224. Measuring viscosity by the Poiseuille Method

The Poiseuille method is based on determination of flow of liquid or gas through a capillary of known sizes due to a certain pressure difference on the ends of the capillary.
Consider a cylindrical tube with a radius R and a length l. Let liquid with a density ρ and a viscosity η flows through this capillary. The pressure difference on both ends of the tube is ΔP.
Imagine a cylindrical part of volume in flowing liquid which is coaxial with the tube and has a radius r and the length l (the same as the tube). It experiences an external force F^e due to the pressure difference:

$$F^{\mathrm{e}} = \pi r^2 \Delta P .$$

If the flow is stationery (with no acceleration), this force is equal to the inner friction force which is defined by the Newton's formula:

$$F^{\mathrm{if}} = -\eta S_{\mathrm{side}} \frac{dv}{dr} = -\eta 2\pi r l \frac{dv}{dr}, \quad F^{\mathrm{if}} = F^{\mathrm{e}} = S_{\mathrm{bas}} \Delta P = \pi r^2 \Delta P ,$$

where S_{side} and S_{bas} are the areas of the side wall and the basis of the imaginary volume element. From this relation we derive the differential equation:

$$dv = -\frac{\Delta P}{2\eta l} r dr .$$

If liquid wets the tube's walls, then we can assume that the layer which is the most close to the wall is motionless. Thus, the speed of the flow changes from 0 at the wall to some value v in the volume of the capillary, and the previous differential equation can be integrated as follows:

$$\int_0^v dv = -\frac{\Delta P}{2\eta l} \int_R^r r dr .$$

Thus, at a given distance r from the axis the liquid flows at the speed of

$$v = \frac{\Delta P}{4\eta l}\left(R^2 - r^2\right) = v_0 - \frac{\Delta P}{4\eta l} r^2 . \qquad (1)$$

Obviously, v_0 is the speed in the central part of the capillary (on the axis).
If now we consider a thin cylindrical layer with the inner radius r and the outer radius ($r + dr$), then liquid in it can be assumed [1] to move at the speed v. Then the flow (the mass of the liquid which flows through its cross-sectional area dS per time unit) can be calculated as

---

[1] If the layer is thin enough, we can neglect the fact that the speeds on its inner wall and the outer wall are slightly different.

$$dQ = \rho \cdot vdS = \rho \frac{\Delta P}{4\eta l}\left(R^2 - r^2\right)(2\pi rdr) .$$

The flow through the whole tube of the radius R is then

$$Q = \frac{\pi \Delta P \rho}{2\eta l}\int_0^R (R^2 - r^2) rdr = \frac{\pi \Delta P \rho R^4}{8\eta l} = \frac{v_0 \rho S}{2} . \qquad (2)$$

This expression is known as the Poiseuille formula (named after an English researcher who studied the flow of liquid). Using this formula, one can determine the viscosity by transmitting liquid through a tube having the length l and the radius R. The parameters which have to be measured are the pressure difference at the tube's ends ΔP and the volume flow Q.

However, one nuance should be stressed. We should be sure that inner friction has a significant effect on the flow value. Too small Q would make the measurement impractically slow; too fast Q will result in a turbulent flow rather than laminar. The minimal radius R is defined by the possibility of making the thin glass tube with an exactly known radius; it is on the order of 1 mm. The length should be about 100 … 1000 times larger than the diameter.

Aim of the work: getting acquainted with the Poiseuille method of measuring viscosity.

Tasks to solve:

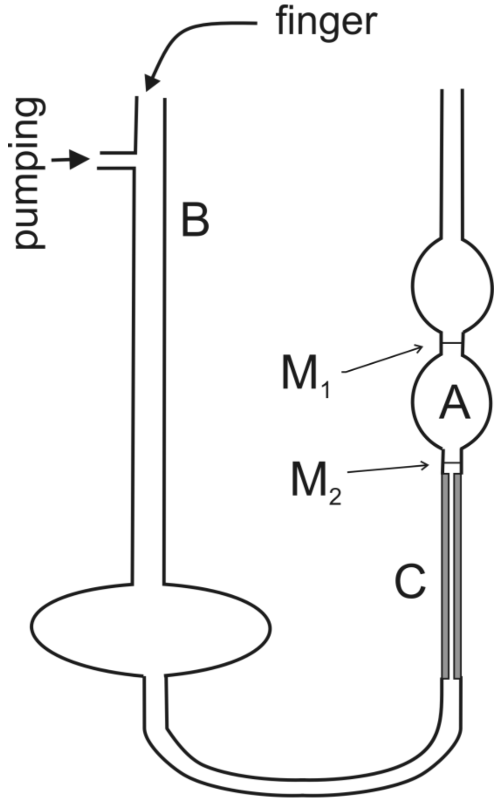
- Learning theoretical basis of the Poiseuille method.
- Getting acquainted with the Ostwald viscometer.
- Calibrating the Ostwald viscometer.
- Measuring the viscosity of ethanol by the Poiseuille method.

An Ostwald viscometer is made of two communicating glass vessels of variable diameters, into which the liquid is poured through the wide neck of the bend B. The principle of working relies on measuring the time which it takes for a standard and investigated liquid samples of identical volumes to pass through identical capillaries C. Using a rubber bulb, liquid is pumped from the bend B to the bulb A above the mark M_1, and then it is let to flow back due to the force of gravity. The time t needed for the liquid meniscus to go between the marks M_1 and M_2 is measured by a stopwatch. Direct using of the Poiseuille equation is difficult since too many parameters should be known. Therefore, it is easier to measure the ratio of the viscosities of the studied and the reference liquids, e.g, water (having the viscosity η_w and passing between the marks of the viscometer in the time t_w). Since the pressure differences on the capillary's ends is proportional to the liquid's densities (ρ and ρ_w), we can write down:

$$\frac{\eta}{\eta_w} = \frac{(\pi t \Delta P r^4)/(8Vl)}{(\pi t_w \Delta P_w r^4)/(8Vl)} = \frac{\Delta P t}{\Delta P_w t_w} = \frac{\rho t}{\rho_w t_w},$$

or

$$\eta = \eta_w \frac{\rho t}{\rho_w t_w}.$$



Instruments

Thermometer, stopwatch, two Ostwald viscometers.

An Ostwald viscometer is a fragile device. For this reason, two identical viscometers already filled with water and ethanol are placed in a transparent casing. A rubber bulb can be attached to a viscometer through a hose.

Algorithm of measurements

1. Close the tube B of the viscometer with your finger and pump the liquid (water) from the bend B into the bulb A higher than the mark M_1. To avoid rough inaccuracies it is necessary that the bulb B was still partially filled with liquid. Note also that the hole above the bulb A is not blocked occasionally with a droplet. In case of problems, call the engineer.

2. Take away the finger. Measure the time t_w during which water will go from mark M_1 to mark M_2.
3. Repeat the measurements several times to find the mean value $\langle t_w \rangle$ and the inaccuracy.
4. Repeat steps 1-3 for the other liquid (ethanol).
5. Measure the temperature in the room. Using reference tables, find the data for water and ethanol (ρ, ρ_w, η_w).
6. Calculate the viscosity of ethanol and estimate the inaccuracy.

Questions

1. Inner friction in liquids. Newton's formula.
2. Physical meaning of the coefficient of dynamical viscosity.
3. Newtonian and non-Newtonian liquids.
4. Which factors define the friction force which resists the flow in a tube?
5. Design of the Ostwald viscometer. Poiseuille method.
6. * Estimate the pressure inside a syringe during the injection.

Hints for finding the inaccuracy

Let the errors in the flow times be Δt_1 and Δt_2. Here 1 and 2 are the numbers of the liquids, and we want to find the viscosity of the first liquid (η_2 is known).

$$\Delta\eta_1 = \left[\left(\frac{\partial\eta_1}{\partial t_1}\Delta t_1\right)^2 + \left(\frac{\partial\eta_2}{\partial t_2}\Delta t_2\right)^2\right]^{0.5} = \left[\left(\frac{\eta_2\rho_1}{\rho_2 t_2}\Delta t_1 \frac{t_1}{t_1}\right)^2 + \left(\frac{\eta_2\rho_1 t_1}{\rho_2 t_2^2}\Delta t_2\right)^2\right]^{0.5}$$

We can take the factor $\eta_1 = \dfrac{\eta_2\rho_1 t_1}{\rho_2 t_2}$ out of the square root, and then

$$\Delta\eta_1 = \eta_1\sqrt{\left(\frac{\Delta t_1}{t_1}\right)^2 + \left(\frac{\Delta t_2}{t_2}\right)^2}\,.$$

In turn, Δt_1 and Δt_2 are found from the statistics of your set of measurements results.