T_2}{T_2} = \frac{\gamma - 1}{\gamma} h_1.$$

The left side in the equation above is actually h_3. If, indeed, we substitute the value of p_3 from Eq. (7) to formula (9) and then express h_3 through other parameters, we obtain

$$h_3 = p_0 \frac{T_1 - T_2}{T_2} \text{ and finally } h_3 = \frac{\gamma - 1}{\gamma} h_1.$$

The working formula is easily derived from the latter expression:

$$\gamma = \frac{h_1}{h_1 - h_3}\,. \qquad (10)$$

Thus, to find the Poisson constant using this method we do not need to calibrate the manometer; the only requirement is that the calibrating dependence $h(p)$ is linear.

Algorithm of measurements

1. Check that the U-shaped manometer is filled with water. Open the valve B and wait for 2-3 minutes. Attach the rubber bulb to the outlet of the valve B. Pump air into the vessel by a quick squeezing of the bulb and close the valve. Repeat the pumping one more time so that the liquid level difference h in the manometer reaches 20-40 cm.
2. Wait for the pressure to stabilize due to heat conductance (usually it takes about 5-6 minutes) and read the value h_1.
3. Open the valve B and close it after the hissing of air coming out is heard no longer. Wait for the pressure to stabilize. Read the value h_3.
4. Repeat steps 1-3 at least 8 times.
5. Calculate γ using Eq. (10) and estimate the inaccuracy assuming that this value is found as a result of direct measurement.
6. Calculate molar heat capacities of air C_V and C_p using formulas (4) and (3).
7. Find the number of degrees of freedom for air using expression (5).
8. Make conclusions about the correctness of kinetic molecular and quantum concepts regarding composition of air molecules.
9. Make conclusions about the number of atoms in an air molecule.

Questions and additional tasks

1. Heat capacity. The value of isochoric capacity.
2. The concept of the number of the freedom degrees of a molecule.
3. Poisson constant and its relation to the number of the degrees of freedom.
4. Mayer’s law.
5. Look at the deriving of the working formula. Which of the relations written here are true only for ideal gas? Which of them are fulfilled for a fixed amount of gas?
6. Present in a qualitative manner the processes in the gas which occur at every stage of the experiment on a single diagram. Write down corresponding equations.
7. Which requirement should be met for the transition from the state 1 to state 2 to be an adiabatic process?
8. Methodic of the experiment. Why is it recommended to wait for several minutes before taking readings from the pressure gauge? What if you will not follow this rule?
9. Is the ratio of the volumes of the rubber bulb and the vessel important?
10. Which requirements should the vessel fulfil? Think of the volume, thickness and rigidity of the wall, colour and transparency of the walls, the shape of the vessel.
11. Analyze the relative inaccuracy of the measurements of γ and C_V. Make conclusions.