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Spline-Interpolation Solution of 3D von Neumann Problem

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Abstract—We present the spline-interpolation approximate solution of the von Neumann problem for the Laplace equation in one class of solids. Our method is based on reduction of the 3D problem to the sequence of 2D problems.

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1. INTRODUCTION

We construct the spline-interpolation approximate solution of the von Neumann problem for the Laplace equation. The basic solution construction is similar to one given in [10]. This equation has wide applications to various gelological problems [1, 3, 4, 5, 7, 12]. The method presented here is applicable for the cylinders, cones and the axisymmetric solids. There exist many methods of approximate solution of this problem. The ordinary approximate method is the finite-element one. The spline-interpolation method is the analog of finite-element method where the discrete boundary nodes are the closed boundary curves and the cells are the layers between two parallel planes. Our method is based on reduction of the 3D problem to the sequence of 2D problems. Note that one can apply the functions of complex variable in the case of 2D problem. The main advantage of the spline-interpolation solution of the 3D von Neumann problem is its continuity in the whole domain up to the boundary even for the case of the linear spline. The piece-wise analytic form of the spline-interpolation solution is also convenient for the applications.

The first and often nontrivial step in application of the ordinary finite element methods is the meshing of the solid [6]. The mesh procedure in our case is the sinples possible, since the finite element is the layer between two planes orthogonal to one of the coordinate axes. Suppose that the finite element step equals δ. Then we deal with $1/\delta^2$ less number of elements than in the case of the classical finite element methods. So the purely arythmetic comutational error in our case is significantly smaller than in the ordinary case.

So this basic element mesh feature allows us to immediately pass to the solution construction. Note that because the basic element is not a simple tethrahedron or a prism, the solution construction in our case is more complicated than in the case of the ordinary finite element methods. We solve less number of problems than in the ordinary case but the problems themselves are more complicated. Nevertheless this approach allows us to consider solids with singularities at the boundary, which seems to be a standing problem for finite element method technique [13]. Naturally the consideration of higher order splines not only provides us with better gluing of the solutions in the adjacent basic elements but also yields better approximation to the exact solution. Note also that the linear spline provides us with the solution which is not only continuous itself but also smoothens as the height of the basic element vanishes.

We present the error estimate of our method. Naturally for the doman M, this estimate given in the terms of the integral square norm $L_2(M)$ similarly to the common finite element methods [14].

We also state that similar methods can be successfully applied to solution of elastisity theory problems. Note that the solution type and approximation etimates are significantly different from that of the Dirichlet problem [9].

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