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Study of the surface of a generalized reduced module for multiply connected domain

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Classical task in the complex function theory concerns with the construction of the conformal mappings $F(w, a) = (w - a)f(w, a)$ from the regions D of the planar type and finite connectivity onto the unit disk with cuts along the arcs of prescribed form, namely, circular concentric arcs, radial slits, or their various disjoint combinations.

I.P. Mityuk [1] has proposed a way to define a generalized reduced modules connected with functions $F(w, a)$. The generalized reduced module,

$$M(w, D) = \frac{1}{2\pi} \ln |f(w, w)|, \tag{1}$$

of a multiply connected domain D at a point w will be called *Mityuk's function* with respect to the distinguished canonical domain.

Connection of the functions (1) with the exterior inverse boundary value problems goes back to F.D. Gakhov [2]. As it has appeared, the non-emptiness of the critical points set of the function $M(w, D)$ is equivalent to the solvability of the suitable exterior problem. The existence of critical points of Mityuk's function in the case of circular concentric slits has been proved by M.I. Kinder [3]. The case of circular and radial slits is studied in the present report: we construct the function $F(w, a)$ and establish the following

Theorem. *Let D be $(n + 1)$-ly connected Jordan domain. Mityuk's function $M(w, D)$ with respect to the unit disk with circular and radial slits has at least one critical point when $n \neq 1$.*

We also discuss the classification problem for the critical points of Mityuk's function and the examples of an absence of such points.

References

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