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A generalised Milne-Thomson theorem for the case of an elliptical inclusion

Yu. V. OBNOSOV and A. V. FADEEV

Prof. Nuzhin Str., 1/37, Kazan 420008, Russia
email: yobnosov@ksu.ru, afadeev@ksu.ru

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An $\mathbb{R}$-linear conjugation problem modelling the process of power fields forming in a heterogeneous infinite planar structure with an elliptical inclusion is considered. Exact analytical solutions are derived in the class of piece-wise meromorphic functions with their principal parts fixed. Cases with internal singularities and with singularities of the given principal parts at the interface are investigated.

Key words: Heterogeneous medium; Elliptic inclusion; $\mathbb{R}$-linear conjugation problem; Analytic functions

1 Introduction

The study of heterogeneous media is of great importance in several branches of mechanics of continua. This is important for manufactured composite materials as well as for investigations of natural ones (soils, aquifers, biological tissues etc.) and designed inhomogeneous objects. Mathematical models describing transport processes are, in general, 3D and transient, and therefore of utmost difficulty for analytical solution. Two-dimensional plane or axisymmetrically heterogeneous structures exposed to steady physical fields (temperature, concentration, fluid pressure, electrostatic stress etc.) are easier to tackle because the Fourier, Fick, Hook, Darcy, Ohm and other 'linear laws' reduce the field problem to the Laplace's equation in the homogeneous and isotropic components of the composite and, therefore, the complex-analysis theory is applicable. Even for a plane structure the possibility to get an explicit solution is problematic: success in analytical treatment depends on the geometry of the composite. Among all plane structures, the most intensively studied, since the seminal contribution of Maxwell [11] and Lord Rayleigh [21], is a medium consisting of an infinite isotropic matrix with an elliptical, in particular circular [12], inclusion. There are numerous papers devoted to this specific topic and only those reporting analytical or semi-analytical solutions are cited below.

Apparently, the works [2, 8] were the first where the problem of determination of the elastic field induced by an elliptical inhomogeneity was considered. Results of the latter paper were generalised in [4, 5, 14] for the 3D case of ellipsoidal inclusions. In particular, Hardiman [8] and Eshelby [5] proved that a uniform stress applied at infinity induces a constant stress state within an elliptical or ellipsoidal inhomogeneity. Eshelby [5] put forward the following conjecture: The field inside a bounded inclusion will be uniform for