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Condensate Fraction of a Fermi Gas in the BCS-BEC Crossover

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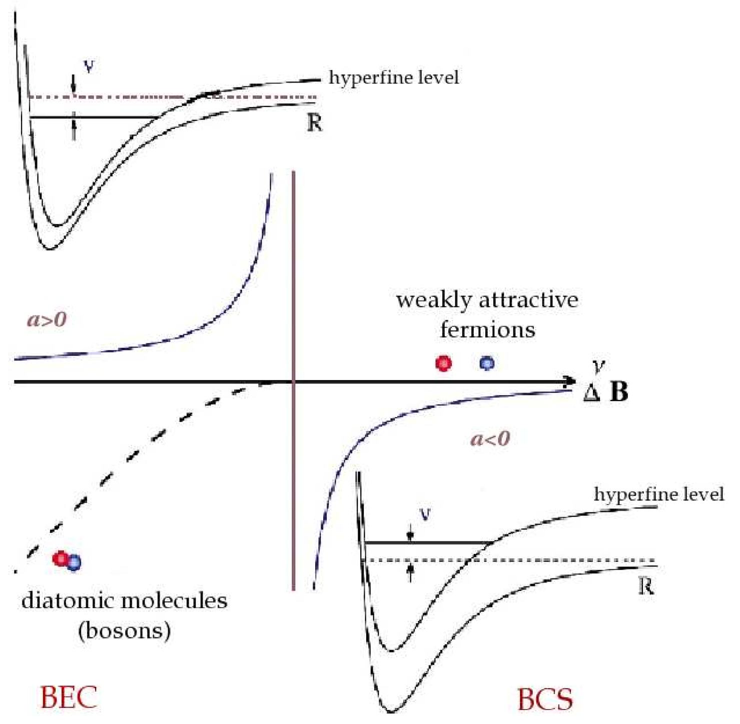
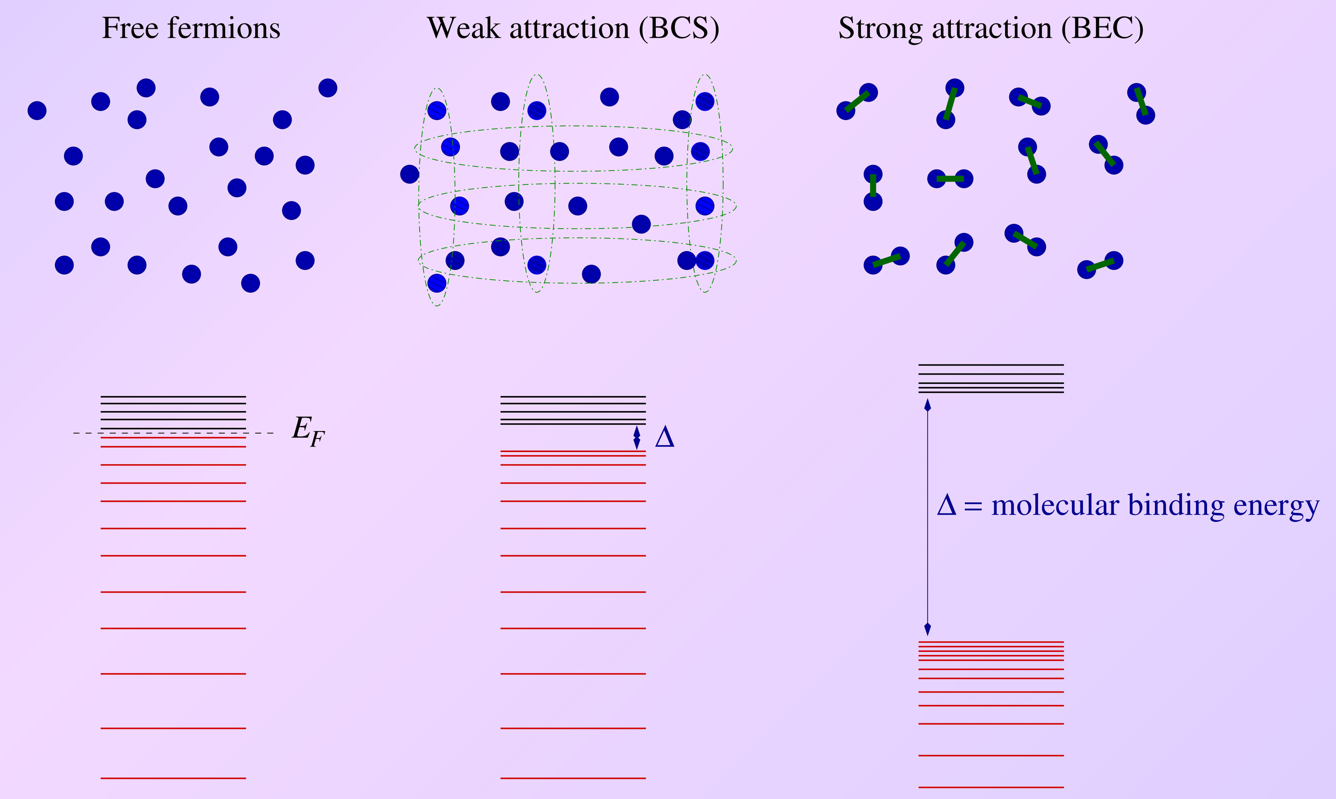
Summary

Consider the problem of the $T = 0$ state of a gas of interacting spin-$\frac{1}{2}$ fermions. As the interaction is turned on and gradually increased, the system evolves continuously from the ideal Fermi gas, to the BCS (Bardeen-Cooper-Schrieffer) state of extremely wide weakly-bound Cooper pairs, to a gas of increasingly tightly bound "molecular" bosonic dimers, to finally reach the limit of an ideal Bose gas in its Bose-Einstein condensate state. As the interaction strength is varied we evaluate the "condensate" density of the fermion pairs, namely those particles contributing to the long-range order of the collective coherent state. In particular, we derive an explicit formula for the condensed fraction as a function of the chemical potential and the superconducting gap [1].



We use this relation to compute the condensate fraction in the crossover from the BCS state to the Bose-Einstein condensate of molecular dimers. By using a local density approximation we study ultracold clouds of several million alkali fermionic atoms in vapor phase, confined in magnetic traps. We compare the observed fraction of condensed atoms [2] with the outcome of our theory, and find good accord.



General Theory for dilute interacting Fermi gas

Consider the Hamiltonian density:

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2m}\sum_{\sigma=\uparrow,\downarrow} \hat\psi_\sigma^+ \nabla^2 \hat\psi_\sigma + g\, \hat\psi_\uparrow^+ \hat\psi_\downarrow^+ \hat\psi_\downarrow \hat\psi_\uparrow$$

Use the BCS variational state [3] ($T = 0$): $|\varphi\rangle = \prod_{\mathbf{k}} \left(u_k + v_k \hat a^+_{\mathbf{k}\uparrow} \hat a^+_{-\mathbf{k}\downarrow}\right)|0\rangle$
where $|0\rangle$ is the vacuum state, and u_k and v_k are variational amplitudes
Energy minimization $\longrightarrow$ gap equation $\longleftarrow$ ultraviolet divergent!!
Solution: introduce the scattering length a_F: $\frac{m}{4\pi\hbar^2 a_F} = \frac{1}{g} + \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{2\epsilon_k}$
Regularized gap equation [4, 5, 6]:

$$-\frac{m}{4\pi\hbar^2 a_F} = \frac{1}{V}\sum_{\mathbf{k}}\left(\frac{1}{2E_k} - \frac{1}{2\epsilon_k}\right).$$

BCS superfluidity of Cooper pairs ($a_F < 0$) $\longrightarrow$ BEC of molecular dimers ($a_F > 0$)
Use dimensionless inverse interaction parameter $y = (k_F a_F)^{-1}$ $\quad(-\infty < y < +\infty)$
For given y, to compute μ and Δ one must solve the coupled equations

$$-\frac{1}{a_F} = \frac{2(2m)^{1/2}}{\pi\hbar^3}\Delta^{1/2} I_1\!\left(\frac{\mu}{\Delta}\right), \qquad n = \frac{N}{V} = \frac{(2m)^{3/2}}{2\pi^2\hbar^3}\Delta^{3/2} I_2\!\left(\frac{\mu}{\Delta}\right)$$

where $I_1(x)$ and $I_2(x)$ are known functions [7].

The condensate fraction of fermionic pairs

It is related to the off-diagonal long-range order (ODLRO) [8] of the system.
For large $|\mathbf{r}_i' - \mathbf{r}_j|$ the two-body density matrix factorizes [9]:

$$\langle \hat\psi_\uparrow^+(\mathbf{r}_1')\hat\psi_\downarrow^+(\mathbf{r}_2')\hat\psi_\downarrow(\mathbf{r}_1)\hat\psi_\uparrow(\mathbf{r}_2)\rangle \to \langle \hat\psi_\uparrow^+(\mathbf{r}_1')\hat\psi_\downarrow^+(\mathbf{r}_2')\rangle\langle\hat\psi_\downarrow(\mathbf{r}_1)\hat\psi_\uparrow(\mathbf{r}_2)\rangle$$

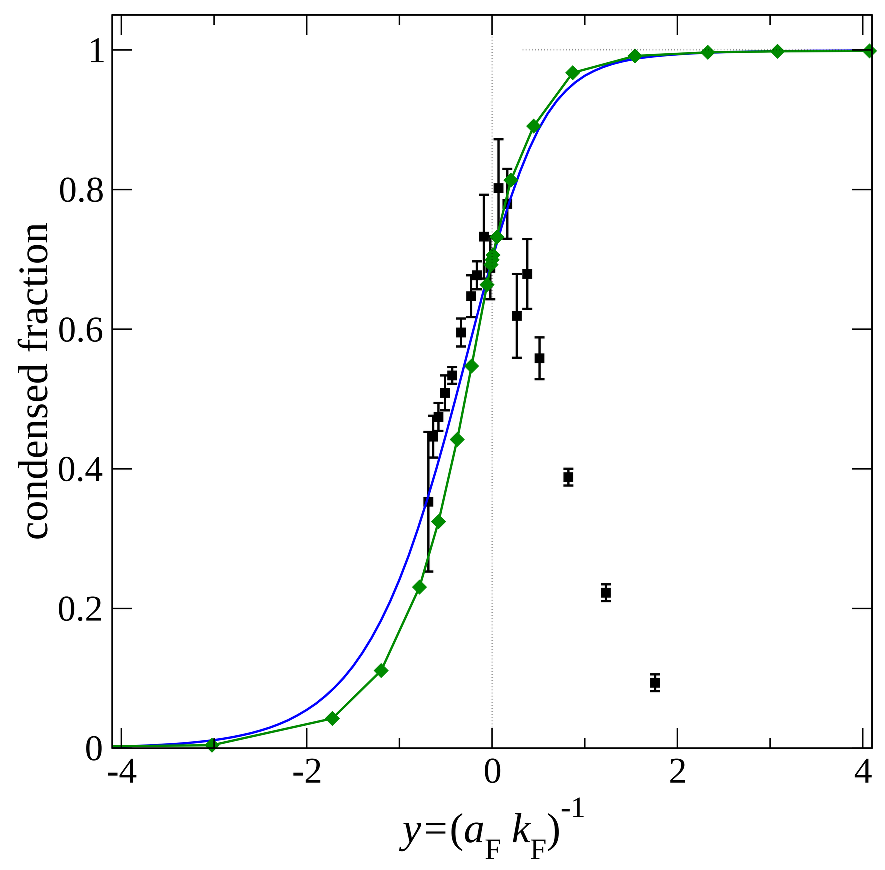
The largest eigenvalue N_0 of the two-body density matrix yields the condensate number of Fermi pairs [9, 10]

$$N_0 = \int d^3\mathbf{r}_1\, d^3\mathbf{r}_2\, |\langle \hat\psi_\downarrow(\mathbf{r}_1)\hat\psi_\uparrow(\mathbf{r}_2)\rangle|^2 = \sum_{\mathbf{k}} u_k^2 v_k^2$$

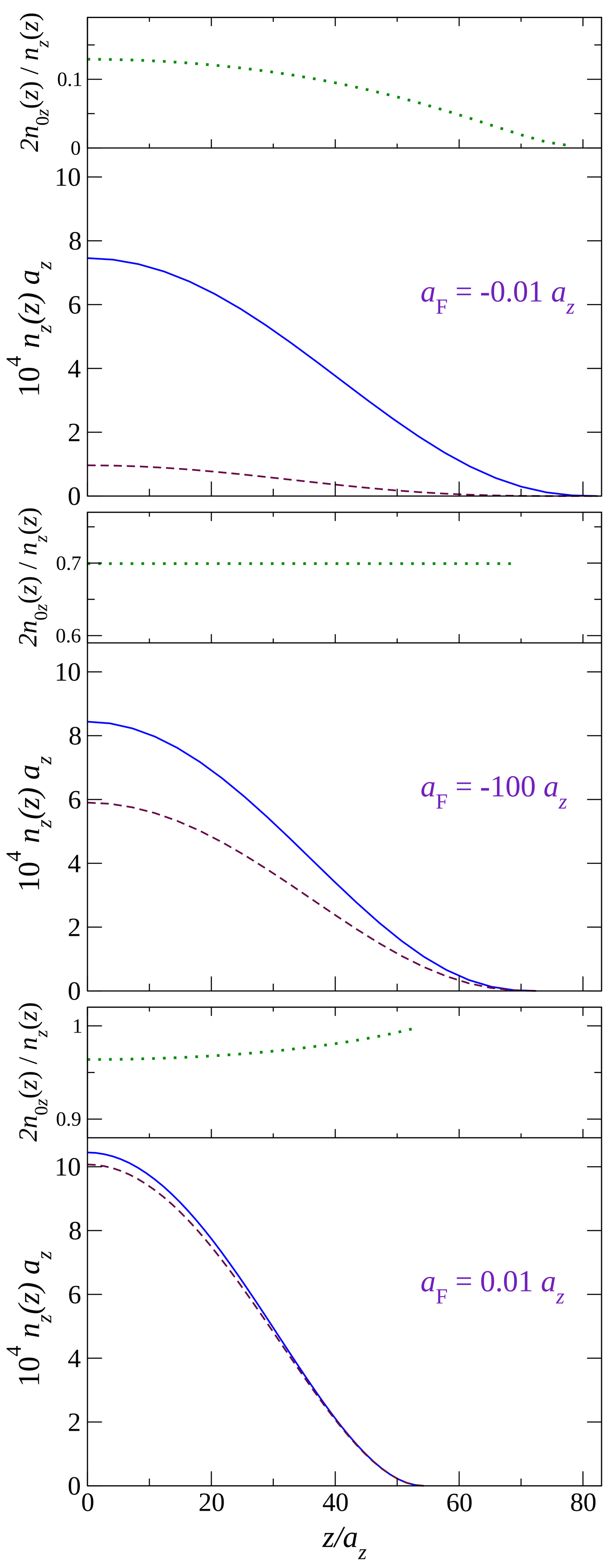
Main result: a formula for the condensate density

$$n_0 = \frac{N_0}{V} = \frac{m^{3/2}}{8\pi\hbar^3}\Delta^{3/2}\sqrt{\frac{\mu}{\Delta} + \sqrt{1 + \frac{\mu^2}{\Delta^2}}}$$

Results



Condensate fraction $N_0/(N/2)$ of Fermi pairs in the uniform two-component dilute Fermi gas as a function of $y = (k_F a_F)^{-1}$.
The same quantity computed in the LDA for a droplet of $N = 6 \times 10^6$ fermions in an elongated harmonic trap with $\omega_\perp/\omega_z = 47$, as in the experiment of Ref. [2], plotted against the value of y at the center of the trap.
Experimentally determined condensed fraction [2].



BCS regime:
at the droplet center $y = -1.2$ ($a_F = -0.01 a_z$)

unitarity regime:
at the droplet center $y = -10^{-4}$ ($a_F = -100 a_z$)

BEC regime:
at the droplet center $y = 0.87$ ($a_F = 0.01 a_z$)

Total axial density [$n_z(z)$] and condensed density [$2n_{0z}(z)$] of a droplet composed by $N = 6 \times 10^6$ fermions in an elongated harmonic trap with $\omega_\perp/\omega_z = 47$, as in the experiment of Ref. [2]. Dotted curves: the axial local condensate fraction $2n_{0z}(z)/n_z(z)$. With ^{6}Li atoms, the harmonic oscillator lengthscale $a_z = \hbar^{1/2}(m\omega_z)^{-1/2} = 12\,\mu$m in the setup of Ref. [2].

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