The Cauchy transform of certain distributions with application*

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Abstract

We research boundary properties of the Cauchy transform of certain distributions with supports on non-rectifiable curves and apply these results for solution of the Riemann boundary value problem.

Introduction.

A number of recent publications (see, for instance, [1, 2, 3]) is dealing with various properties of the Cauchy transforms of measures. If μ is a finite measure on the complex plane, then its Cauchy transform is integral

$$\mathcal{C}\mu := \frac{1}{2\pi i}\int \frac{d\mu(\zeta)}{\zeta - z}.$$

In particular, if support S of the measure μ is rectifiable curve, $d\mu = f(\zeta)d\zeta$ and $f(\zeta)$ is integrable (with regard to the length of S) function, then we obtain the Cauchy integral

$$\mathcal{C}(f(\zeta)d\zeta) = \frac{1}{2\pi i}\int_\Gamma \frac{f(\zeta)d\zeta}{\zeta - z}. \tag{1}$$

On the other hand, if φ is a distribution with compact support S on the complex plane, then its Cauchy transform is defined by equality

$$\mathcal{C}\varphi := \frac{1}{2\pi i}\langle\varphi, \frac{1}{\zeta - z}\rangle,$$

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