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Semiclassical Long Wormhole Throats[1]

A. A. Popov*

Institute of Mathematics and Mechanics, Kazan Federal University, Kremlevskaya ul. 18, Kazan, 420008 Russia

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Abstract—Vacuum polarization of a quantized scalar field with non-conformal coupling ξ is calculated in the background of a long wormhole throat. It is shown that the stress-energy tensor of vacuum fluctuations in this spacetime is determined by the local geometry of spacetime only. Self-consistent solutions of the semiclassical Einstein field equations describing a long throat of a traversable wormhole are obtained.

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1. INTRODUCTION

By definition, a wormhole is a bridge connecting two asymptotically flat regions. Usually such a construction is a classical object and should satisfy the Einstein equations. The topology of 4D wormholes is that of a direct product of Minkowski plane and a unit sphere. Static traversable wormholes could be threaded by "exotic matter" that violates certain energy conditions at least at the throat [1]. As an example of such matter, one can consider the vacuum of quantized fields. This approach makes it possible to consider a wormhole metric as a self-consistent solution of semiclassical gravity. In the realm of this theory, vacuum fluctuations of quantized fields are the source of spacetime curvature[2]:

$$G_\nu^\mu = 8\pi \langle T_\nu^\mu \rangle_{\rm ren}. \tag{1}$$

The main difficulty in semiclassical gravity theory is that the effects of a quantized gravitational field are ignored. A popular solution of this problem is to justify ignoring the gravitational contribution by working in the limit of a large number of fields, in which the gravitational contribution is negligible. Another problem is that the vacuum polarization effects are determined by the topological and geometric properties of spacetime as a whole, or by the choice of a quantum state in which the expectation values are taken. It means that calculation of the functional dependence of $\langle T_\nu^\mu \rangle_{\rm ren}$ on the metric tensor in an arbitrary spacetime presents a formidable difficulty. Only in some spacetimes with high degrees of symmetry for conformally invariant fields, $\langle T_{\mu\nu} \rangle_{\rm ren}$ can be computed, and Eqs. (1) can be solved exactly [3]. Let us stress that the only parameter of length dimensionality in such a problem is the Planck length l_{Pl}. This implies that the characteristic scale l of spacetime curvature (which corresponds to a solution of Eqs. (1) can differ from l_{Pl} only if there is a large dimensionless parameter. As an example of such a parameter, one can consider the number of fields whose polarization is a source of spacetime curvature[3]. In some cases $\langle T_{\mu\nu} \rangle_{\rm ren}$ is determined by the local properties of spacetime, and it is possible to calculate the functional dependence of the renormalized expression for the vacuum expectation value of the stress-energy tensor operator of the quantized fields on the metric tensor approximately. One of the most widely known examples of such a situation is the case of a very massive field. In this case, $\langle T_\nu^\mu \rangle_{\rm ren}$ can be expanded in powers of the small parameter

$$1/(ml) \ll 1, \tag{2}$$

where m is the mass of the quantized field and l is the characteristic scale of spacetime curvature [4, 5]. Using the first nonvanishing term of this expansion for minimally or conformally coupled scalar field, Taylor, Hiscock and Anderson [6] showed that Eqs. (1) have no wormhole solution for some class of static spherically symmetric spacetimes. Let us stress that in this case the existence of an additional parameter of length dimensionality $1/m$ does not increase the

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*E-mail: apopov@ksu.ru

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[2] Throughout we use such units that $c = \hbar = G = 1$.

[3] Here and below it is of course assumed that the characteristic scale of change of the background gravitational field is sufficiently greater than l_{Pl}, so that the very notion of classical spacetime is still meaningful.

All these conditions are valid for $\xi < 0$, $|\xi| \gg 1$. A particular solution of the set 0f equations (26), (27) is

$$\xi = -10^4, \quad m^2 = 10^3, \quad r \simeq 101.49. \tag{33}$$

Let us note that the stress-energy of the fields considered here has the "exotic" properties (in the sense of Morris and Thorne [1]) needed to support a long wormhole throat:

$$p_r = -\epsilon = \frac{1}{4\pi^2 r^4}\left[0.00310 + \frac{1}{720}\ln\left(m_{DS}^2 r^2\right) + \frac{1}{m^2 r^2}\left(-\frac{\xi^3}{6} + \frac{\xi^2}{12} - \frac{\xi}{60} + \frac{1}{630}\right)\right] - \frac{Q^2}{8\pi r^4} < 0, \tag{34}$$

where p_r is the radial pressure, ϵ is the energy density, while r and Q are determined by the expressions (30) and (31).

4. CONCLUSIONS

We have succeeded in finding a solution of semiclassical gravity which describes a long wormhole throat. Such objects are created by an electrostatic field and vacuum fluctuations of quantized scalar fields. The stress-energy tensor of these fluctuations in the considered spacetime is determined by the local geometry of spacetime only. The spacetime geometry far from the throat is not described by the resulting zero-order (with respect to the small parameter $L_\star/L$) solution. An obvious defect of such solutions is as follows: the value of $|\xi|$, which corresponds to a large (compared to the Planck length) value of the throat radius is also large (compared to 1). The latter seems unlikely. Nevertheless, in the "large N" case, where the number of matter fields is large, the throat radius r is proportional to $\sqrt{N}$ (or $N^{1/4}$ for very massive fields), and the values of r satisfying the condition $r \gg 1$ (i.e., large compared with the Planck length) can be obtained for a value of $|\xi|$ smaller than the one considered above.

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