Robust Adaptive Multi-Agent Coverage Control for Flood Monitoring

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***Abstract*—This paper presents a control strategy for monitoring a dynamically changing flood zone by using a group of unmanned aerial vehicles (UAVs) and an unmanned surface vehicle (USV). The strategy requires to allocate UAVs in the flood zone, achieving an optimal coverage efficiency. As the flight duration of UAVs is limited, they need to be called back to the USV for recharging or for battery swapping. Therefore, the UAVs are required to be allocated near the USV while covering the flood area. A robust adaptive control controller is proposed to implement the aforementioned strategy, the validity of which is tested under simulations.**

***Index Terms*—Coverage control, multi-agent system, robust adaptive control.**

I. Introduction

Disaster robotics has drawn an increasing attention in the recent decade [1], [2]. This paper deals with an important problem in the field of disaster robotics: utilizing UAVs to monitor a dynamic flood area [3]. Assume that a group of UAVs and a USV are involved in the flood monitoring mission. As the flight duration of UAVs is limited, they need to be called back to the USV for recharging or for battery swapping. Therefore, the UAVs are required to be allocated near the USV while covering the flood area, the shape of which are assumed to the known.

The allocation of UAVs in the flood area formulates a coverage problem [4]. It is concerned with planning an optimal configuration of the agent network for the coverage of an area of interest, and with driving each agent to the desired position to realize the planned configuration. To tackle the coverage problem, several Voronoi-based approaches [4]–[8] have been proposed. In these approaches, a centroidal Voronoi configuration has been generated over the coverage region, which maximizes the coverage efficiency by driving agents to the centroids in corresponding Voronoi cells.

Regarding these approaches, in [5], arbitrary target patterns were represented with an optimal agent deployment. However, the focus of this work has largely been on static environments. The distributed coverage controller proposed in [6] can react to the dynamic environment, tracking moving targets using the Centroidal Voronoi Tessellation (CVT). Although it has been verified under simulations and experiments that the CVT can be achieved by using this controller, a formal proof of convergence was not presented. In [7], [8], the stability of multi-agent systems under the proposed controllers has been established, but uncertainties in the control process have not been taken into consideration. The research conducted in [9]–[11] addressed the adaptive coverage control problems for network systems with uncertainties. However, they were either based on the model predictive control [9] or machine learning algorithms [10], [11] for which the asymptotic stability of the control system is difficult to prove.

To address both adaptiveness and stability, in this paper, we propose an FATII based coverage controller, the asymptotic stability of which is established, and the adaptiveness is shown by simulations in the presence of rather large time-varying uncertainties. In addition, as one consider the scenario of battery swapping for UAVs, unlike the aforementioned research which only concerned about fixed number of UAVs, in this paper, a control strategy is proposed to achieve optimal configuration, while the number of UAVs varies during the monitoring process.

The rest of the paper is organized as follows. In Section II the flood monitoring problem by heterogenous robotic systems is stated. In Section III, a robust adaptive control algorithm is proposed, and its validity is tested under simulations in Section IV. Finally, conclusions are drawn in Section V.

II. Control problem statement

This paper aims to design a control strategy for monitoring a flood area by using a USV and multiple UAVs. As the flight duration of UAVs is limited, they need to be called back to the USV for recharging or for battery swapping. Therefore, the UAVs are required to be allocated near the USV while covering the flood area. The deployment, charging, and redeployment process of UAVs is illustrated by Fig. 1.

For the design of control strategy, a kinematic model for the UAVs is adopted in this paper. It is described by

$$\dot{\boldsymbol{p}}_i = \boldsymbol{u}_i + \boldsymbol{\xi}_i,\ i \in \{1, 2, ..., n\}, \tag{1}$$

where n is the number of UAVs, $\boldsymbol{p}_i \in \mathbb{R}^2$ represents the state (horizontal displacements) of a single UAV, $\boldsymbol{u}_i \in \mathbb{R}^2$ denotes the velocity controller, and $\boldsymbol{\xi}_i \in \mathbb{R}^2$ is the time-varying uncertainties.

The allocation of UAVs in the flood area formulates a coverage problem [4]. It is concerned with planning an optimal configuration of the agent network for the coverage of an area of interest, and with driving each agent to the desired position to realize the planned configuration. To achieve an optimal coverage configuration, a control method based on the CVT is proposed in this paper.

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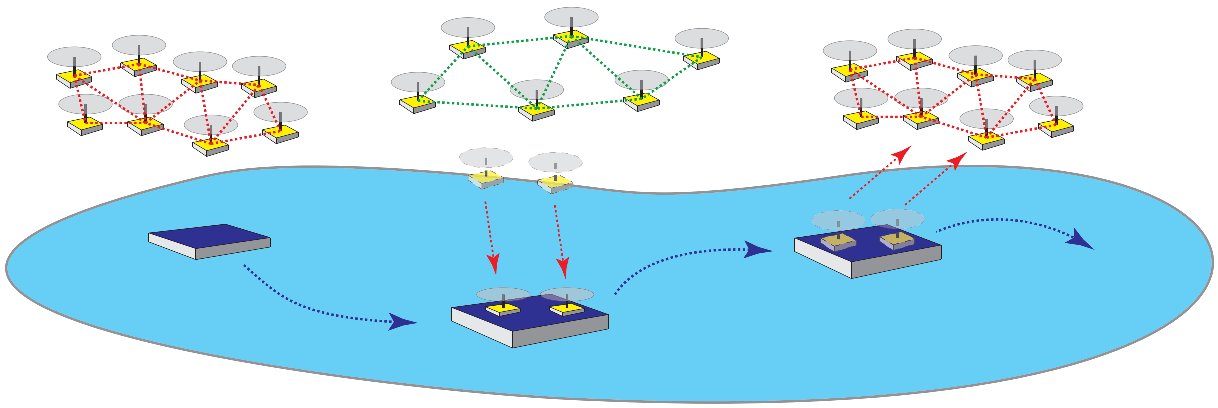

Fig. 1. Sketch of the deployment, charging, and redeployment process of UAVs.

1) CVT: Let $\mathcal{D} \subset \mathbb{R}^2$ be a 2-D dynamic region to be covered, represented by the blue polygon in Fig. 1, and ϕ be the associated density function, which is assumed to be positive, bounded, and continuously differentiable. It captures the relative importance of a point $\boldsymbol{x} \in \mathcal{D}$ at time t [6].

Let $\boldsymbol{p}_i \in \mathcal{D}$ represented by the black dots in Fig. 1, be the position of the i-th UAV and $\boldsymbol{p} = \{\boldsymbol{p}_i\}$ where $\{.\}$ denotes a collection of functions. The coverage problem is concerned with placing n UAVs in $\mathcal{D}$, dividing $\mathcal{D}$ into regions of dominance of the i-th UAV. Thus, a Voronoi tessellation is formulated, denoted by

$$V_i(\boldsymbol{p}) = \{\boldsymbol{x} \in \mathcal{D} \mid \|\boldsymbol{x} - \boldsymbol{p}_i\| \leq \|\boldsymbol{x} - \boldsymbol{p}_j\|, i \neq j\}, \tag{2}$$

such that the i-th UAV is in charge of each subregion V_i.

To measure the performance of the coverage for the multi-agent system (how well a given point $\boldsymbol{x} \in \mathcal{D}$ is covered by the i-th UAV at position $\boldsymbol{p}_i \in \mathcal{D}$), one can define the locational cost $\mathcal{H}(\boldsymbol{p}, t) = \sum_{i=1}^{n} \int_{V_i} \|\boldsymbol{x} - \boldsymbol{p}_i\|^2 \phi(\boldsymbol{x}, t) \, \mathrm{d}\boldsymbol{p}$, where

$$\dot{\mathcal{H}} = \sum_{i=1}^{n} \frac{\partial \mathcal{H}}{\partial \boldsymbol{p}_i} \dot{\boldsymbol{p}}_i + \frac{\partial \mathcal{H}}{\partial t}. \tag{3}$$

By assuming the cost function $\mathcal{H}$ varies rather slowly with respect to time, one has $\frac{\partial \mathcal{H}}{\partial t} = 0$ [4]. In [6], it was shown that

$$\frac{\partial \mathcal{H}}{\partial \boldsymbol{p}_i} = 2m_i(\boldsymbol{p}_i - \boldsymbol{c}_i)^\top. \tag{4}$$

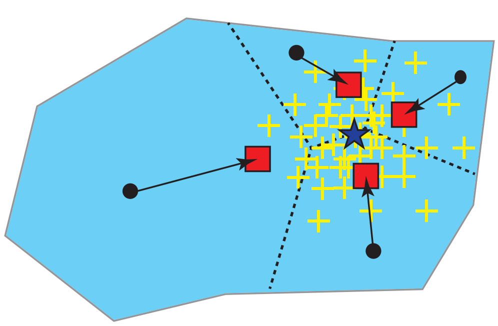

Fig. 2. Illustration of the coverage problem. Here, the red and black dots respectively denote the desired and actual positions of the UAVs. The star marks the position of the USV, and the surrounding yellow crosses represent the density function.

where the mass m_i and center of mass $\boldsymbol{c}_i$ of the i-th Voronoi cell V_i, represented by the red blocks in Fig. 1, are defined as

$$m_i(\boldsymbol{p}, t) = \int_{V_i} \phi(\boldsymbol{x}, t) \, \mathrm{d}\boldsymbol{x}, \;\; \boldsymbol{c}_i(\boldsymbol{p}, t) = \frac{\int_{V_i} \phi(\boldsymbol{x}, t) \, \boldsymbol{x} \mathrm{d}\boldsymbol{x}}{m_i}.$$

Note that ϕ strictly positive implies $m_i > 0$.

At a given time t, an optimal coverage for the domain $\mathcal{D}$ requires a configuration of UAVs $\boldsymbol{p}$ to minimize $\mathcal{H}$. From (4), one can see that a critical point is

$$\boldsymbol{p}_i(t) = \boldsymbol{c}_i(\boldsymbol{p}, t). \tag{5}$$

Remark 1: When (5) is satisfied, an agent network is said to be in a locally optimal coverage configuration [12]. The corresponding $\boldsymbol{p}$ defines the CVT.

Remark 2: The density function ϕ is visualized by the yellow crosses in Fig. 2. The center of the density function denotes the position of the USV. The UAVs require to stay close to the yellow region for the purpose of battery charging.

By defining the state error $\boldsymbol{e}_i = \boldsymbol{p}_i - \boldsymbol{c}_i$, (1) becomes

$$\dot{\boldsymbol{e}}_i = \boldsymbol{u}_i + \boldsymbol{d}_i - \dot{\boldsymbol{c}}_i. \tag{6}$$

The control problem can then be stated as constructing an asymptotically stabilizing law $\boldsymbol{u}_i$ for the multi-agent system to achieve a CVT ($\lim_{t\to\infty} \boldsymbol{e}_i(t) = \boldsymbol{0}$), in the presence of the time-varying uncertainty $\boldsymbol{d}_i$.

III. Controller Design

To deal with time-varying uncertainties in the multi-agent system, we propose an FATII based coverage control technique, in which the uncertainty $\boldsymbol{d}_i$ in (6) is approximated as

$$\boldsymbol{d}_i(t) = \sum_{j=1}^{N} \boldsymbol{d}_{ij} \psi_j(t), \tag{7}$$

where $\boldsymbol{d}_{ij}$ denotes unknown constant vectors, $\psi_j(t)$ is the basis function, selected as the Fourier series [13], [14] in this paper.

Substituting (7) into (6) yields

$$\dot{\boldsymbol{e}}_i = \boldsymbol{u}_i + \sum_{j=1}^{N} \boldsymbol{d}_{ij} \psi_j(t) - \dot{\boldsymbol{c}}_i. \tag{8}$$

Remark 3: In the FATII controller design [15], to remove the unknown term $\boldsymbol{d}_{ij}$ from the expression of $\dot{\boldsymbol{p}}_i$, one defines in the extended space $(\boldsymbol{p}_i, \hat{\boldsymbol{d}}_{ij})$ the manifold

$$\mathcal{M}_i = \{(\boldsymbol{p}_i, \hat{\boldsymbol{d}}_{ij}) \in \mathbb{R}^2 \mid \boldsymbol{d}_{ij} - \hat{\boldsymbol{d}}_{ij} - \boldsymbol{\beta}_{ij} = \boldsymbol{0}\}, \tag{9}$$

where $\hat{\boldsymbol{d}}_{ij} \in \mathbb{R}^{n\times 1}$ (the estimation of $\boldsymbol{d}_{ij}$) and $\boldsymbol{\beta}_{ij}(\boldsymbol{p}_i, t) \in \mathbb{R}^{n\times 1}$ are functions to be specified. By defining the off-the-manifold variable $\boldsymbol{z}_{ij} = \boldsymbol{d}_{ij} - \hat{\boldsymbol{d}}_{ij} - \boldsymbol{\beta}_{ij}$ where $\boldsymbol{z}_{ij} \in \mathbb{R}^{n\times 1}$, (8) is transformed to

$$\dot{\boldsymbol{e}}_i = \boldsymbol{u}_i + \sum_{j=1}^{N} \big(\boldsymbol{z}_{ij} + \hat{\boldsymbol{d}}_{ij} + \boldsymbol{\beta}_{ij}\big)\psi_j - \dot{\boldsymbol{c}}_i, \tag{10}$$

Here, $\boldsymbol{z}_{ij} = \boldsymbol{0}$ implies that for each agent i, the system dynamics stays on the manifold $\mathcal{M}_i$.

The FATII based coverage controller is then designed as

$$\begin{aligned}
\boldsymbol{u}_i &= -k_i \boldsymbol{e}_i - \sum_{j=1}^{N} (\hat{\boldsymbol{d}}_{ij} + \boldsymbol{\beta}_{ij})\psi_j, \\
\dot{\hat{\boldsymbol{d}}}_{ij} &= -\boldsymbol{e}_i \dot{\psi}_j + (k_i \boldsymbol{e}_i + \dot{\boldsymbol{c}}_i)\psi_j, \\
\boldsymbol{\beta}_{ij} &= \boldsymbol{e}_i \psi_j,
\end{aligned} \tag{11}$$

where $k_i = k + \frac{m_i}{4}$ and k is a positive constant.

Theorem 1: The closed loop system, formulated by (10) and (11), is asymptotically stable.

Proof 1: Substituting (11) into (10) yields

$$\dot{\boldsymbol{e}}_i = -k_i \boldsymbol{e}_i + \sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j - \dot{\boldsymbol{c}}_i. \tag{12}$$

The derivative of $\boldsymbol{z}_i$ is computed as

$$\begin{aligned}
\dot{\boldsymbol{z}}_{ij} &= -\dot{\hat{\boldsymbol{d}}}_{ij} - \frac{\partial \boldsymbol{\beta}_{ij}}{\partial \boldsymbol{e}_i}\dot{\boldsymbol{e}}_i - \frac{\partial \boldsymbol{\beta}_{ij}}{\partial t} \\
&= \boldsymbol{e}_i \dot{\psi}_j - k_i \boldsymbol{e}_i \psi_j - \dot{\boldsymbol{c}}_i \psi_j \\
&\quad -\psi_j \Big(- k_i \boldsymbol{e}_i + \sum_{k=1}^{N} \boldsymbol{z}_{ik}\psi_k - \dot{\boldsymbol{c}}_i \Big) - \boldsymbol{e}_i \dot{\psi}_j \\
&= -\psi_j \Big(\sum_{k=1}^{N} \boldsymbol{z}_{ik}\psi_k \Big).
\end{aligned} \tag{13}$$

To prove the stability of the closed-loop system, the Lyapunov candidate function is chosen as

$$V = \frac{1}{2}\mathcal{H} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{N} \boldsymbol{z}_{ij}^\top \boldsymbol{z}_{ij}, \tag{14}$$

the derivative of which is calculated as

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n} \frac{\partial \mathcal{H}}{\partial \boldsymbol{p}_i}\dot{\boldsymbol{p}}_i + \sum_{i=1}^{n}\sum_{j=1}^{N} \boldsymbol{z}_{ij}^\top \dot{\boldsymbol{z}}_{ij} \\
&= \sum_{i=1}^{n} m_i \boldsymbol{e}_i^\top (\dot{\boldsymbol{e}}_i + \dot{\boldsymbol{c}}_i) + \sum_{i=1}^{n}\sum_{j=1}^{N} \boldsymbol{z}_{ij}^\top \dot{\boldsymbol{z}}_{ij}.
\end{aligned} \tag{15}$$

Substituting (12) and (13) into (15) yields

$$\begin{aligned}
\dot{V} =& \sum_{i=1}^{n} m_i \boldsymbol{e}_i^\top \left(-k_i \boldsymbol{e}_i + \sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j \right) \\
& - \sum_{i=1}^{n} \Big(\sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j \Big)^\top \Big(\sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j \Big) \\
=& \sum_{i=1}^{n} \left(m_i \boldsymbol{e}_i^\top \Big(- k_i \boldsymbol{e}_i + \sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j \Big) + \frac{1}{4} m_i^2 \boldsymbol{e}_i^\top \boldsymbol{e}_i \right. \\
& \left. - \frac{1}{4} m_i^2 \boldsymbol{e}_i^\top \boldsymbol{e}_i - \Big(\sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j \Big)^\top \Big(\sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j \Big) \right) \\
\leq& -\sum_{i=1}^{n} \left(m_i \Big(k_i - \frac{m_i}{4} \Big) \|\boldsymbol{e}_i\|_2^2 + \left\| \frac{m_i}{2}\boldsymbol{e}_i - \sum_{j=1}^{N} \boldsymbol{z}_{ij}\psi_j \right\|_2^2 \right).
\end{aligned}$$

By selecting $k_i = k + \frac{m_i}{4}$ where $k > 0$, $\dot{V}$ is negative semi-definite. According to the Barbalat's lemma [16, Lemma 4.3], both the state and the estimation error converge to zero.

IV. Case study

The simulation results are presented in Fig. 3, where the blue polygon represents a dynamically changing flood region $\mathcal{D} \subset \mathbb{R}^2$. The speeds of vertices of the polygon vary randomly between 0m/s and 1.5m/s along x and y directions (samples are uniformly drawn from $[0, 1.5)$). The star marks the position of the USV, which also denotes the center of the density distribution. At the beginning, there are 8 UAVs in the flood area, where the red and black dots respectively denote the desired and actual positions of the UAVs. The initial positions of the dots are set as shown in Fig. 3(a). After 30s, two UAVs leave for charging and the desired positions for the rest 6 UAVs are reconfigured, represented by the green dots. Finally, after charging for 30s, the 2 UAVs return to the multi-agent network and the new configuration of the network is denoted by the red dots again. An optimal coverage of the flood region is achieved if the distances between the red (green) and black dots converge to zero (see Remark 1).

The gain value for the feedback controller adopted in the simulation is $k_i = 40$, and the number of the basis functions is selected as $N = 5$. The disturbance $\boldsymbol{\xi}_i$ to the control system is chosen as

$$\boldsymbol{\xi}_i = \begin{bmatrix} 20\cos(\pi t) & 20\sin(\pi t) \end{bmatrix}^\top, \tag{16}$$

which is sufficiently large.

Given $\boldsymbol{x} \in \mathcal{D}$ where $\boldsymbol{x} = \begin{bmatrix} x & y \end{bmatrix}^\top$, the time-varying density function $\phi(\boldsymbol{x}, t)$ is defined by a bivariate normal distribution, as

$$\phi = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{1}{2}\Big(\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} \Big) \right). \tag{17}$$

The coefficients of (17) are chosen as $\sigma_x = \sigma_y = 30$, $\mu_x = 100 + 2t$, $\mu_y = 150 + 2t$. It implies that the center of the density distribution moves along straight lines while its radius remains constant.

The density function (17) is visualized in Fig. 3. The UAVs are expected to adaptively allocated around the center of the density function (the position of the USV) while covering the flood region. In the simulation results, the multi-agent system converges to an optimal configuration (distances between the red (green) and black dots converge to zero (see Remark 1)) in the presence of the disturbance (16).

V. Conclusions

In this paper, a control strategy of multiple UAVs has been proposed for covering a dynamic flood regions with time-varying density functions in the presence of time-varying uncertainties. An optimal coverage configuration is generated though a Voronoi-based algorithm, during which process the change of UAVs' number are taken into consideration when some UAVs leave the network for charging. The UAVs are adaptively driven to the generated configuration by the proposed FATII based controller. The stability of the proposed controller has been established and its validity has been tested under simulations.

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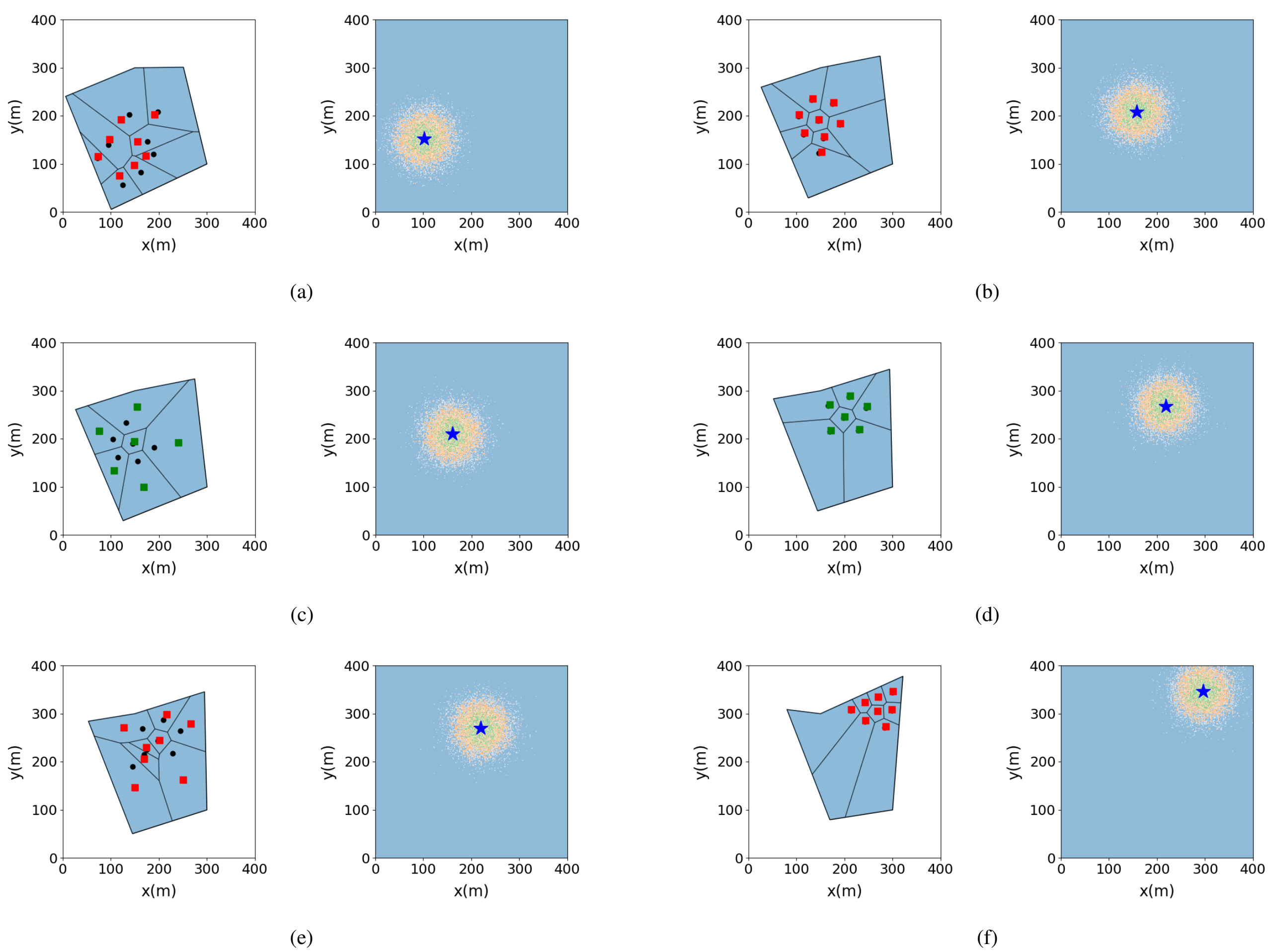

Fig. 3. Coverage of a dynamic flood area with a time-varying density function in the presence of time-varying disturbances: (a) t=0s, (b) t=30s, (c) t=31s, (d) t=60s, (e) t=61s, (f) t=100s. The number of UAVs changes from 8 to 6 when t=31s, and from 6 to 8 when t=61s.

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