Exact Nonlocal Boundary Conditions in the Theory of Dielectric Waveguides

R. Z. Dautov and E. M. Karchevskiy
Kazan State University, Russia

Abstract— Optical waveguides are dielectric cylindrical structures that can conduct electromagnetic energy in the visible and infrared parts of the spectrum. The waveguides used in optical communication are flexible fibers made of transparent dielectrics. The cross section of a waveguide usually consists of three regions: the central region (core) is surrounded by a cladding which, in turn, is surrounded by a protective coating. The refractive index of the core can be constant or can vary over the cross section; the refractive index of the cladding is usually constant. The coating is optically isolated from the core; for this reason, it is usually neglected in mathematical models, and it is assumed that the cladding is unbounded from the outside. We use the classical model, in which the waveguide is assumed to be unbounded and linearly isotropic.

The original problem is formulated for the entire cross-sectional plane and (except for a point spectrum) has a continuous fragment of the spectrum. Although the location of this fragment is known exactly, a numerical solution requires that false approximate solutions be detected and discarded. The original problem using exact nonlinear boundary conditions is reduced to a linear parametric eigenvalue problem in a circle, which is convenient for numerical solution. This problem has no continuous spectrum. Moreover, its spectrum is identical to the point part of the spectrum of the original problem. The examination of solvability of the obtained problem is based on the spectral theory of compact self-adjoint operators. The existence of surface waves is proved, and properties of the dispersion curves are investigated. An algorithm for the numerical solution of the problem based on the discretization of the equations using the finite element method is proposed. The convergence of the numerical method is proved. Numerical results are discussed.