R.G. Salahudinov (Kazan, Russia)
rsalakhud@gmail.com,

Monotone functionals on level sets of the distance function and inequalities for Euclidean moments

Let G be a simply connected domain in the plane. Applying the standard notation in the theory of estimates on level lines, consider the following functional

$$i_s(\mu) := s\int_{\mu}^{\rho(G)} t^{s-1} a(t)\, dt,$$

defined on the level set $G(\mu)$. The functional $I_s(G) := i_s(0)$ is called the Euclidean moment of the domain G of order s.

Theorem 1. *Let G be a simply connected domain in the plane such that $I_s(G) < \infty$. If G does not coincide with a Bonnesen-type domain, then the function*

$$\frac{i_p(\mu) - p\pi\int_0^{\rho(G)t^{\alpha-1}(\rho(G)-t)^2} dt}{p\pi\int_0^{\rho(G)t^{\alpha-1}(\rho(G)-t)} dt}$$

is strictly monotone decreasing on $[0, \rho(G)]$.

This theorem generalizes one of the statements in [1]. Theorem 1 and its analogs lead to the following inequalities.

Theorem 2. *Let G be a convex domain in a plane of finite area. Let $s < p < q$, then the inequality*

$$\frac{I_q(G)}{q(q-s)\rho(G)^{q+2}} - \frac{I_p(G)}{p(p-s)\rho(G)^{p+2}} \geq C_1(s,p,q)\frac{I_s(G)}{\rho(G)^{s+2}} + C_2(p,q)\frac{sl(\rho(G))}{\rho(G)}.$$

Equality is achieved if and only if $G \in \Gamma$.

Список литературы

[1] Salakhudinov R. G. An isoperimetric monotonicity of euclidian moments of simply connected domain // Russ. Math. (Iz. VUZ). - 2013. - V. 57 (8). - P. 57-69 (DOI: 10.3103/S1066369X13080070)