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INNOVATION MANAGEMENT *FROM* FRACTAL INFINITE PATHS INTEGRAL POINT OF VIEW

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Abstract. *While a mastery of management innovation is crucial for the future of the economy, to date, there is no theory able to base with objectivity the management of creativity and entrepreneurship. This absence is not due to the lack of methods but to ignorance of mathematical foundations which justify the paradigmatic transgression. These foundations exist nevertheless. It can be mentioned the fractal geometry and the role played by the singularities and correlations over long distances. In the set theory, let us mention Cohen's forcing methods and its engineering consequences through CK theory. In the categories theory, we can mention the principles of Kan extension herein applied by the mean of holomorphic analysis and the analytical extensions. All these methods are based on the recognition of the incompleteness of any structure axiomatically closed (Goedel). At the junction between the physics and the economy, the goal of the present work is to show that the lack of recognition of the role of singularities in this science must be searched in mental biases and the paradigms that affect our concept of equilibrium. We show that this concept must be generalized. If the criticism of the concept of equilibrium in economics is already known, it does not lead, quite as much, to a theory of innovation. We would like to address the issue of creativity by backing the reasoning by the questioning of the concept of equilibrium, using an analogy coming from the physics in fractal structures. The idea is to consider the equilibrium as some steady state limit of a fractional dynamics. The fractional dynamics is a dynamics controlled by non integer fractional equation. These equations will be considered in the Fourier space and by the means of their hyperbolic geodesics. Due to the intrinsic incompleteness of the fractality and of its cardinality, the thickening of the infinite will be used to show that there is no even physical balance but only pseudo-equilibria. The practical use of this observation leads to the design of a dynamic model of creativity, named DQPl (Dual Quality Planning), giving a topologic content to the innovation process. New principles of management of innovation emerge in naturally.*

Keywords: *Creativity, Innovation, Management, Fractal and Hyperbolic Geometry, Path Integrals, Categories theory.*

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1. Introduction

All human societies are organized to pursue the general interest while maximizing everyone's utility [1]. Nevertheless this dual objective is rationally impossible to be reached. As it has been shown by Arrow, -after Condorcet in 1785-, (Condorcet, Arrow's impossibility theorem), there is no aggregation function, except dictatorial [2,3], able to validates the hypothesis of Pareto optimality (also a competitive steady state). Global equilibrium [4-7] cannot meet the independence of individual choices [8,9]. The transfer of transitivity of these individual choices to the transitivity of overall social choice is highly problematic [10-13]. If the set, which has to be optimized, contains many agents – game and economic theory-, the overall equilibrium often leads a local, sub-optimality [7,14]. Because of the multiplicity of individual choices, even embedded in an overall equilibrium (considered as a smooth Euclidean manifold E characterized by state variables), the local metric between agents defines almost surely a hyperbolic structure F for their internal relationships [15]. For instance, insurances at disposal leads the society to an increase for taking risks and the hyperbolic set of risks leads to chaotic and fractal behaviors [16,17]. Fortunately, according to a Nash's theorem, we can assert the existence of an isometric application between E and F, which can then live together almost in harmony [18,19]. Might this isometric application and embedding, explain the emergence of overall laws of behavior? The absence of demonstration of this hypothesis involves that the emergence of an equilibrium, and macroscopic laws for the overall of numerous complex system, remains mainly misunderstood [20] and also far from individual interest.

Conversely, compared to the above distribution (E overall, and F local) the innovation management requires the consideration of a dual interdependency [17]. The overall manifold must now be associated with the hyperbolic metric F* (global chaos with a growth) and a local state, associated with a Euclidean class of space E* (local steady state). Thus, according to the Nash theorem, it must be a paradoxical isometric application embedding the overall into the local (Voronin's universal theorem). The closure of the system may then lead to the scaling properties [21]. Due to overall correlations (and of the memory), the system is then in an over-optimal state (creative and innovative) and new matters, devices and organizations are able to emerge. The diagram shown in Figure 1 can illustrate our hypothesis.

Dedicated to the management of the creativity the above approach is the symmetric of the traditional approach of complex systems. It is characterized by intrinsic local irreversibility no more due to traditional statistical effects but to the will of the engineers. Initially designed for batteries optimization, this approach has been generalized in physics [22] using the concept of transfer of energy upon fractal interface (TEISI model) and transfer function Z(s).

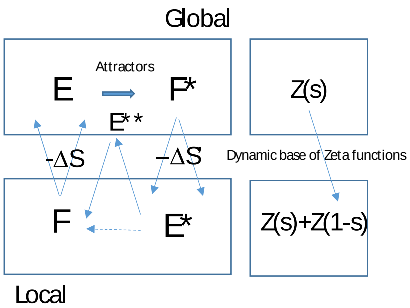

Figure 1. Diagram showing the relationship between local and global metric depending on 'equilibrium' location and attractor fields. Z(s) points out the integer or fractional geodesics of the dynamics related to reciprocal applications (arrows).

This analysis was later extended to econophysics via a model of Dual Quality Planning (DQPl model). This model was practically applied [23] for the development of the pedagogy of the innovation and sustainable development. Playing with the balance of negentropy production this tool uses the lever of a paradoxical and singular' pseudo-equilibrium' E** (Figure 1), as a source of the innovative processes and of the economic growth.

Let us observe that the fractal geometry plays an important role in the models especially through a theory of the measurement [24]. The very general presence of scaling properties in the metric of discreet set of relationship characterized by a 1D order, -that is to say identical to N-, finds their origins in a couple of constraints: (i) the addition (+) of some agent must be considered as a constructivist action, while (ii) the deconstruction or the partition of a set, must be related to the arithmetical division (/). Because this partition cannot be done without a remainder in general situation, the only solution to remove it and to close the system, is to introduce scaling properties [21]. This categorical requirement explains the universality of the fractal geometry of nature [16].

2. Constraints for steady states

Games and economic theories: we cannot consider in details herein the multiplicity of the specific cases, described within games and economic theories [7]. In games, the gain-factor is an ‘arbitral variable’ while in economy the optimality is based on a ‘utility function’ [25, 26]. The Sisyphean task required by the analysis of the multiplicity of the possible axioms and of the maturity issues, leads to the question of the very relative universality of the assumptions and, therefore, of the rational conclusions, that can be inferred [7,27]. In both cases the assumption of an income obtained by a rational player taking an added value sourced in risks taking, leads to a misfit between the optimization of the collective interests (judging criteria isomorph to the natural numbers N), and the requirements of individual interest, (infinite set of choices characterized by an higher cardinality than N). This contradiction renders out of order the simplest idea of a set of choices distributed as N... except as if, a drastic reduction of the problematic is clearly announced and performed by the econometric models. The inability to merge without drastic reduction, the main set of choices (local and overall) leads scientists to release some constraints for obtaining plausible mathematical solutions. These last are certainly globally ‘optimal’ but locally ‘under-optimal’ leading even to social disasters (mass unemployment, dictatorship, social partition ...). One of the constraints that it is the most amazing to have been released is the rationality of the choices [27-29]. In many games one can show that the rationality can be clearly cons-productive [6].

Steady state, attractors and quality planning: the surprise with regard to the *impossibility theorems* (to reach equilibriums) can be overcome by noting that the dynamic attraction fields (attractor) to the zeroes of a non-monotone function (a zero mimics mathematically a states of equilibrium) have the fractal structure of a Julia sets F. The convergence to a steady state E given by a zero (Newton-Raphson method) is highly dependent on initial conditions (boundary) and on the space-time scale accuracy. This point of view is not natural and requires a mathematical analysis to be clearly understood.

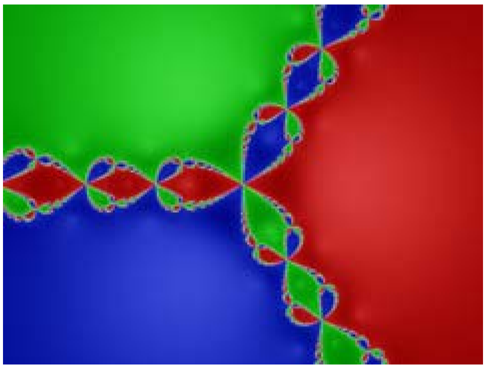

Figure 2. Classical example of Julia attractor for the automorphism $z \to z^3 - 1$ in the complex plan. We observe that the field of attraction for the cubic root of the unit has a very complex boundary that mixes together the different domains according to a fractal structure. The Euclidean roots are related to hyperbolic structure of the attractor. This type of property is general.

The surjection from E (zeroes) into F (attractors), that controls the dynamics, may be related to the Nash’s theorem [18], asserting that there exists an isometric application between the Riemannian manifold (associated to the individual set of choices or location in F) and embedding Euclidean space (optimum E). Despite this recognition of the role of the fractality, the libertarians, who think in the frame of a linear simple rationality, ignore the theoretical constraint of the epimorphism (surjection). For them, the base of measure of the set of instability is nil. At an upper level of cognition and faced to chaos, libertarians think that a complex system can be reduced to its statistical components (invisible hand of the market) without singular extremum or black swan. It is now well known that this hypothesis is almost surely untrue [16,30,31]. In spite of an exceptional deepness, the Schumpeterian concept of ‘creative-destruction’ [32] is written in the same ‘language’. Despite the need for a step of divergence in all creative process (destruction) it is clear that this process must be strongly rooted in the environment and in the history with which it must resonate. Creation and thermodynamic self-organization, can clearly find their origin in the entropy production, but the associated mess must be fractal. It must contain memory and correlations which partly change the stochastic standards: Fractional Brownian Motion, non-Gaussian Distribution, Levy flight etc.

Dynamics, path integrals and fractional differential equations: Some recent works in statistical thermodynamics can be used to illustrate the assumption of a debt with regard to the ancients (memory). One can borrow in [33] the model of the ‘Equilibrium with Growth' and

Morishima’s reserves concerning the Schumpeterian hypothesis of ‘destruction’ in the model of innovation.

The Badiali's concepts concerning the ‘Thermodynamic Paths Integrals', may be used to enlighten the Morishima’s issues. Badiali’s model [34] is based upon a local irreversible transition. As developed by Feynman, he sums this transition all along the infinite number of possible paths leading from state A (t = 0) to state B (t = 1). In spite of very singular way of access to thermodynamic solutions, under the condition of the existence of a concept of energy (Noether theorems) and using a standard statistical analysis, he obtains, from very simple computations, a diffusive type of behavior (Fokker Planck and/or Smoluchowski). This conclusion reminds the result of the computation of the Black and Sholes equations. His model founds easily the quantum mechanics, that is to say, the existence of the set of steady states, which explain the stability of molecules, atoms, etc (zeroes of an universal wave function). At this step, his main idea concerns the concept of fluctuations of closed paths (A = B). The global steady state is the set of paths for which the internal energy (introduced into the system) is equal to the average energy of closed paths between the time t = 0 and t = 1. This point of view introduces a conceptual breakthrough concerning the idea of what is the equilibrium. In certain conditions, there is no thermodynamic state. The equilibrium must be ‘forced’ [35] by the experimentalist. Obviously, a homotopy interpretation is implicitly involved: all closed paths that do not round a singularity, -i.e. a steady state-, are strictly equivalent and aside with regards to the zeroes.

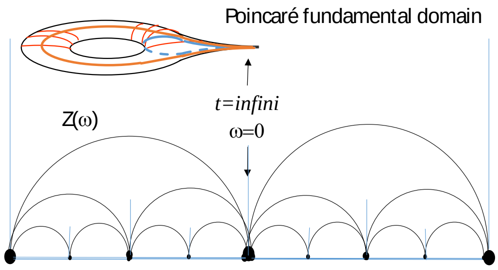

Figure 3. Poincaré traditional fundamental domain. This figure is based upon the canonical first order transfer function in Fourier space. The semi-circle parameterized with frequency is considered herein as the boundary of a fundamental domain of a complex plan whose diagonal is an ordered discrete set. This *N* set at infinity, represent the equilibrium states of a discrete 1D space. The automorphism points out the unicity of this equilibrium E. The topology of this automorphism is given by the punctuated torus. The angle at cusp is nil because the automorphism is related to integer differential equations. The fractionality modifies this situation (Figure 4 and 5).

This equivalence can be represented for instance using Poincaré's fundamental domains where the hyperbolic boundary may canonically be given by a first order dynamical transfer function (semi-circle in Fourier space). The singularities are then located at the boundaries of an automorph dynamic groups controlled only by one parameter: the time (frequency in Fourier space). We can consider the research of equilibria as equivalent to the integral path-loops upon hyperbolic space determined by the folding of the fundamental domain (punctuated torus in Figure 3) as the sum of all level of approximation of the horizontal axis [15].

The hyperbolic space is the complementary of the space of zeroes (of a function of utility ie of energy). As the attractors (Julia set) of the Euclidean geometry E, the hyperbolic space is characterized by a Fractal geometry F* (scaling properties) and there exist a dual relationship between the set of equilibrium (E) and the stationary dynamics inside of the fractality. The isometric transformation F (integrals of paths) in E (all stable states) is associated to the interferential interaction between the paths, and equally that of E in F* (the interference of the paths using the phase, when we withdraw all zeroes in the loops).

As suggested above, this analysis allows us to reverse the terms of the problem (Figure 1). The hyperbolic metric, F* may be attributed to the overall structure characterized by a chaos in evolution (growth). By duality, a Euclidean metric must be attributed at a hypothetical local (individual) level E*. For instance, the Zipf law encourages to explore this inversion; equally the Fermi distribution in quantum mechanics. Nevertheless, the definition of the status of E* is extremely problematic. Due to the infinite development of the automorphism, E* is likely to be think like an abstract an artificial reference, required just for justify the mathematical symmetries! Except in a very specific case E* ($d = 1$) is not related to any type of equilibrium. More generally, it cannot possess any simple physical status! The problem is identical for performing the CK model of innovation [36,37]. The status of the extended concept C* (conceptual extension) is never defined in his documents. To unveil this ‘status’, we have to consider the intermediation of an E** equivalent to E starting from a standard dynamic situation. As shown in Figure 1, E** may then appear like a specifically designed equilibrium (it is design like E). It is qualified pseudo equilibrium because the dynamic within correlated chaos, is not able to possess any type of equilibrium. It is for example affected by strong fluctuations during its asymptotic position. This intermediate ‘state’ is what Husserl named, a Zeit Objekt [38]. Explaining the dynamic status of E** shall allow us to qualify E** as a 'weak-equilibrium'.

3. Fractional dynamics and fractality

Implicitly, the TEISI model, developed 30 years ago [22,17] takes place in the class of models of path integrals (sum of state at different frequency). Its foundation takes into account not only the interface (equivalent to a full path F) but all interfaces at all scales (that is to say the whole full paths F*). The fractal set F* must consider a steady states, that can no longer be written in N but in a set of upper cardinality than N (like Fractal structures). This extension must have physical consequences; among these consequences: the presence of a phase angle at infinity and therefore the role of interferences which veils the presence of what should be the natural level of equilibrium E*.

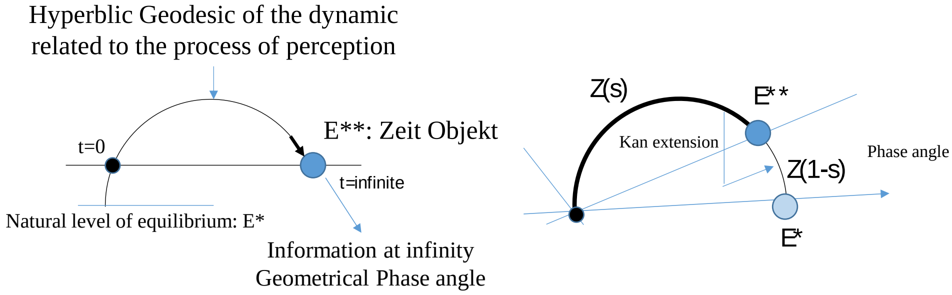

Figure 4. The main conclusion of the TEISI model, is herein interpreted in terms of perception of pseudo steady states. It points out these states at thick infinite time (nil frequency). If the structure is hyperbolic (for instance fractal) the model proves the presence of a remainder information at infinity (phase angle for fractal geometry). A rotation of the axis of representation discloses the need for complex time and the meaning of the Kan extension with respect to E* and E**.

Beyond mere physical, and especially for social sciences, it is important to unveil the hidden meaning of E * (as pseudo equilibrium). We will not repeat herein the analysis leading to formalize the fractal-transfer by using of non-integer differential equations, by cons we emphasize the fact that in Fourier space, the dynamics is then represented by a hyperbolic geodesic which is nothing more than a circular arc whose Euclidean extension is related to the fractal dimension, that is to say, the correlations within the chaos. We note that the nil frequency, usually equivalent to steady state (here infinite time, but herein in the absence of convergence) defines what is named above a Zeit Objekt (tempo object). The end point of the geodesic defines an object because it defines physically its phenomenological perception (through experiences) via an irreversible process. Nevertheless this object is a Zeit Objekt, because even if the frequency is nil (the time is at infinity), the spectrum of frequencies, i.e. the set of its 'qualities', leaves aside a remainder. The Zeit Objekt is an object but this object is not really in equilibrium. Due to a geometrical phase at infinity this object carries with its a transcendental momentum (living, social and in general complex systems [39]). Let us observe that, without additional conditions, it is unable to define an equilibrium except for d = 1. Fortunately when d = 2 even if E** is not at equilibrium, some useful symmetries appear and the process is well known to be related with a statistical object (for instance, the Fermi distribution). This observation can serve as a guide for asserting that, even below the physical horizon, E* should play the same role as a connected set of steady states used as reference. How to overcome, at this step, the apparent paradox of the transgression of the traditional paradigm asserting that a steady state, is a state without evolution, while the series which define our object, does not converge? To do so we have to follow a new idea:

If the dynamics is fractal the arc defining the Zeit Objekt matches (geometrically and partly) an underlying deterministic process (d = 1) whose the virtual steady state E * is located under the experimental horizon of F* (Figure 4). E=E* for d=1. We then have to distinguish two cases: (i) simple d = 2 with symmetries (ii) the requisite extension 1 <d <2.

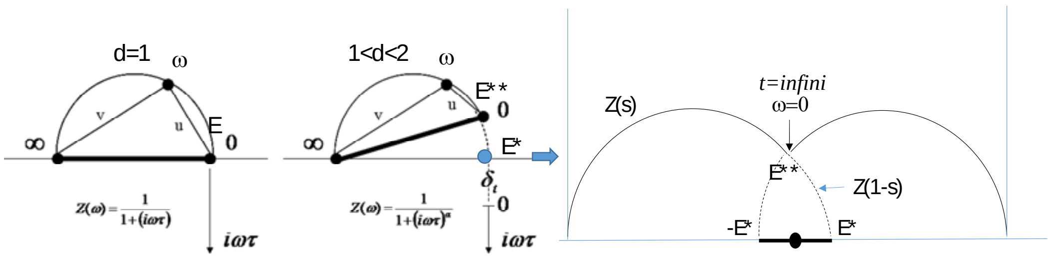

Figure 5. Transformation of the first order transfer function Z(s) into fractional canonical transfer function. Geometrical phase angle appears. Equilibrium E is split in into E* and E**. The linear hyperbolic distance (u/v) vs standard time relationship turns into a power law. Noether homogeneity is recovered by embedding time in the complex space. Poincaré fundamental domain is modified and the limit equilibrium is replaced by a group of symmetry [40]. The equilibrium becomes a dynamic process requiring Kan extension and therefore producing negentropic effects.

As far as the metric of the interface is given by d = 2, we can construct a very simple morphism between time and space (external algebra). Due to a degeneration shown in the figure 5 there exists a différomorphisme (Laplacian) between deterministic process (d=1) and a fractional ½ stochastic process. In practice, the average distance of this elementary hyperbolic process, is a linear form of the square root of the time variable. This characteristic, -associable to the diffusion and to a solvable Galois group-, gives access to equilibrium states. Similar to an optimization principle, the equilibrium emerges naturally from stochastic phenomenon such as those assumed for the free market [26].

Initially under the experimental horizon, steady state can be found, like a solution of an *ad doc* automorphism. We can read this automorphism as a curvature of the dynamics that is carried by the Zeit Objekt d = 1/2 on the axis-states (Figure 6). This operation is represented by fundamental Poincaré's group of the torus (Euclidean) with a nil angle at cusp (hyperbolicity at infinity).

One observes that in this case it is also possible to combine the Zeit Objekt to a homotopy class, which asserts to the absence of singularity other than equilibrium states projected onto the line at infinity.

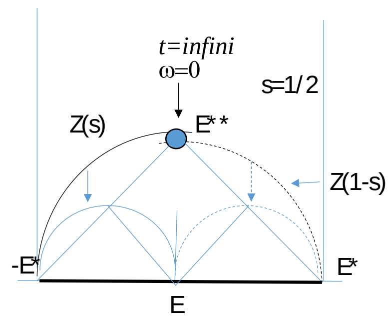

Figure 6. Degeneration of the fractal dynamic when d=2 and a=1/d. The generalized Poincaré fundamental domain pointed out in figure 3 and 5 disappears herein. Nevertheless the structure of the diagram authorizes a couple of projections (Nash theorem) leading an isometric relationship of E** and E. The reconstruction of the automorphism according to external algebra and the Laplacian operators is then very simple. As it is well known diffusion equation can easily be built from first order equation as seen in the figure after Nash theorem application.

Even if the economists measure the limitations of their models and attempt to introduce amendments, the statistical and dynamical constraints related to degeneration involved by the geometry are never considered and the Peano geometric structure for F* stays implicitly the basis of the general theories of economy, therefore their limits. As shown above this situation contains many underlying hypotheses and some confusions due to the equivalence of Z(s) and Z(1–s) when s = 1/d+i with in this case d = 2.

It is no longer the same if d is fractional. Instead of two data for defining the partial arc of semi-circle, the dynamic then requires 3 data. Therefore, the parameterization must be expressed using a complex time. The final state can never be reduced to the initial state and the definition of the application via a homotopic simple loop is impossible. In this case the hyperbolic curvature splits the infinity into a couple of non-symmetric parts $E \to E^{*} + E^{**}$and the simplicity of the differomorphism obtained for d = 2, between Euclidean process E* and hyperbolic one F* is broken. The Nash's morphism, strictly non-linear, non-differential and non-convergent, causes a shift from the ordinality of the problem toward its cardinality. The line at infinity becomes thick. The standard state becomes a string. The information at infinity cannot be put aside. The associated Galois groups are almost certainly not solvable. We cannot find the zeroes, and thus the traditional steady states. Nevertheless, due to geodesic structures of the dynamics, their virtual equivalents E* located under the experimental horizon continue to exist. But it is no longer able to be easily reduced by automorphism and compactification, unlike the case d = 2. Transposed in the field of economy, the irreducible curvature at infinity prohibits the existence of a Pareto optimality and an impossibility theorem appears naturally. The concept of connex derivation is then required. It combines the global fractal character F* (chaotic) and the behavior locally smooth E ** (individual stability). At this step, the assertions concerning the link between fractional differential dynamics and Riemann Zeta universal functions can play an extremely useful heuristic role. We shall see that these functions are naturally associated to the model de creativity whose acronym is DQPl (dynamic model for innovation). The acronym DQPl (décupler in French) means increase, amplify, extend, and overcome etc., in other words: creation.

4. Zeta functions and DQPl model

Let us recall herein the surprising relationship between (i) the well-known Riemann zeta function Z(s) with s = a + it, (ii) the concept of fractal path integrals related with statistical thermodynamic [34] and (iii) the 'fractional dynamics' related to d-fractal geometry [39,40]. In the frame of these relationships, with a=1/d, Riemann's hypothesis (that is to say a =1/2) ensures the existence of a close link between the set of the zeroes of the zeta function; the duality between (i) addition, multiplication and fractality [21]; (ii) the spectral relevance of stochastic analysis; (iii) the existence of a linear time (subject to the assumption of the existence of a Noetherian energy invariance); and (iv)the quantum mechanics (*i.e.*, its steady states). As this universal function possesses a set of zeroes if and only if a=1/2 (related with the set of primes), we can assert that the constraint d = 2 plays a central role in the establishment of an overall steady state for all type of complex system, which depends of a 1/2 fractional dynamic. But, let us ask, what happens in the other physical situations, for instance for a general type hyperbolic dynamic? The local ½ fractal chaos of associated with primes, leads to the emergence of overall deterministic and/or stochastic laws. What become these rules when d is not integer [42] (Tayurskii *et al.*, 2012). Obviously, the rule which must be revisited is the rule of the equilibrium! An analogy with the problematic above, suggests a main distinction between (i) the status of equilibrium for local games or economies, precisely characterized by steady states (or a set of zeroes of a situation that looks like a stochastic situation), and (ii) the fractional case for 1/2 <a <1, where the absence of zero for the zeta function does not lead to a simple definition of the equilibrium but to a more fundamental concept (equilibrium becomes a specific case). The analogy with zeta function may therefore enlighten the problematic of a concept of a generalized equilibrium even for the economy. The concept is derived from the properties of the Riemann function but under its more general form, that is to say when the value of 'a' is not the half of the unit. In this case, the absence of zeroes indicates that without adding any additional criterion, there does not exist a steady state in general for fractional complex systems.

The DQPl model addresses the issue of these criteria. Indeed, like for the set of attractors of the zeroes of a non-monotonic function, the analysis of the set {Z (s), Z(1–s)} allows us to imagine a generalization of the surjection from Z (s) into Z(s–1). This surjection can be interpreted as a Kan extension in categories theory. This extension is similar to a 'local factorization' in distributions theory, -rather than a traditional factorization of a function (using zeroes)-. This generalized factorization comes just before any path integration in Fourier space. The integrated set is then obviously not factorable (path integral). We named this new situation, 'spectral equilibrium'. The Cohen's forcing of the 'spectral equilibrium' taking into account E* in the context of an absence of steady state for E**, can be well understood, in this frame. The 'forcing' and Kan extension takes the same meaning. Thus complete information on this complex system requires a new type of merging between Z (s) and Z (1–s). As shown by the DQPl model the topology authorizes this merging by gluing.

This operation consisting to 'fill up' increases the cardinal of the set of the given data and leads to a mixing of information and therefore to an exponential growth of entropy. The topologic DQPl model, is specifically designed to understand how the mixing entropic term is reversed into a negentropy term when playing with the incompleteness of the circle (here the circle may represents the knowledge). The opening of the circle represents the incompleteness, while the gluing the lips of the open sector, compels the system to take into account the incompleteness, in achieving an operational gluing. Doing so, the gluing performs a 'Cohen forcing' and solves the paradox of the use of the incompleteness as a lever of a performance (action on innovation).

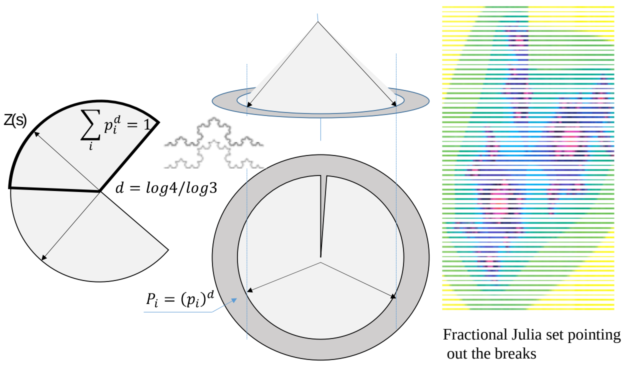

Figure 7. Development of fractional dynamics into DQPl model. The circle is deprived of a sector (uncompleteness). It shows (i) the partial arc of semi-circle pointing out the relation with a fractional dynamic, (ii) the splitting between different types of statistics. The relation with the fractal geometry is given. The gluing raises a new axis: the axis of innovation. The analysis of the fractional attractors points out cuts and attractor slips within the Julia set when, far from integer, the fractionality is involved.

The function Z(s) and the distribution Z(s) + Z(1–s) can be associated to couple of arcs of circles (enlarged dynamics) whose the open sector would have been first recognized like relevant. It is removed to compel the action. The comparison of both structures -before and after gluing- discloses the presence of two classes of competing probability. The resolution of that competition in the frame of a conceptual synthesis is precisely related to the negentropic process named: innovation. By gluing, the innovation appears geometrically as the emergence of an orthogonal direction with regard to the complex plan of the initial knowledge (the plan of the circles). The 3D representation points out the fact that the incompleteness is precisely a variable of the problem of innovation.

The emergence of a third dimension tells us that the merging of the two classes of probabilities is constructive. Assuming that the circle whose a sector is removed, schematizes a knowledge which includes the existence of an incompleteness (full circle excludes this incompleteness), the ability to freely assume the incompleteness, that is to say, to take into account a transcendental equilibrium E* related to this incompleteness, leads to think the physical environment by taking into account this abstract object. This posture leads to design new steady states E' = f(E*,E**), ie new objects. Unexpected in the field of initial knowledge these new objects require, for being achieved, a transitory situation named Zeit Objekt (it is an object in mind). The new object originates from the entropic mess, is created precisely through a Kan extension. The structure of the uncompleted circle is related to the d value in fractional dynamics. If we consider otherwise the automorphism associated with fractional operators, like a fields of attractors, -fractal Julia attractors-, the fractionarity involves the emergence of many lines of cutting (Figure 7) which are continuous and derivable into the traditional Julia diagram. The generalized Julia set, points out, the presence of new deterministic correlations, between close attractors. These lines, also characterized by scaling properties of the attractor fields, are the proof that creative process is partly chaotic and partly determined by a capability of new coupling, inside the chaos. Obviously these new correlations are naturally and probably implicitly used by the innovators and the entrepreneurs. The new concepts of fractal equilibria can help us to overcome the traditional way of thinking the management and therefore summons the closure of the methods of management based upon short term accountancy and reporting.

5. Conclusion: the paradoxical management of innovation

Highlighting (i) the strange character of an equilibrium that is no more a state but a transformation group ((E**, E*, –E*) ie a stationary loop), and (ii) the need to express this extension by using a temporal singularity, the TEISI model and then the DQPl model provide a mathematic content to the concept of Kairos as a singular reaction to an external excitation. Both model lead to think the ‘Equilibrium Growth’ as a tangible reality, and not as a paradox. Both concepts lead to enlarge the

Schumpeterian concept the destructive-creation. Associated with distributions with long tails (long range correlations), the entanglement between local and overall states, which naturally characterizes the majority of the incomplete complex situations, explains the conceptual deficiencies which render inefficient the performances of the traditional management, when it is based upon the only consideration of the transitivity of collective choices (1D ordered set as instrument of the accountancy). The topological operation of gluing in the DQPl model is equivalent to an overcoming of the managerial tradition that always assumes a perfectly defined initial ordered knowledge within an unknown environment. In the traditional managerial framework, the state of the knowledge may be tested and structured but, except marginal progresses, the creative process is then almost prohibited. This situation is qualified as an academism. As pointed out by Mandelbrot in his memories [43], the management of innovation must be based upon different methodologies. Taking into account the scaling correlations, the knowledge must be recognized as always uncompleted. It can be 'forced' through and far from its boundaries using the incompleteness as a lever. The goal of the DQPl strategy is not to fill the incompleteness but to overcome it. The multiplicative factor introduced by the lever is a negentropic term. The gluing is the expression of this lever. The gluing is performed from the upper level of cognition (the brain for the will; the leader for social structure; the macrostructure in living systems). It is able to integrate all the local chaotic paths and the multiple fields of attractors into a sole vision and a singular motion. Born from a path-integral point of view, this posture is clearly holistic and merges, like for the zeta function, the concept constructivist (the addition) and the concept of partition (the division). Fractal geometry then emerges naturally as the underlying geometry for solving the paradoxes. The concept of quality is no more in its detail. The reference factor for innovating is the merging between detail and overall point of view. For doing so, the main reference is the 'differance' emerging from the correlations between different levels of approximation of the reality, or between the qualities used to define them. The quality, which has not to be forgotten, is clearly the incompleteness. Due to this forgot, the traditional constraint of maximization of individual choices always is proved suboptimal. Conversely, as shown the role of motivation, the choice to abandon the search for a Pareto optimum by the acceptation of freedom axis , is often revealed as creative on-optimal choice; equally, the choice of move the accountancy rationality into a fractional dynamic rationality using Kan extension methods.

Let us recall herein how the great economist Arrow, was accused of political anarchy. The accusation was not gratuitous and the DQPl model, gives a mathematical foundation of his position. What is the meaning of the word, 'anarchy' in our model? To finish our analysis we would like to match this word with the word ‘chaos’ for giving a weak to it definition. Starting from the physical meaning of DQPl model, the 'weak-chaos' puts in conjunction -not in competition- at least a couple of fields of Julia attractors. In this frame the lever of the innovation management is the understanding of what means the concept of forcing in set theory, or the concept of extension in categories theory. All procedure of creativity that, explicitly, matches fractional divergent dynamics, operates implicitly by controlling the extensions of the set of knowledge into its own incompleteness. This incompleteness bases the weak chaos. In practice, the uncertainty associated with non-integer states, is expressed among others by the need to make (policy) choices in a situation where a possible equilibrium should be located below the physical horizon. The lever for creativity is then associated to an embedment in an enlarged set, characterized by an upper cardinality of the set of the basic and sure knowledge. The main question is precisely to define what means the word sure! The understanding of the paradoxes involved by the embedding a sure knowledge into an uncompleted set, is solved within the topological DQPl because it matches a fractal dynamic. Its physical meaning is given by TEISI through energy exchange upon fractal structure. It is characterized precisely by the presence of an upper cardinality with respect to the traditional N order used by the simple accounting. Its ability to take into account the remainder of the knowledge at boundary and the existence of Zeit Objekt, help us for grasping the meaning of the pseudo-equilibriums. Doing so, the DQPl model also improves the effectiveness of management and, on the way, the capability for innovating. It is exactly what is perceived by the greatest among the economists who try to improve their models unfortunately in the frame of a vision given by integer numbers. We show herein, that the fractionality and its underlying fractality might be a way to reach the same objective with a great economy of means, indeed to generalize their models.

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