Universal Algorithm for the Accurate Computation of the Modal Characteristics of Arbitrary-Shape Optical Fibers

E.M. Karchevskii

Kazan State University, Department of Applied Mathematics
18, Kremlevskaia street, Kazan, 420008, Russia
e-mail: Evgenii.Karchevskii@ksu.ru

Introduction. This paper is devoted to determining the complex propagation coefficients of surface-wave and leaky-wave eigenmodes of arbitrary-shape optical fibers. The original physical problem is reduced to a nonlinear spectral problem for Fredholm system of integral equations. We propose Galerkin's method for the calculating of approximate values of the complex propagation coefficients. A practical efficiency of this method is been shown by comparing of solutions of some problems of the theory of electromagnetic waves with experimental data and the results obtained by other methods.

Formulation of the Problem. The original physical problem of determining the complex propagation constants of surface-wave and leaky-wave eigenmodes of arbitrary-shape optical fibers is reduced to the nonlinear problem for the system of Helmholtz equations on the plane [1], with Reichardt conditions on infinity [2]:

(1) $$\Delta u+\chi_j^2(\beta)u=0,\ \Delta v+\chi_j^2(\beta)v=0,\quad M\in\Omega_j,\ j=1,2,$$

(2) $$u^+=u^-,\quad \chi_1^{-2}(\beta)\left(\beta\frac{\partial v}{\partial\tau}+\varepsilon_1\omega\frac{\partial u^-}{\partial n}\right)=\chi_2^{-2}(\beta)\left(\beta\frac{\partial v}{\partial\tau}+\varepsilon_2\omega\frac{\partial u^+}{\partial n}\right),\quad M\in\Gamma,$$

(3) $$v^+=v^-,\quad \chi_1^{-2}(\beta)\left(\beta\frac{\partial u}{\partial\tau}-\mu_0\omega\frac{\partial v^-}{\partial n}\right)=\chi_2^{-2}(\beta)\left(\beta\frac{\partial u}{\partial\tau}-\mu_0\omega\frac{\partial v^+}{\partial n}\right),\quad M\in\Gamma,$$

(4) $$u=\sum_{k=-\infty}^{\infty}\alpha_k H_k^{(1)}(\chi_2(\beta)r)\exp(ik\varphi),\quad v=\sum_{k=-\infty}^{\infty}\gamma_k H_k^{(1)}(\chi_2(\beta)r)\exp(ik\varphi),\quad r\ge R.$$

Here, $\chi_j^2=k_0^2n_1^2-\beta^2$; $k_0^2=\omega^2\varepsilon_0\mu_0$; ω is the frequency of electromagnetic oscillations; ε_0 and μ_0 are the permittivity and permeability of vacuum, respectively; $\varepsilon_j=\varepsilon_0 n_j^2$; n_1 and n_2 are the refractive indexes of the fiber and environment; Ω_1 is a bounded domain with the boundary $\Gamma=\{M\in R^2: r=r(t), t\in[0,2\pi]\}$, $r(t)\in C^2$; $\Omega_2=R^2\setminus\bar{\Omega}_1$; $\partial u/\partial n$ is the derivative normal to the contour Γ; $\partial u/\partial\tau$ is the derivative tangential to the contour Γ; f^+ (f^-) is the limit value of the function f from the exterior (interior) of the contour Γ; r and φ are the polar coordinates of the point M; $H_n^{(1)}$ is Hankel function of the first kind and n-th order; and $u,v\in C^2(\Omega_1\cup\Omega_2)\cap C^1(\bar{\Omega}_1)\cap C^1(\bar{\Omega}_2)$.

The axial propagation coefficient β is an unknown complex parameter, $\beta\in H_1\cap H_2$, H_j is Reimann surface of the function $\ln\chi_j(\beta)$. The surface H_j has an infinite number of the complex sheets. The "proper" one (H_j^0) is specified by the conditions: $-\pi/2<\arg\chi_j(\beta)<3\pi/2$, $\operatorname{Im}\chi_j(\beta)\ge 0$. The propagation coefficients β at $\Lambda_0=H_1^0\cap H_2^0$

0-7803-5637-3/99/\$10.00 ©1999 IEEE

may belong only (see [3]) to $G=\{\beta\in\Lambda_0:\operatorname{Im}\beta=0,\ k_0n_2<|\beta|<k_0n_1\}$ and to $\Lambda_0\setminus(\{\operatorname{Re}\beta=0\}\cup\{\operatorname{Im}\beta=0\})$.

Regularization of the Problem. We use the representation of the functions u and v in form of single-layer potentials:

$$u=\int_\Gamma \Phi_j(\beta;M,M_0)\varphi_j(M_0)dl_{M_0},\quad v=\int_\Gamma \Phi_j(\beta;M,M_0)\psi_j(M_0)dl_{M_0},\quad M\in\Omega_j,$$

$$\Phi_j(\beta;M,M_0)=\frac{i}{4}H_0^{(1)}\left(\chi_j(\beta)|M-M_0|\right),\quad \varphi_j,\psi_j\in C^{0,\alpha}(\Gamma).$$

We obtain a nonlinear spectral problem for the following set of integral equation (see [4]):

(5)
$$A(\beta)z\equiv\left(C(\beta)+R(\beta)\right)z=0,$$
$$C(\beta)=\left[C_{i,j}(\beta)\right]_{i,j=1}^4,\quad R(\beta)=\left[R_{i,j}(\beta)\right]_{i,j=1}^4,\quad z=\left[z_i\right]_{i=1}^4,$$
$$C_{1,1}=C_{2,2}=I,\quad C_{1,2}=C_{1,3}=C_{1,4}=C_{2,1}=C_{2,3}=C_{2,4}=0,$$
$$C_{3,1}=\omega\varepsilon_2\chi_2^{-2}I,\quad C_{3,2}=C_{4,1}=\beta\varepsilon_2\chi_2^{-2}S,\quad C_{3,3}=-\omega\left(\varepsilon_1\chi_1^{-2}+\varepsilon_2\chi_2^{-2}\right)I,$$
$$C_{3,4}=C_{4,3}=\beta\left(\chi_1^{-2}-\chi_2^{-2}\right)S,\quad C_{4,2}=-\omega\mu_0\chi_2^{-2}I,\quad C_{4,4}=\omega\mu_0\left(\chi_1^{-2}+\chi_2^{-2}\right)I,$$
$$R_{1,1}=R_{2,2}=L^{-1}R_2^{(1)}(\beta),\quad R_{1,3}=R_{2,4}=L^{-1}\left(R_1^{(1)}(\beta)-R_2^{(1)}(\beta)\right),\quad R_{1,2}=R_{1,4}=0,$$
$$R_{2,1}=R_{2,3}=0,\quad R_{3,1}=-\omega\varepsilon_2\chi_2^{-2}R_2^{(2)}(\beta),\quad R_{3,2}=R_{4,1}=\beta\chi_2^{-2}R_2^{(3)}(\beta),$$
$$R_{3,3}=-\omega\left(\varepsilon_1\chi_1^{-2}R_1^{(2)}(\beta)+\varepsilon_2\chi_2^{-2}R_2^{(2)}(\beta)\right),\quad R_{4,4}=\omega\mu_0\left(\chi_1^{-2}R_1^{(2)}(\beta)+\chi_2^{-2}R_2^{(2)}(\beta)\right),$$
$$R_{3,4}=R_{4,3}=\beta\left(\chi_1^{-2}R_1^{(3)}(\beta)-\chi_2^{-2}R_2^{(3)}(\beta)\right),\quad R_{4,2}=\omega\mu_0\chi_2^{-2}R_2^{(2)}(\beta),$$
$$Sx=\frac{1}{2\pi}\int_0^{2\pi}\operatorname{ctg}\frac{t_0-t}{2}x(t_0)dt_0+\frac{i}{2\pi}\int_0^{2\pi}x(t_0)dt_0,\quad Lx=-\frac{1}{2\pi}\int_0^{2\pi}\ln\left|\sin\frac{t-t_0}{2}\right|x(t_0)dt_0,$$
$$R_j^k(\beta)x=\frac{1}{2\pi}\int_0^{2\pi}h_j^{(k)}(\beta;t,t_0)x(t_0)dt_0,\quad h_j^{(1)}(\beta;t,t_0)=2\pi\Phi_j(\beta;M,M_0)+\ln\left|\sin\frac{t-t_0}{2}\right|,$$
$$h_j^{(2)}(\beta;t,t_0)=4\pi|r'(t)|\frac{\partial\Phi_j(\beta;M,M_0)}{\partial n_M},\quad h_j^{(3)}(\beta;t,t_0)=2|r'(t)|\frac{\partial h_j^{(1)}(\beta;M,M_0)}{\partial\tau_M}-i,$$
$$z_1(t)=\left(\varphi_1(M)-\varphi_2(M)\right)|r'(t)|,\quad z_2(t)=\left(\psi_1(M)-\psi_2(M)\right)|r'(t)|,$$
$$z_3(t)=\varphi_1(M)|r'(t)|,\quad z_4(t)=\psi_1(M)|r'(t)|.$$

Spectral problem (1) - (4) is equivalent to the problem (5) for all $\beta\in\Lambda_0$. The operator-valued function $A(\beta):H\to H$, $H=C^{0,\alpha}\times C^{0,\alpha}\times C^{0,\alpha}\times C^{0,\alpha}$, is Fredholm holomorphic in Λ operator-valued function. Thus, the spectrum of the problem (1) - (4) may consists of only isolated points.

Numerical Algorithm. For a numerical solution of the problem (5), a discretized matrix equation is derived by Galerkin's method based on trigonometric basis functions. The singularities of the kernels of the integral operators are separated analytically. The singular operators $L^{-1}: C^{1,\alpha} \to C^{0,\alpha}$ and $S: C^{0,\alpha} \to C^{0,\alpha}$ has a known spectrum:

$$\lambda_k^{L^{-1}} = \{1/\ln 2, \text{ for } k = 0; \quad 2|k|, \text{ for } k \neq 0\},$$
$$\lambda_k^S = \{i, \text{ for } k = 0; \quad i\,\mathrm{sign}(k), \text{ for } k \neq 0\}$$

with the trigonometric eigenfunctions

$$\exp(ikt), \quad k = 0, \pm 1, \pm 2, \ldots, t \in [0, 2\pi].$$

Determinant zeros β_n of the matrix $A_n(\beta)$ of this system (n being the number of basis function) are assumed as the approximation values of the propagation coefficients β. The convergence of this method has been studied in [5].

Numerical Results. In order to access the efficiency of the described method, we solved the problem (1) - (4) for waveguides of circular, elliptic, rectangular, and triangular cross- section.

The dispersion characteristics of the complex EH_{11} modes of a circular cross-section dielectric fiber were constructed by the proposed method. It was found, that even for the number of basis functions $n = 1$, they coincided exactly with the dispersion characteristics [6] obtained by the eigenfunction expanded method.

The dispersion characteristics of the fundamental modes of an elliptic cross-section waveguide with the ratio of semiaxes equal to 1.31 were constructed. It was found, that even for the number of basis function $n = 2$, they coincided exactly with the dispersion characteristics [7] obtained by the method of separation of variables. A further increase in n did not improve the computational accuracy.

The solution of the problem (1) - (4) for the waveguide of rectangular cross-section was based on the approximation of the contour by the curve

$$r(t) = \left[\left(\frac{\cos t}{a}\right)^{2N} + \left(\frac{\sin t}{b}\right)^{2N}\right]^{-(2N)^{-1}}, \quad t \in [0, 2\pi].$$

As $N \to \infty$, this curve tends to a rectangle with the sides $2a$ and $2b$.

As in [8], we obtained the dispersion characteristics: the dependence of $h = \beta / k_0$ on $p = 2b / \lambda$, $\lambda = 2\pi / \omega$, for the fixed values of ε_1, ε_2, and a / b. The results of computations for $\varepsilon_1 = 2.08$, $\varepsilon_2 = 1$, and $a / b = 1.5$ are shown in Fig. 1. by a solid curve for the fundamental modes and by a dashed curve for higher-order modes. The stars indicate the experimental data of [8]. The results demonstrated in Fig. 1 were obtained for $n = 3$. The method exhibits a stable internal convergence. For instance, the modulus of the difference between the values of h obtained for $n = k$ and $n = k + 1$ did not exceed $\Delta \approx 10^{-2}$ for $k = 2$, $\Delta \approx 10^{-4}$ for $k = 3$, and $\Delta \approx 10^{-6}$ for $k = 4$. All computations were carried out for $N = 20$. A further increase in N did not influence the accuracy of the computations.

We also considered the problem (1) - (4) for a waveguide with a cross-section in the form of an equilateral triangle. The contour was approximated by a curvilinear triangle:

$$x(t) = a\left(\sin t - \frac{\sin(2t)}{3}\right), \quad y(t) = -a\left(\cos t + \frac{\cos(2t)}{3}\right), \quad t \in [0, 2\pi], \quad M(t) = (x(t), y(t)).$$

The results of calculations for the fundamental modes were compared with those obtained by the point-matching method in [9], and by the constant field approximation method in [10]. We obtained the dispersion characteristics: the dependence of $q = (h^2 - \varepsilon_2)/(\varepsilon_1 - \varepsilon_2)$ on $v = 2/\sqrt{3}\, a k_0 (\varepsilon_1 - \varepsilon_2)^{1/2}$, for the fixed values of $\varepsilon_1 = 2.31$, and $\varepsilon_2 = 2.25$. The results of computations for $n = 2$ are shown in Fig. 2. by a solid curve. The stars indicate the point-matching solution [9]. The dashed curve indicate the constant field approximation solution [10]. The internal convergence of the method was the same as in the previous case.

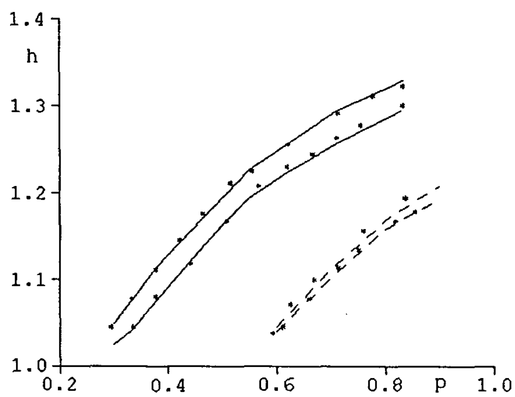

Fig. 1.

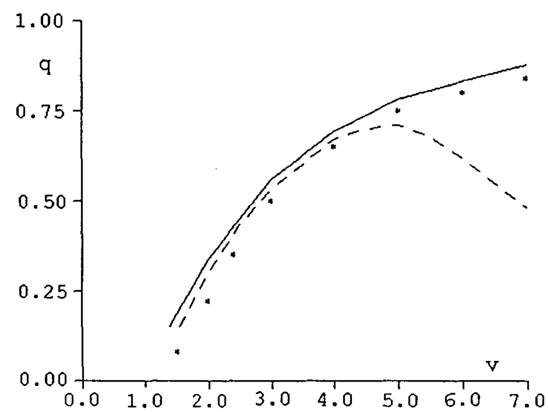

Fig. 2.

References

1. Voitovich N.N., Katsenelenbaum B.Z., Sivov A.N., and Shatrov A.D., Eigenwaves of Dielectric Waveguides of Compound Cross - Sectional Shape (Review), Radiotekh. Electron. (Moscow), vol. 24, no. 7, pp. 1247-1263, 1979.
2. Reichardt H., Ausstrahlungsbedingungen fur die Welengleihung, Abh. Mathem. Seminar, Univ. Hamburg, vol. 24, pp. 41-53, 1960.
3. Nosich A.I., Excitation and propagation of waves on cylindrical microstrip and slot lines, and other open waveguides, D. Sc. dissertation, Dept. Radio Physics, Kharkov State University, 1990 (in russian).
4. Karchevskii E.M., Determination of the Propagation Constants of Dielectric-Waveguide Eigenmodes by Methods of Potential Theory, Computational Mathematics and Mathematical Physics, vol. 38, no. 1, pp. 132-136, 1998.
5. Karchevskii E.M. Study of the Numerical Method for Solving Spectral Problem in the Theory of Dielectric Waveguides, Izv. Vyssh. Uchebn. Zaved., Mat. no. 1, 1999 (accepted for publication).
6. Jablonski T.F. Complex modes in open lossless dielectric waveguides, J. Opt. Soc. Am. A/Vol. 11, no. 4, pp. 1272-1282, 1994.
7. Lyubimov L.A., Veselov G.I., and Bei N.A., Dielectric Waveguide of Elliptic Cross- Section, Radiotekh. Electron. (Moscow), vol. 6, no. 11, pp. 1871-1880, 1961.
8. Goncharenko A.M. and Karpenko V.A., Osnovi teorii opticheskih volnovodov (Fundamentals of Optical Waveguide Theory), Minsk: Nauka i Tekhnika, 1983 (in russian).
9. James J.R. and Gallet I.N.L., Modal Analysis of Triangular-cored Glassfibre waveguid, IEE Proc., vol. 120, no.11, pp. 1362-1370, 1973.
10. Keuster E.F., Pate R.C., Fundamental Mode Propagation on Dielectric Fibers of Arbitrary Cross-Section, IEE PROC-H, vol. 126, no. 1, pp. 41-47, 1980.