Variation of Nongravitational Parameters for Comet Encke as a Result of its Decay

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Abstract. An assumption is made that variations of nongravitational parameters of comet Encke in Marsden's model are caused by its secular decay. To account for decrease of nongravitational parameters three variants of physical processes are proposed: 1) deposition of a substantial nonvolatile mass not impeding the sublimation; 2) generation of a little-massive mantle shrinking the effective area linearly; 3) generation of a little-massive mantle shrinking the area exponentially. The corresponding equations are derived. The adequacy of the first two models over more than 150 years is shown.

Many models [1] explain the behavior of Marsden's parameters [2] of comet Encke [3] by the rotation pole precession of the spotty nucleus as proposed by Whipple and Sekanina [4]. Since the comet decay is not considered, they are not self-consistent. Other shortcomings are reviewed by Chernetenko [5].

In the present work we examine three models for secular decay of comet Encke assuming a constant shape of the nucleus. (Keeping nearly constant shape of a cometary nucleus was confirmed by Medvedev [6].) Since not all the mass is ejected in the same direction, Meshcersky's equation for the reactive force acting on the comet nucleus of mass m is given by

$$\vec{F}_r = \lambda \vec{u} \frac{dm}{dt}, \tag{1}$$

where λ is the anisotropy factor and $\vec{u}$ is the velocity of the matter escaping from the nucleus in the orbital coordinate system; in Marsden's model [2] both being considered as constants. The number of particles in mass m is given by

$$N = mN_A / M , \tag{2}$$

where N_A is the Avogadro number and M is the mean molecular mass. In Marsden's model the number of ejected particles from a unit pure area during unit time is given by

$$-\frac{1}{\beta S}\frac{dN}{dt} \equiv Z = Z_0 g(r(t)) , \tag{3}$$

where S is the geometrical area, $0 \le \beta \le 1$ is the ratio of the effective area to geometrical area, $r(t)$ is the heliocentric distance (in AU), and

$$g(r) = 0.111262 \cdot 10^{-8} (r / 2.808)^{-2.15} (1 + (r / 2.808)^{5.093})^{-4.6142} . \tag{4}$$

In preliminary calculations one can use $\langle g(r(t)) \rangle = g(a, \quad e)$ depending on the size and shape of the orbit. One obtains the acceleration in orbital coordinates (*i* = 1; 2; 3 are radial, transverse, and normal directions):

$$w_i = -\frac{\lambda u_i M Z_0 \beta S}{N_A m} g(r) . \tag{5}$$

By definition, one finds Marsden's parameters (units are AU/(104 days)2):

$$A_i = -\frac{\lambda u_i M Z_0 \beta S}{N_A m} . \tag{6}$$

1. Assume a substantial nonvolatile mass is deposited. To continue this deposition, it should not impede the sublimation ($\beta = 1$). Before Style II Marsden's model was introduced, a similar case was considered by Sekanina [7] but it was not developed. If the nucleus shape is constant, then

$$\frac{S}{m_{\text{ice}}} = \frac{\varphi}{\rho_{\text{ice}} R} , \tag{7}$$

where φ depends on the shape ($\varphi = 3$ for a sphere), ρ_{ice} and m_{ice} are the density and the mass of the ice, R is its mean radius defined as

$$R = \sqrt[3]{\frac{3m_{\text{ice}}}{4\pi\rho_{\text{ice}}}} . \tag{8}$$

Analogically R_{end} is defined using the nonvolatile mass $m_{\text{end}} = m - m_{\text{ice}}$:

$$R_{\text{end}} = \sqrt[3]{\frac{3m_{\text{end}}}{4\pi\rho_{\text{end}}}}. \tag{9}$$

Hence Marsden's parameters and their variations due to the ablation are

$$A_i = -\frac{\lambda u_i M Z_0 \varphi}{N_A \rho_{\text{ice}}} \frac{R^2}{R^3 + (\rho_{\text{end}} R_{\text{end}}{}^3 / \rho_{\text{ice}})}, \tag{10}$$

$$\frac{dA_i}{dR} = -\frac{\lambda u_i M Z_0 \varphi}{N_A \rho_{\text{ice}}} \frac{2R(R^3 + (\rho_{\text{end}} R_{\text{end}}{}^3 / \rho_{\text{ice}})) - 3R^4}{(R^3 + (\rho_{\text{end}} R_{\text{end}}{}^3 / \rho_{\text{ice}}))^2}. \tag{11}$$

And also their combination is

$$\frac{dA_i}{A_i dR} = \frac{2}{R} - \frac{3R^2}{R^3 + (\rho_{\text{end}} R_{\text{end}}{}^3 / \rho_{\text{ice}})}. \tag{12}$$

Combining (2), (3), (7), and (8) if $\beta = 1$, one obtains

$$\frac{dR}{dt} = -\frac{\varphi M Z_0}{3 N_A \rho_{\text{ice}}} g(r). \tag{13}$$

Designating $\sqrt[3]{\frac{\rho_{\text{ice}}}{\rho_{\text{end}} R_{\text{end}}{}^3}} R = \chi \geq 0$ and $\frac{\phi M Z_0}{3 N_A \rho_{\text{ice}}{}^{2/3} \rho_{\text{end}}{}^{1/3} R_{\text{end}}} = \alpha \geq 0$ in (12) and (13), one writes the final set

$$\begin{cases} \frac{dA_i}{d\chi} = A_i\left(\frac{2}{\chi} - \frac{3\chi^2}{1+\chi^3}\right), \\ \frac{d\chi}{dt} = -\alpha g(r(t)), \\ \alpha = \text{const.} \end{cases} \tag{14}$$

Table 1. Parameters of solution in Fig. 1 for beginning and ending dates of model

Year	A_2	α, day^{-1}	χ
1786	–0.0461	$3.24 \cdot 10^{-5}$	0.945
2032	0		0

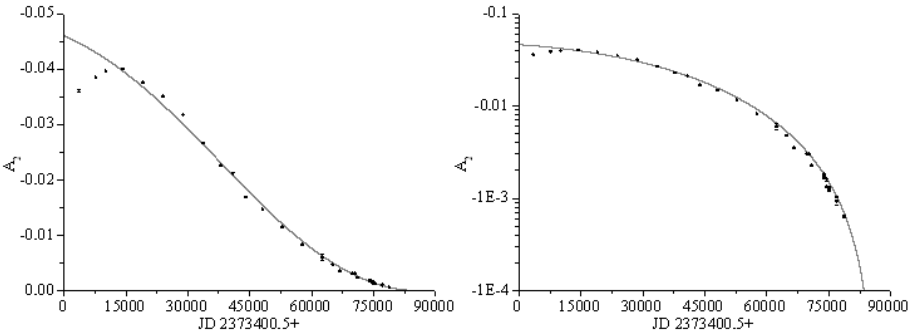

Fig. 1. Formal solution of set (14) on linear and logarithmic scales.

The formal (because input data are A_2 with mean errors from [2, 3, 8] and other sources, not astrometry) solution of (14) for A_2 (which is much more accurate than both A_1 and A_3) is in Fig. 1, its parameters are in Tab. 1. Here and below elements *a*, *e*, are assumed as equal to their mean values. The accuracy of A_2 increased with time when its value decreased, thus the graph on the logarithmic scale is a better representation for the weighted accuracy. One can see that the model reproduces A_2 adequately over more than 170 years and predicts the total decay about 2022.

2. Suppose a little-massive mantle is generated shrinking the effective area linearly with the thickness. This is equivalent to Shul'man's assumption that the mantle formation is completed when the volume containing the covering area equal to the nucleus area has sublimated [9]. In analogy to (7) one has

$$\frac{S}{m} = \frac{\varphi}{\rho R}, \tag{15}$$

where ρ is the nucleus density (assumed uniform), *R* is the mean radius:

$$R = \sqrt[3]{\frac{3m}{4\pi\rho}}. \tag{16}$$

Hence Marsden's parameters and their variations due to the ablation are

$$A_i = -\frac{\lambda u_i M Z_0 \varphi\beta}{N_A \rho R}, \tag{17}$$

$$\frac{dA_i}{dR} = -\frac{\lambda u_i M Z_0 \varphi}{N_A \rho}\frac{d(\beta / R)}{dR}. \tag{18}$$

And also their combination is

$$\frac{dA_i}{A_i dR} = \frac{R}{\beta}\frac{d(\beta / R)}{dR} = \frac{1}{\beta}\frac{d\beta}{dR} - \frac{1}{R}. \tag{19}$$

By the model assumption, one gets

$$\beta = 1 - h / h_0, \tag{20}$$

where h_0 is the mantle thickness terminating the sublimation,

$$h = \frac{f(m_{\text{begin}} - m) / \rho}{S} \tag{21}$$

is its current thickness, m_{begin} is the initial mass, and f is the nonvolatiles bulk part. Substituting (15) and (16) one has

$$h = \frac{f}{\varphi}\frac{R_{\text{begin}}{}^3 - R^3}{R^2}. \tag{22}$$

Derivating (20) with respect to *R*, considering (22), and substituting R_{begin} from them, one obtains

$$\frac{d\beta}{dR} = \frac{3f}{\varphi h_0} + \frac{2(1-\beta)}{R}. \tag{23}$$

Hence one sets (19) to:

$$\frac{dA_i}{dR} = A_i\left(\frac{1}{\beta}\frac{3f}{\varphi h_0} + \frac{2}{\beta R} - \frac{3}{R}\right). \tag{24}$$

Combining (2), (3), (15), and (16), one obtains

$$\frac{dR}{dt} = -\frac{\varphi M Z_0}{3 N_A \rho}\beta g(r). \tag{25}$$

Designating $\frac{3f}{\varphi h_0} R = \chi$ and $\frac{M Z_0 f}{N_A \rho h_0} = \alpha$ in (24), (23), and (25), one writes

$$\begin{cases} \dfrac{dA_i}{d\chi} = A_i\left(\dfrac{1}{\beta} + \dfrac{2}{\beta\chi} - \dfrac{3}{\chi}\right), \\ \dfrac{d\beta}{d\chi} = 1 + \dfrac{2(1-\beta)}{\chi}, \\ \dfrac{d\chi}{dt} = -\alpha\beta g(r(t)), \\ \alpha = \text{const.} \end{cases} \tag{26}$$

The formal solution of (26) for A_2 is in Fig. 2, its parameters are in Tab. 2. It reproduces A_2 adequately over more than 150 years and suggests that on discovery the comet surface was nearly pure ice ($\beta > 1$ makes no sense).

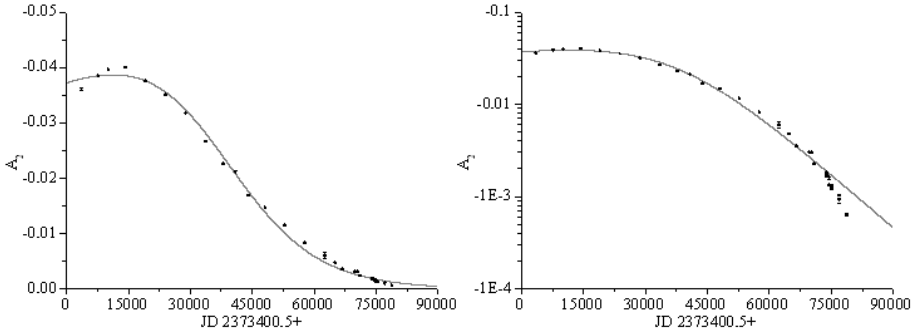

Fig. 2. Formal solution of set (26) on linear and logarithmic scales.

Table 2. Parameters of solution in Fig. 2 for beginning and ending dates of model

Year	A_2	α, day^{-1}	β	χ
1786	–0.0372	$3.32 \cdot 10^{-5}$	0.99	0.637
2032	–0.000455		0.00547	0.288

3. A case of the generation of a little-massive mantle shrinking the effective area exponentially was also considered. It was shown that this model represents the variation of A_2 only qualitatively and is of no interest.

Solutions obtained in the present work leave significant offsets. Causes of offsets may be not only in model assumptions, but also out of them. These are the solution procedure formality, the cometary stochasticity, and accidental errors in A_2 multiplied by the correlation with little-significant A_1.

References

1. *Yeomans D. K., Chodas P. W., Sitarski G., Szutowicz S., Królikowska M.* Cometary Orbit Determination and Nongravitational Forces / Eds M. C. Festou, H. U. Keller, H. A. Weaver. Comets II. Tucson: Univ. Arizona Press, 2004. P. 137–151.
2. *Marsden B. G., Sekanina Z., Yeomans D. K.* Comets and nongravitational forces. V. // Astron. J. 1973. Vol. 78. P. 211–225.
3. *Marsden B. G., Sekanina Z.* Comets and nongravitational forces. VI. Periodic comet Encke 1786–1971 // Astron. J. 1974. V. 79. P. 413–419.
4. *Whipple F. L., Sekanina Z.* Comet Encke: precession of the spin axis, nongravitational motion, and sublimation // Astron. J. 1979. Vol. 84. P. 1894–1909.

5. *Medvedev Yu. D., Sveshnikov M. L., Sokolsky A. G., Timoshkova E. I. et al.* / Ed. A. G. Sokolsky. Asteroid-Comet Hazard. St. Petersburg: ITA RAS, 1996 (in Russian).
6. *Medvedev Yu. D.* Effekty sublimatsii v orbital'nom i vrashchatel'nom dvizhenii kometnogo yadra. Avtoref. diss… dokt. fiz.-mat. nauk. (Effects of sublimation in orbital and rotational motion of cometary nucleus. Abstr. ScD thesis.) St. Petersburg: IAA RAS, 1995 (in Russian).
7. *Sekanina Z.* Dynamical and Evolutionary Aspects of Gradual Deactivation and Disintegration of Short-Period Comets // Astron. J. 1969. Vol. 74. P. 1223–1234.
8. *Kozlov E. A., Medvedev Yu. D., Zamarashkina M. D., Pittichová J. et al.* Catalogue of short-period comets. Bratislava: Astronomical Institute, Slovak Academy of Sci. 2005.
9. *Shul'man L. M.* Dinamika kometnykh atmosfer. Neitral'ny gaz. (Dynamics of cometary atmospheres. Neutral gas.) Kiev: Naukova dumka, 1972 (in Russian).