A.N. Turanov

PROBLEMS AND EXERCISES IN MEDICAL PHYSICS WITH EXAMPLES AND SOLUTIONS

$$E=mc^2$$

For English-speaking students of medical, biomedical and pharmaceutical fields of study

2023

KAZAN (VOLGA REGION) FEDERAL UNIVERSITY
INSTITUTE OF PHYSICS
DEPARTMENT OF MEDICAL PHYSICS

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"Problems and Exercises in Medical Physics with Examples and Solutions" is intended for students of foreign groups of medical specialties of the university. Example problems and exercises with brief theoretical background and complete solution are presented for five domains of physics. Mechanics, Molecular Physics, Electricity and Magnetism, Sound and Waves, and Optics. Tasks for independent study are given at the end of the brochure.

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Contents

List of literature

When compiling this collection of problems, the following list of literature sources was used:

Aganov, A.V. Medical Physics Part 1. Mechanics. Molecular physics / A.V. Aganov, K.S. Usachev. – Kazan.: Kazan University Press, 2022. – 280 p.

Young, H.D. University Physics with Modern Physics / H.D. Young, R.A. Freedman. – Pearson Education, Inc.; 15th edition, 2019. – 1600 p.

Serway, R.A. Physics for Scientists and Engineers with Modern Physics / R.A. Serway, J.W. Jewett, Jr. – Cengage Learning, 2018. –1484 p.

Introduction

Solving basic and even elementary problems in physics is an important element of training when studying the discipline "Medical Physics" by the first-year students of medical specialties of foreign groups of Kazan Federal University. Different students` levels, wide spectrum of educational curriculums, variety of systems of units adopted in their home countries, and even different systems of mathematical notation result in substantial challenges that students and teachers have to get over together. A detailed analysis of problems and their solutions given in this collection, with an emphasis on units of measurement and the correct formalization, that are given in this brochure, will help students who have arrived from abroad not only to master one of the basic training courses for a future physician, but also to adapt to study at a Russian university.

I thank the reviewers of this text. Special thanks to the authors of the books cited above in the List of literature.

I welcome any feedback from students and professors, especially concerning errors or deficiencies. Please, feel free to contact me: Alex Turanov, e-mail: anturanov@kpfu.ru

1. MECHANICS

Example 1.1.

The speed of a body is 36 km/h (kilometers per hour). Convert km/h to m/s (meters per second).

Solution Unit conversations:

1 km = 1000 m,

1 h = (60 min/h)·(60 s/min) , "min" means minute.

Thus:

υ = 36 km/h = $\frac{(36\ \mathrm{km/h})\cdot(1000\ \mathrm{m/km})}{(60\ \mathrm{min/h})\cdot(60\ \mathrm{s/min})}$ =$\frac{36000\ \mathrm{m/h}}{3600\ s/\mathrm{h}}$ = 10 [m/s].

Answer: 10 m/s.

Example 1.2.

The lengths of the vectors $\vec{A}$ is 2 and $\vec{B}$ is $\sqrt{2}$; the angle between them is 45º. Calculate their scalar product $\vec{A}\cdot\vec{B}$.

Solution

cos(45°) = $\frac{\sqrt{2}}{2}$

$\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cdot\cos(\Theta_{AB})$ = $2\cdot\sqrt{2}\cdot\frac{\sqrt{2}}{2}$ = 2

Answer: 2.

Example 1.3.

A body moves along a straight line equally slowly down with a constant deceleration (a < 0). The deceleration is 2 m/s^2, the initial speed is 50 m/s, the time interval of deceleration is 10 s. Calculate the speed of the body at the end of this time interval.

Solution For movement along a straight line, the definition of deceleration:

$a = \frac{\upsilon_f-\upsilon_i}{t}$,

units of measurement for deceleration (and acceleration) are m/s^2, a > 0 in a case of acceleration, and a < 0 in a case of deceleration; υ_i is the initial speed, units are m/s; υ_f is the final speed, units are m/s; t is time interval of speed changes, measured in units s (seconds). Then,

$\upsilon_f = \upsilon_i + a \cdot t$ = (50 m/s) – (2 m/s^2)·(10 s)= 30 [m/s].

Answer: 30 m/s.

Example 1.4.

The body moves along a straight line with a constant acceleration (a > 0), and covers a distance of 100 m in 2 s. The initial speed is 30 m/s. Calculate the acceleration of this body.

Solution The magnitude of displacement of a body moving along a straight line with a constant acceleration is given by the expression:

$$S = \upsilon_i \cdot t + \frac{a \cdot t^2}{2},$$

where S is a magnitude of displacement of the body, units are m; a is a body`s acceleration, units are m/s^2, $a > 0$; υ_i is its initial speed, units are m/s; t is time of acceleration, units are s . Then,

$$a = \frac{(S - \upsilon_i \cdot t) \cdot 2}{t^2} = \frac{((100\ \mathrm{m}) - (30\ \mathrm{m/s}) \cdot (2\ \mathrm{s})) \cdot 2}{(2\ \mathrm{s})^2} = 20\ [\mathrm{m/s^2}].$$

Answer: 20 m/s^2.

Example 1.5.

Two forces, F_1 , 50 N (newtons), and F_2 , 30 N, act on a body in opposite directions toward each other. The mass of a body is 2 kg (kilograms). Calculate the acceleration of the body.

Solution Newton`s second law of dynamics gives

$$\sum_i \vec{F_i} = m \cdot \vec{a},$$

where $\sum_i \vec{F_i}$ is the net force, units are N, m is the body`s mass, units are kg, $\vec{a}$ is the acceleration, units are m/s^2. The projections of forces acting on the body have opposite directions, and the direction of the acceleration is the same as the direction of the greater force. Therefore,

$$a = \frac{F_1 - F_2}{m} = \frac{(50\ \mathrm{N}) - (30\ \mathrm{N})}{2\ \mathrm{kg}} = 10\ [\mathrm{m/s^2}].$$

Answer: 10 m/s^2.

Example 1.6.

A mass m (10 kg) hangs on a spring that has spring constant k (200 N/m). Calculate the extension of the string Δx when the mass attached to the string is at rest and at static force equilibrium.

Solution The standard acceleration of free fall g = 9.8 m/s^2. Newton`s second law:

$\sum_i \vec{F_i} = m \cdot \vec{a}$.

There are two forces acting on the mass: its force of gravity $m \cdot g$, and the tension force of the spring, $k \cdot \Delta x$ (Hooke`s law); k is the spring constant, units are N/m; Δx is the elongation (deformation) of the spring, units are m.

If a mass is at static equilibrium, then its acceleration a = 0. The forces have opposite directions

$m \cdot g - k \cdot \Delta x = 0$

$\Delta x = \frac{m \cdot g}{k} = \frac{(10\ \mathrm{kg}) \cdot (9.8\ \mathrm{m/s^2})}{200\ \mathrm{N/m}}$ = 0.49 [m].

Answer: 0.49 m.

Example 1.7.

A body (mass m is 5 kg) is at the height h of 2 m relative to the surface of the Earth. Calculate its potential energy U.

Solution The standard acceleration of free fall g = 9.8 m/s^2. Potential energy U of a body in gravitational field, units are J - joules, is

$U = m \cdot g \cdot h$,

where m is a mass, units are kg, h is a height, units are m. Thus,

U = (5 kg)·(9.8 m/s^2)·(2 m) = 98 [J].

Answer: 98 J.

Example 1.8.

A body fell down from the height h of 20 m. Calculate its speed to the second power (υ^2) at the moment of the collision with the ground if its initial speed was zero.

Solution The standard acceleration of free fall g = 9.8 m/s^2. The principle of mechanical energy conservation states:

$(K_i + U_i) – (K_f + U_f) = A$,

where A is a work of conservative forces, units are J, subscripts i and f denote initial and final states, correspondingly, K is the kinetic energy, units are J: $K = \frac{m \cdot v^2}{2}$, m is a mass of a body, υ is its speed; U is its potential energy.

In this example: v_i=0, K_i = 0, $U_i = m \cdot g \cdot h$, $K_f = \frac{m \cdot v^2}{2}$, U_f = 0, A = 0 . So,

$0 + m \cdot g \cdot h = \frac{m \cdot v^2}{2} + 0$

$v^2 = 2 \cdot g \cdot h = 2 \cdot (9.8\ \mathrm{m/s^2}) \cdot (20\ \mathrm{m}) = 392\ [\mathrm{m^2/s^2}]$.

Answer: 392 m^2/s^2.

Example 1.9.

A 100-watt lamp worked for 30 s. Calculate the energy consumed by the lamp.

Solution A power can be evaluated as

$P = \frac{W}{t}$,

where P is a power, units are W – watts, W is an energy, units are J, t is a time interval, units are s.

$W = P \cdot t$ = (100 W)·(30 s) = 3000 [J].

Answer: 3000 J.

Example 1.10.

A power is 1492 W. Convert W to Horsepower [HP].

Solution Unit conversations:

1 HP = 746 W.

P = 1492 W = $\frac{1492\ W}{746\ \mathrm{W/HP}}$ = 2 [HP].

Answer: 2 HP.

Example 1.11.

A body moves along a straight line at the constant speed υ (100 m/s) under the action of the force F (5 N). The directions of the velocity and the force are the same. Calculate the power related with this process.

Solution We can use the equation:

$P = F\cdot\upsilon$,

where P is a power, F is a force, υ is a speed.

P = (5 N)·(100 m/s) = 500 [W].

Answer: 500 W.

Example 1.12.

Two particles are at a distance r_1 under the gravitational attraction F_1. Calculate the force of gravitational attraction F_2 between the same particles, if the distance between them is increased 2 times.

Solution Universal gravitational law reads as:

$F = \mathrm{G}\cdot\frac{m\cdot M}{r^2}$,

where F is a magnitude of the gravitational force acting on either particle, m and M are their masses, r is the distance between them, G is the gravitational constant, in SI units, 6.67×10^{-11} N·m^2/kg^2 . For both distances: $F_1 = \mathrm{G}\cdot\frac{m\cdot M}{r_1^2}$ and $F_2 = \mathrm{G}\cdot\frac{m\cdot M}{r_2^2}$, then

$\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2}$.

Since $r_2 = 2\cdot r_1$

$\frac{F_2}{F_1} = \frac{r_1^2}{(2\cdot r_1)^2} = \frac{1}{4}$.

$F_2 = F_1/4$

Answer: $\frac{1}{4}$.

Example 1.13.

A particle (mass 1 kg) slides down at the speed of 5 m/s. Calculate the momentum of the particle.

Solution The momentum p (units are kg·m/s) is:

$p = m \cdot \upsilon$,

where m is a mass, units are kg, υ is a speed, units are m/s.

p = (1 kg)·(5 m/s) = 5 [kg·m/s].

Answer: 5 kg·m/s.

Example 1.14.

A point particle (mass 1 kg) moves on a circle trajectory with the radius of 2 m under the angular acceleration 5 rad/s^2 (rad is radian). Calculate the torque associated with this motion.

Solution A torque (or a moment of force) τ, units are N·m

$\tau = I \cdot \alpha$,

where the moment of inertia (or rotational inertia) about a rotational axis for a point particle is $I = m \cdot R^2$, units of I are kg·m^2 ; R is a radius of rotation, units are m; α is an angular acceleration, units are rad/s^2. Therefore,

$\tau = m \cdot R^2 \cdot \alpha$ = (1 kg)·(2 m)^2·(5 rad/s^2) = 20 [N·m] (*note that the units of torque are* N·m, *not* J - *joule*!).

Answer: 20 N·m.

Example 1.15.

One load (mass m_1 is 2 kg) lies on the left shoulder (radius R_1 is 1 m) of a see-saw. Another load (mass m_2) lies on the right shoulder (radius R_2 is 2 m). The see-saw is at balance. Evaluate mass m_2.

Solution The system is at equilibrium. Therefore, magnitudes of torque for left and right shoulders of the see-saw are equal:

$\tau_1 = \tau_2$,

$F_1 \cdot R_1 = F_2 \cdot R_2$,

where F_1 and F_2 are the weights of the loads:

$m_1 \cdot g \cdot R_1 = m_2 \cdot g \cdot R_2$

$m_2 = \frac{m_1 \cdot R_1}{R_2} = \frac{(2\ \text{kg}) \cdot (1\ \text{m})}{2\ \text{m}}$ = 1 [kg].

Answer: 1 kg.

Example 1.16.

A child (mass m is 20 kg) slides down from the height of 5 m and reaches its end with the speed 2 m/s. Calculate the thermal energy generated in this process due to friction.

Solution The standard acceleration of free fall g = 9.8 m/s^2. The principle of mechanical energy conservation:

$(K_i + U_i) – (K_f + U_f) = Th$,

where Th is a thermal energy, units are J, subscripts i and f denote initial and final states, correspondingly, K is a kinetic energy, U is a potential energy.

In this example:

$U_i = m{\cdot}g{\cdot}h$, $K_i = 0$, $U_f = 0$, $K_f = \frac{m{\cdot}v^2}{2}$.

So,

$m{\cdot}g{\cdot}h + 0 = 0 + \frac{m{\cdot}v^2}{2} + Th$

$Th = m{\cdot}g{\cdot}h - \frac{m{\cdot}v^2}{2} = (20\ \text{kg}){\cdot}(9.8\ \text{m/s}^2){\cdot}(5\ \text{m}) - \frac{(20\ \text{kg}){\cdot}(2\ \text{m/s})^2}{2} =$

$= 980 – 40 = 940$ [J].

Answer: 940 J.

Example 1.17.

The first ball (mass m_1 is 2 kg) moving east at the speed of 10 m/s collides head-on with the second ball (mass m_2 is 1 kg) that was at rest. Calculate the velocities (speed and direction) of each ball after an elastic collision.

Solution The system is isolated. This means that under these circumstances the law of momentum conservation applies as:

$m_1{\cdot}\overrightarrow{v_{\iota 1}} + m_2{\cdot}\overrightarrow{v_{\iota 2}} = m_1{\cdot}\overrightarrow{v_{f1}} + m_2{\cdot}\overrightarrow{v_{f2}}$ (1)

where $\overrightarrow{v_{\iota 1}}$ is the velocity of the first ball before the collision, $\overrightarrow{v_{\iota 2}}$=0 is the velocity of the second ball before the collision, since this ball was initially at rest, $\overrightarrow{v_{f1}}$ is the velocity of the first ball after the collision, $\overrightarrow{v_{f2}}$ is the velocity of the second ball after the collision.

The collision of these balls is elastic. This means that under these conditions the law of mechanical energy conservation applies as:

$$\frac{m_1 \cdot v_{i1}^2}{2} + \frac{m_2 \cdot v_{i2}^2}{2} = \frac{m_1 \cdot v_{f1}^2}{2} + \frac{m_2 \cdot v_{f2}^2}{2} \qquad (2)$$

Let us align the coordinate axis with the east direction since both balls shall, presumably, move east after the collision. In this case both $v_{f1} \geq 0$ and $v_{f2} \geq 0$. If this assumption is wrong, then signs of v_{f1} and/or v_{f2} will be negative. From (1) one may conclude that

$m_1 \cdot v_{i1} = m_1 \cdot v_{f1} + m_2 \cdot v_{f2}$ and $v_{f2} = \frac{m_1 \cdot v_{i1} - m_1 \cdot v_{f1}}{m_2}$.

Substitution of this expression into (2) gives:

$$\frac{m_1 \cdot v_{i1}^2}{2} = \frac{m_1 \cdot v_{f1}^2}{2} + \frac{m_2}{2} \cdot \left(\frac{m_1 \cdot v_{i1} - m_1 \cdot v_{f1}}{m_2}\right)^2 .$$

The binomial expansion for the second power is:

$(a-b)^2 = a^2 - 2ab + b^2$, then

$$\frac{m_1 \cdot v_{i1}^2}{2} = \frac{m_1 \cdot v_{f1}^2}{2} + \frac{m_2}{2} \cdot \frac{m_1^2 \cdot v_{i1}^2 - 2 \cdot m_1^2 \cdot v_{i1} \cdot v_{f1} + m_1^2 \cdot v_{f1}^2}{m_2^2} .$$

Multiplication of the above expression by $\frac{2}{m_1}$ gives:

$$v_{i1}^2 = v_{f1}^2 + \frac{m_1}{m_2} \cdot v_{i1}^2 - \frac{m_1}{m_2} \cdot 2 \cdot v_{i1} \cdot v_{f1} + \frac{m_1}{m_2} \cdot v_{f1}^2$$

$$(10\ \mathrm{m/s})^2 = v_{f1}^2 + \frac{2\ \mathrm{kg}}{1\ \mathrm{kg}} \cdot (10\ \mathrm{m/s})^2 - \frac{2\ \mathrm{kg}}{1\ \mathrm{kg}} \cdot 2 \cdot (10\ \mathrm{m/s}) \cdot v_{f1} + \frac{2\ \mathrm{kg}}{1\ \mathrm{kg}} \cdot v_{f1}^2 .$$

Collecting terms of the same power with respect to v_{f1} one obtains the following equation for v_{f1}: $3 \cdot v_{f1}^2 - 40 \cdot v_{f1} + 100 = 0$.

The above is a quadratic equation which has two sets of solutions (roots) for v_{f1} and v_{f2} as follows:

1) $v_{f1} = 10$ [m/s]; $v_{f2} = 0$ [m/s]; These roots are related to a formal mathematical solution which describes a situation that is impossible in real life: it assumes that the first ball must go through the second without interaction with the latter.

2) $v_{f1} = \frac{10}{3}$ [m/s]; $v_{f2} = 13\frac{1}{3}$ [m/s].

<u>Answer:</u> $\frac{10}{3}$ m/s east; $13\frac{1}{3}$ m/s east.

2. MOLECULAR PHYSICS

Example 2.1.

An ideal gas (the amount of substance ν = 1 mol, mol is mole unit) is in the vessel of $V = 100\ l$ (l is liter) at the temperature of $T = +27$ ºC. Calculate the gas pressure, P, in the vessel.

Solution Unit conversations:

$V = 100\ (l) = (100\ l)\cdot(10^{-3}\ \mathrm{m}^3/l) = 0.1\ [\mathrm{m}^3]$,

$T = +27$ ºC $= 273 + 27$ (K) $= 300$ [K].

The ideal gas law is:

$P\cdot V = \nu\cdot R\cdot T$,

where P is a pressure, units are Pa – pascal, V is gas volume, units are m^3; T is an absolute temperature, units are K – kelvins; ν is the amount of substance, units are mol – mole; $\nu = \frac{N}{Na} = \frac{m_{\mathrm{tot}}}{M}$, where N is a total number of molecules; $Na = 6.022\times10^{23}$ molecules/mol is the Avogadro`s number; m_{tot} is a total mass of the gas, units are kg, M is a molar mass of the gas, units are kg/mol; R is the ideal gas constant, $R = 8.31$ J/mol·K.

$P = \frac{\nu\cdot R\cdot T}{V} = \frac{(1\ \mathrm{mol})\cdot(8.31\ \mathrm{J/mol\cdot K})\cdot(300\ \mathrm{K})}{0.1\ \mathrm{m}^3} = 24930$ [Pa].

Answer: 24930 Pa.

Example 2.2.

A force, $F = 10$ N, is acting normally on the surface with the area $A = 2\times10^4\ \mathrm{cm}^2$ (cm is centimeter). Calculate pressure, P, at the surface.

Solution Unit conversations:

$A = 2\times10^4\ (\mathrm{cm}^2) = 2\times10^4\times10^{-4}\ (\mathrm{m}^2) = 2\ [\mathrm{m}^2]$.

The definition of pressure:

$P = \frac{F}{A}$,

where P is a pressure, units are Pa; F is a normal force, units are N; A is a surface area, units are m^2.

$P = \frac{10\ \mathrm{N}}{2\ \mathrm{m}^2} = 5$ [Pa].

Answer: 5 Pa.

Example 2.3.

Calculate an oceanic pressure, P, at depth $L = 0.01$ km. The atmospheric pressure $P_0 = 10^5$ Pa. The density of water ρ = 1000 kg/m.

Solution The standard acceleration of free fall $g \approx 10$ m/s^2. Unit conversations:

$L = 0.01$ (km) $= 0.01 \times 10^3$ (m) $= 10$ (m).

A pressure underwater is:

$P = P_0 + \rho \cdot g \cdot L$

where P is a pressure, units are Pa; P_0 is the atmospheric pressure; ρ is the density of fluid, units are kg/m^3; g is the standard acceleration of free fall; L is the depth, units are m.

$P \approx 10^5$ Pa + $(10^3$ kg/m$^3) \cdot (10$ m/s$^2) \cdot (10$ m) $= 2 \times 10^5$ [Pa].

Answer: 2×10^5 Pa.

Example 2.4.

A hydraulic lift has two pistons with cross section areas $A_1 = 0.1$ m^2 and $A_2 = 2$ m^2, which support masses $m_1 = 1$ kg and m_2, respectively. Calculate m_2.

Solution The pressure that is applied to both cylinders is the same, thus,

$P_1 = P_2$,

$\frac{m_1 \cdot g}{A_1} = \frac{m_2 \cdot g}{A_2}$,

$m_2 = m_1 \cdot \frac{A_2}{A_1}$,

$m_2 = (1 \text{ kg}) \cdot \frac{2 \text{ m}^2}{0.1 \text{ m}^2} = 20$ [kg].

Answer: 20 kg.

Example 2.5.

The diameter of syringe\`s cylinder is $D_1 = 1.0$ cm. The diameter of syringe\`s needle is $D_2 = 1.0$ mm (mm is the millimeter). The syringe is full of liquid. A nurse moves the syringe\`s plunger with the speed of $\upsilon_1$. Calculate the speed of flow of liquid, $\upsilon_2$, from the syringe\`s needle.

Solution Unit conversations:

1 cm = 10 mm.

The equation of continuity for incompressible liquid is:

$\upsilon_1 \cdot A_1 = \upsilon_2 \cdot A_2$,

where A_1 and A_2 are areas of cross sections, units are m^2; υ_1 and υ_2 are speeds of liquid through these sections, units are m/s. The area of a circle $A = \frac{\pi \cdot D^2}{4}$, where D is the diameter of circle, units are m. Thus,

$$\upsilon_1 \cdot \frac{\pi \cdot D_1^2}{4} = \upsilon_2 \cdot \frac{\pi \cdot D_2^2}{4} ,$$

$$\upsilon_1 \cdot D_1^2 = \upsilon_2 \cdot D_2^2 ,$$

$$\upsilon_2 = \upsilon_1 \cdot \left(\frac{D_1}{D_2}\right)^2 = \upsilon_1 \cdot \left(\frac{10\ \text{mm}}{1\ \text{mm}}\right)^2 = 100 \cdot \upsilon_1 .$$

Answer: $100 \cdot \upsilon_1$.

Example 2.6.

A fluid flows laminar through a pipe under the pressure difference $\Delta P = 10^6$ Pa. The viscosity of the fluid $\eta = 0.314$ Pa·s. The length of the pipe $L = 12.5$ m, the radius of the pipe $R = 10$ cm. Calculate a volumetric flow rate Q.

Solution Unit conversations:

$R = 10$ (cm) $= 10 \times 10^{-2} = 0.1$ [m].

Hagen-Poiseuille equation is:

$$Q = \frac{\pi \cdot R^4}{8 \cdot \eta \cdot L} \cdot \Delta P ,$$

where Q is a volumetric flow rate, units are m^3/s, R is the pipe`s radius, units are m, η is the viscosity of the fluid, units are Pa·s, $L$ is the pipe`s length, units are m, ΔP is pressure difference at two ends of pipe, units are Pa.

$$Q = \frac{\pi \times (0.1\ \text{m})^4}{8 \cdot (0.314\ \text{Pa} \cdot \text{s}) \cdot (12.5\ \text{m})} \times (10^6\ \text{Pa}) = \frac{10 \times 10^{-4} \times 10^6}{100} = 10\ [\text{m}^3/\text{s}].$$

Answer: 10 m^3/s.

Example 2.7.

The speed of flow in a circular pipe is $\upsilon = 1$ m/s. Diameter of the pipe $D = 0.1$ m. The density of the liquid is $\rho = 1000$ kg/m^3. The viscosity of the liquid $\eta = 0.1$ Pa·s. Calculate Reynolds number, *Re*, of the flow.

Solution Reynolds number is defined as:

$$Re = \frac{\rho \cdot \upsilon \cdot D}{\eta},$$

where ρ is the density of the fluid, units are kg/m^3, υ is the flow speed, units are m/s, η is kinematic viscosity of the fluid, units are Pa·s, D is a characteristic linear dimension, units are m (when liquid flows within a circular pipe then D is the inside diameter of the pipe, or D is diameter of a ball moving in liquid).

$$Re = \frac{(10^3\ \mathrm{kg/m^3})\cdot(1\ \mathrm{m/s})\cdot(0.1\ \mathrm{m})}{0.1\ \mathrm{Pa \cdot s}} = 1000 \text{ [dimensionless or unitless]}.$$

Answer: 1000.

Example 2.8.

A temperature reading $T = +50$ ºC. Convert this value to an absolute temperature value.

Solution

T (K) ≈ 273 + T (ºC) = 273 +50 = 323 [K].

Answer: 323 K.

Example 2.9.

An energy value is $W = 5$ cal (cal is the calorie). Convert this value to Joules.

Solution Unit conversations:

1 cal = 4.186 J.

W = (5 cal)·(4.186 J/cal) = 20.93 [J].

Answer: 20.93 J.

Example 2.10.

A gas initially contained in a vessel of the volume $V_i = 1\ m^3$ is compressed to the volume $V_f = 0.5\ m^3$ by the constant pressure $P = 1$ atm (atm is the atmosphere). Calculate work, performed on the gas, W_g .

Solution Unit conversations:

1 atm = 10^5 Pa.

The work is done *by* the gas (when $V_f > V_i$) or performed *on* the gas (when $V_f < V_i$)

$W = P\cdot(V_f - V_i)$,

where W is a work performed on/by the gas, units are J, P is a pressure in this system, units are Pa, V_f and V_i are the final and initial volumes of the gas, correspondingly, units are m^3.

$W = (10^5\ Pa)\cdot(0.5\ m^3 - 1\ m^3) = -5\times10^4$ [J]

The work *on* gas is

$W_g = -W = 5\times10^4$ [J].

Answer: 5×10^4 J.

Example 2.11.

A system received a heat $Q = 10$ J and performed a work $W = 5$ J. Calculate the change of energy, ΔU, of the system.

Solution The first law of thermodynamics

$\Delta U = Q - W$,

where ΔU is a change of the internal energy of the system, units are J, Q is a quantity of heat, units are J, W is the work done by the system, units are J.

$\Delta U = 10$ J- 5 J = 5 [J].

Answer: 5 J.

Example 2.12.

An initial pressure of the ideal gas is P_i. The volume of the gas decreased twofold as a result of an isothermal process. Calculate the final pressure, P_f, of the gas.

Solution The equation of an isothermal process for the ideal gas is:

$P_i \cdot V_i = P_f \cdot V_f$,

where P_i and P_f are the initial and the final pressures of the gas, units are Pa, V_i and V_f are the initial and the final volumes of the gas, units are m^3.

Since $V_i = 2 \cdot V_f$, then $P_i \cdot 2 \cdot V_f = P_f \cdot V_f$,

$P_f = 2 \cdot P_i$.

Answer: $2 \cdot P_i$.

Example 2.13.

A heat engine consumes from a hot reservoir the heat Q_h = 10 J per each thermodynamic cycle. It converts some of the consumed heat to a work and transfers the rest of the heat, Q_c = 2 J, to a cold reservoir. Calculate the efficiency ϵ of this heat engine [%].

Solution The efficiency of a heat engine is:

$\epsilon = 1 - \left|\frac{Q_c}{Q_h}\right|$,

where Q_h and Q_c are the quantities of heat transferred from a hot reservoir to a cold reservoir and from a cold reservoir to a hot reservoir, respectively, units are J.

$\epsilon = 1 - \frac{2\ \mathrm{J}}{10\ \mathrm{J}} = 0.8 = 80$ [%].

Answer: 80 %.

Example 2.14.

A liquid is at the height h = 0.1 m in a capillary tube, the capillary radius is R = 1 mm. The density of the liquid is ρ = 800 kg/m^3. The standard acceleration of free fall g = 10 m/s^2. Calculate the surface tension, σ, of the liquid.

Solution Unit conversations:

R = 1 (mm) = 1×10^{-3} (m).

The height of a liquid in a capillary is:

$h = \frac{2 \cdot \sigma}{\rho \cdot R \cdot g}$, where h is the height of liquid in a capillary, units are m, σ is the surface tension at the liquid`s surface, units are N/m, ρ is the density of the liquid, units are kg/m^3, R is the capillary radius, units are m.

$$\sigma = \frac{\rho \cdot h \cdot R \cdot g}{2} \approx \frac{(800\ \mathrm{kg/m^3}) \cdot (0.1\ \mathrm{m}) \cdot (1 \times 10^{-3}\,\mathrm{m}) \cdot (10\ \mathrm{m/s^2})}{2} = 0.4\ [\mathrm{N/m}].$$

Answer: 0.4 N/m .

Example 2.15.

What volume, V, of helium is needed in a balloon if this balloon is to lift a load of m = 180 kg (including the weight of empty balloon)? The density of helium is ρ_{He} = 0.179 kg/m^3, the density of air is ρ_{air} = 1.29 kg/m^3.

Solution Second Newton`s law is:

$$\sum_i \vec{F_i} = m \cdot \vec{a} .$$

There are three forces acting on the balloon: the force of gravity of helium $m_{He} \cdot g$, the force of gravity of the empty balloon $m \cdot g$, and the buoyant force. Archimedes` principle is:

$$F_{Arch} = \rho \cdot g \cdot V ,$$

where F_{Arch} is the buoyant force, units are N, ρ is the density of the fluid, units are kg/m^3, V is the volume of a body immersed in the fluid (liquid or gas), units are m^3, the standard acceleration of free fall g = 9.8 m/s^2.

The system is in static equilibrium, consequently a = 0. Directions of both force of gravity forces are opposite to the direction of the buoyant force. Thus,

$$m_{He} \cdot g + m \cdot g - \rho_{air} \cdot g \cdot V = 0 ,$$

since $m_{He} = \rho_{He} \cdot V$, then

$$\rho_{He} \cdot V + m - \rho_{air} \cdot V = 0$$

$$V = \frac{m}{\rho_{air} - \rho_{He}} = \frac{180\ \mathrm{kg}}{1.29\ \mathrm{kg/m^3} - 0.179\ \mathrm{kg/m^3}} = 162\ [\mathrm{m^3}].$$

Answer: 162 m^3 .

3. ELECTRICITY AND MAGNETISM

Example 3.1.

Two point charges, q_1 = +25 nC (n is nano prefix, this prefix denotes a factor of 10^{-9}, C is the coulomb) and q_2 = -75 nC, are separated by the distance of 3 cm. Calculate the electric force between the charges.

Solution Unit conversations:

q_1 = +25 nC = +25×10^{-9} C,

q_2 = -75 nC = -75×10^{-9} C,

r = 3 cm = 0.03 m.

Coulomb`s law:

$F = k \cdot \frac{|q_1 \cdot q_2|}{r^2}$,

where F is an electric force, units are N, q_1 and q_2 are two point charges, units are C (coulomb), r is the distance between the charges, units are m, the Coulomb constant (the electric force constant) $k = 9\times10^9$ N·m^2/C^2. Thus,

$F = (9\times10^9\ \mathrm{N \cdot m^2/C^2}) \cdot \frac{|(25\times10^{-9}\ \mathrm{C}) \cdot (-75\times10^{-9}\ \mathrm{C})|}{(0.03\ \mathrm{m})^2} \approx 0.019$ [N].

Answer: 0.019 N.

Example 3.2.

A positive point charge q = +3 μC (μ is micro prefix, this prefix denotes a factor of 10^{-6}) is surrounded by a sphere with the radius r = 0.2 m. Calculate the electric flux Φ (N·m^2·C^{-1}) through the sphere`s surface.

Solution The Coulomb constant $k = 9\times10^9$ N·m^2/C^2. Unit conversations:

q = +3 μC = +3×10^{-6} C.

Gauss`s law:

$\Phi = \frac{q}{\epsilon_0}$,

where Φ is the total electric flux, units are V·m (V is volt), q is the charge, units are C, $\epsilon_0 = 8.854\times10^{-12}$ C^2·N·m^2 is the vacuum permittivity.

$\Phi = 4 \cdot \pi \cdot k \cdot q \approx 4 \cdot 3.14 \cdot (9\times10^9\ \mathrm{N \cdot m^2/C^2}) \cdot (3\times10^{-6}\ \mathrm{C}) = 3.4\times10^5$ [N·m^2/C].

Answer: 3.4×10^5 N·m^2/C .

Example 3.3.

A parallel-plate capacitor (the insulator is air) has capacitance of C = 1 F (F is farad). If the plates of the capacitor are 1 mm apart, then what is the area A of the plates?

Solution The Coulomb constant $k = 9\times10^{9}$ N·m^2/C^2. Unit conversations:

d = 1 mm = 1×10^{-3} m.

The capacitance of a parallel-plate capacitor in vacuum

$C = \epsilon_0 \frac{A}{d}$,

where C is the capacitance, units are F, A is the area of its plates, units are m^2, d is the separation (distance) between the plates, units are m, the vacuum permittivity $\epsilon_0 = 8.854\times10^{-12}$ C^2·N·m^2 or $\epsilon_0 = \frac{1}{4\pi\cdot k}$. Thus,

$A = \frac{C\cdot d}{\epsilon_0} = 4\pi\cdot k\cdot C\cdot d =$

$= 4\cdot3.14\cdot(9\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\cdot(1\ \text{F})\cdot(1\times10^{-3}\ \text{m}) = 1\times10^{8}\ [\text{m}^2]$.

Answer: 1×10^{8} m^2.

Example 3.4.

Calculate the equivalent capacitance, when two capacitors C_1 = 1 F and C_2 = 3 F are connected in series.

Solution The equivalent capacitance of a series capacitors` combination is:

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1\ \text{F}} + \frac{1}{3\ \text{F}} = \frac{4}{3}\ [\frac{1}{\text{F}}]$,

$C = \frac{3}{4} = 0.75$ [F].

Answer: 0.75 F.

Example 3.5.

Calculate the equivalent capacitance, when two capacitors C_1 = 1 F and C_2 = 3 F are connected in parallel.

Solution The equivalent capacitance of the parallel capacitors` combination is:

$C = C_1 + C_2 = 1\text{ F} + 3\text{ F} = 4$ [F].

<u>Answer:</u> 4 F.

<u>Example 3.6.</u>

The difference of electric potentials between two ends of a conductor is $V = 2$ V. The current through the conductor is $I = 0.5$ A (A is ampere). Calculate the resistance, R, of the conductor.

<u>Solution</u> Ohm`s law

$R = \frac{V}{I}$

where R is the resistance, units are Ω, V is the potential difference, units are V, I is the current, units are A.

$R = \frac{2\text{ V}}{0.5\text{ A}} = 4$ [Ω].

<u>Answer:</u> 4 Ω.

<u>Example 3.7.</u>

A wire has the diameter $d = 1$ mm, its length $l = 314$ m, and its resistivity $\rho = 2\times10^{-8}$ Ω·m. Calculate the resistance R of the wire.

<u>Solution</u> Unit conversations:

$d = 1\text{ mm} = 10^{-3}$ m.

The relationship between resistance and resistivity is

$R = \rho\cdot\frac{l}{A}$

where R is the resistance, units are Ω, ρ is the resistivity, units are Ω·m, l is the length, units are m, A is the cross-section area, units are m^2. The area of a circle $A = \frac{\pi\cdot d^2}{4}$, where d is the diameter of the circle, units are m. Thus,

$R = \rho\cdot\frac{l}{\pi\cdot\frac{d^2}{4}} = (2\cdot10^{-8}\ \Omega\cdot\text{m})\cdot\frac{314\text{ m}}{3.14\cdot\frac{(10^{-3}\text{ m})^2}{4}} \approx 8$ [Ω].

<u>Answer:</u> 8 Ω.

Example 3.8.

Calculate the equivalent resistance, when two resistors $R_1 = 1\ \Omega$ and $R_2 = 3\ \Omega$ are connected in parallel.

Solution The equivalent resistance of a parallel combination of resistors is:

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{1\ \Omega} + \frac{1}{3\ \Omega} = \frac{4}{3} [\frac{1}{\Omega}]$,

$R = \frac{3}{4} = 0.75\ [\Omega]$.

Answer: 0.75 Ω.

Example 3.9.

Calculate the equivalent resistance, when two resistors $R_1 = 1\ \Omega$ and $R_2 = 3\ \Omega$ are connected in series.

Solution The equivalent resistance of series combination of resistors is:

$R = R_1 + R_2 = 1\ \Omega + 3\ \Omega = 4\ [\Omega]$.

Answer: 4 Ω.

Example 3.10.

Calculate the rate of energy dissipation (the conversion of electrical energy into heat), P, in the resistor $R = 10\ \Omega$, if current $I = 2$ A.

Solution The rate of energy dissipation is:

$P = V{\cdot}I$ (1),

where P is a power (rate of energy dissipation), units are W, V is the potential difference, units are V, I is the current, units are A. Ohm`s law

$R = \frac{V}{I}$ (2),

where R is the resistance. From (2): $V = R{\cdot}I$, substituting this into (1) gives

$P = I^2{\cdot}R = (2\ \text{A})^2{\cdot}(10\ \Omega) = 40\ [\text{W}]$.

Answer: 40 W.

Example 3.11.

A proton moves within a uniform magnetic field with the magnitude $B = 5$ T. Proton\`s speed $\upsilon = 2\times10^5$ m/s. Proton\`s velocity and direction of the magnetic field constitute the angle $\alpha = 30°$. Calculate the magnitude of magnetic force acting on the moving proton.

Solution The proton charge is $q = 1.6\times10^{-19}$ C.

sin(30°) = 0.5

The magnetic force acting on a moving charged particle is

$\vec{F} = q\cdot\vec{\upsilon} \times \vec{B}$,

where $\vec{F}$ is the magnetic force, units are N, q is the charge, units are C, $\vec{\upsilon}$ is the velocity vector, units are m/s, $\vec{B}$ is the magnetic field, units are T – tesla. The magnitude of magnetic force is:

$F = q\cdot\upsilon\cdot B\cdot\sin(\alpha) = (1.6\times10^{-19}\text{ C})\cdot(2\times10^5\text{ m/s})\cdot(5\text{ T})\cdot0.5 = 8\times10^{-14}$ [N].

Answer: 8×10^{-14} N.

Example 3.12.

Two parallel straight superconducting cables lay 4.5 mm apart and carry equal currents of $I = 15000$ A. The length of each cable is $L = 1$ m. Calculate the magnitude of the interaction force F between these cables.

Solution Unit conversations:

$d = 4.5\text{ mm} = 4.5\times10^{-3}$ m.

The magnitude of the interaction force between two long straight parallel current-carrying conductors is

$F = \frac{\mu_0\cdot I_1\cdot I_2\cdot L}{2\cdot\pi\cdot d}$

where F is the magnitude of the force, units are N, the vacuum magnetic permeability $\mu_0 = 4\cdot\pi\times10^{-7}$ (T·m/A), I_1 and I_2 are the currents through the conductors 1 and 2, units are A, L is the length of conductors, units are m, d is the distance between conductors, units are m.

$$F = \frac{(4 \cdot \pi \times 10^{-7}\ \text{T} \cdot \text{m/A}) \cdot (15000\ \text{A}) \cdot (15000\ \text{A}) \cdot (1\ \text{m})}{2 \cdot \pi \cdot (4.5 \times 10^{-3}\ \text{m})} = 1 \times 10^4\ [\text{N}].$$

Answer: 1×10^4 N.

Example 3.13.

A solenoid consists of a helical winding of a wire on a cylinder. The solenoid has $n = 10^7$ turns of the wire per meter and carries the current $I = 2$ A. Find the magnitude of the magnetic field B at the middle of the solenoid`s length.

Solution From Ampere`s law

$B = \mu_0 \cdot n \cdot I$

where B is the magnetic field, units are T, the vacuum magnetic permeability $\mu_0 = 4 \cdot \pi \times 10^{-7}$ (T·m/A), n is the number of turns of the wire per unit length, units are m^{-1}, I is the current, units are A.

$B = (4 \cdot \pi \times 10^{-7}\ \text{T} \cdot \text{m/A}) \cdot (10^7\ \text{m}^{-1}) \cdot (2\ \text{A}) = 25.12\ [\text{T}].$

Answer: 25.12 T.

4. SOUND AND WAVES

Example 4.1.

The frequency of a sound wave is $f = 500$ Hz. Calculate its period T.

Solution For a simple harmonic motion

$T = \frac{1}{f}$,

where T is the period of sound vibration, units are s, f is the frequency, units are Hz – hertz.

$T = \frac{1}{500\ \text{Hz}} = 0.002$ [s].

Answer: 0.002 s.

Example 4.2.

The speed of sound in air at 20 °C is $\upsilon = 344$ m/s. The frequency of a sound wave is $f = 172$ Hz. Calculate its wavelength λ.

Solution Wavelength is:

$\lambda = \frac{\upsilon}{f}$,

where λ is the wavelength, units are m, υ is the speed of the wave, units are m/s, f is the frequency, units are Hz.

$\lambda = \frac{344\ \text{m/s}}{172\ \text{Hz}} = 2$ [m].

Answer: 2 m.

Example 4.3.

The distance to the point source of sound increased doubled. Calculate how many times the intensity of sound has decreased.

Solution The intensity of sound vibrations originated from a point source of sound at a distance r is:

$I = \frac{P}{4 \cdot \pi \cdot r^2}$,

where I is the intensity of sound, units are W/m^2, P is the power of a point source, units are W, r is the distance to the point source, units are m.

$r_2 = 2 \cdot r_1$, $I_1 = \frac{P}{4 \cdot \pi \cdot r_1^2}$, $I_2 = \frac{P}{4 \cdot \pi \cdot r_2^2}$, therefore,

$\frac{I_1}{I_2} = \frac{P \cdot 4 \cdot \pi \cdot r_2^2}{P \cdot 4 \cdot \pi \cdot r_1^2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2 \cdot r_1}{r_1}\right)^2 = 4$.

Answer: 4 times.

Example 4.4.

The intensity of sound is I = 1 W/m^2. Calculate the power, P, of the sound passing through the window with the surface area 3 m^2.

Solution The sound intensity is

$I = \frac{P}{A}$,

where I is the sound intensity, units are W/m^2, P is the sound power, units are W, A is the area, units are m^2.

$P = I \cdot A$ = (1 W/m^2)·(3 m^2) = 3 [W].

Answer: 3 W.

Example 4.5.

The speed of sound in ideal gas at T_1 = 127 °C is υ_1 = 100 m/s. Calculate the speed, υ_2, of sound at T_2 = -173 °C.

Solution Unit conversations:

T_1 = 127 °C = (273 + 127) K = 400 K

T_2 = -173 °C = (273 – 173) K = 100 K

The speed of sound in ideal gas is

$\upsilon = \sqrt{\frac{\gamma \cdot R \cdot T}{M}}$,

where υ is the speed of sound, units are m/s, the ideal gas constant R = 8.31 J/mol·K, T is an absolute temperature of the gas, units are K, M is molar mass of the gas, units are kg/mol, γ is the adiabatic index (dimensionless), $= \frac{i+2}{i}$, where i is the number of degrees of freedom of the gas molecules motions (3 for a monatomic gas, 5 for a diatomic gas etc.). Thus,

$\upsilon_1 = \sqrt{\frac{\gamma \cdot R \cdot T_1}{M}}$, $\upsilon_2 = \sqrt{\frac{\gamma \cdot R \cdot T_2}{M}}$, then $\frac{\upsilon_2}{\upsilon_1} = \frac{\sqrt{\frac{\gamma \cdot R \cdot T_2}{M}}}{\sqrt{\frac{\gamma \cdot R \cdot T_1}{M}}} = \sqrt{\frac{T_2}{T_1}}$

$\upsilon_2 = \upsilon_1 \cdot \sqrt{\frac{T_2}{T_1}}$ = (100 m/s)$\cdot\sqrt{\frac{100\text{ K}}{400\text{ K}}}$ = 100$\cdot\sqrt{\frac{1}{4}}$ = $\frac{100}{2}$ = 50 [m/s].

Answer: 50 m/s.

Example 4.6.

The intensity of sound is $I = 100\cdot I_0$. The threshold of hearing at 1000 Hz is $I_0 = 10^{-12}$ W/m^2. Calculate the loudness of sound (sound intensity level) β .

Solution The definition of sound intensity level is:

β [dB] = $10\cdot\log\left(\frac{I}{I_0}\right)$ (1).

Let us recall that "log" means logarithm to the base 10, dB is decibel.

$I = 100\cdot I_0$ substitution of this into (1) leads to:

$\beta = 10\cdot\log\left(\frac{100\cdot I_0}{I_0}\right) = 10\cdot\log 10^2 = 10\cdot 2\cdot\log 10$ = 20 [dB].

Answer: 20 dB.

Example 4.7.

A car traveling at the speed υ_s = 360 km/h creates a sound with the frequency f_s = 1 kHz. The car approaches a standing person. Calculate the frequency of the sound f_L that this person hears.

Solution Unit conversations:

υ_s = 360 km/h = $\frac{(360\text{ km/h})\cdot(1000\text{ m/km})}{60\cdot 60\text{ s/h}}$ = 100 [m/s].

The equation of Doppler effect is

$f_L = f_s \cdot \frac{\upsilon+\upsilon_L}{\upsilon-\upsilon_s}$,

where f_L is the frequency heard by a listener, units are Hz, f_s is the frequency of source, units are Hz, υ = 343 m/s is the speed of sound in air, υ_L is the speed of the listener, units are m/s, υ_s is the speed of the source, units are m/s.

f_L = (1000 Hz)$\cdot\left(\frac{343\text{ m/s}+0\text{ m/s}}{343\text{ m/s}-100\text{ m/s}}\right) = 1412$ [Hz].

Answer: 1412 Hz.

5. OPTICS

Example 5.1.

The speed of light in a substance is $\upsilon = 1.5\times10^8$ m/s. Calculate the refractive index n of the substance`s material.

Solution The speed of light in vacuum $c = 3\times10^8$ [m/s]. The refractive index is

$n = \frac{c}{\upsilon} = \frac{3\times10^8 \text{ m/s}}{1.5\times10^8 \text{ m/s}} = 2$ [dimensionless].

Answer: 2.

Example 5.2.

The distance between an object and a thin lens is $o = 12$ cm. The distance from the lens to the object`s image is $i = 6$ cm. Calculate the focal length f .

Solution Object-image relation for a thin lens is:

$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$,

where o and i are the object and image distances, f is the focal length of the lens, units are m or other units of length, but they must be the same for all tree values.

$\frac{1}{f} = \frac{1}{12 \text{ cm}} + \frac{1}{6 \text{ cm}} = \frac{3}{12} [\frac{1}{\text{cm}}]$,

$f = \frac{12}{3} = 4$ [cm].

Answer: 4 cm.

Example 5.3.

The focal length of a thin lens is $f = 10$ cm. Calculate the power of the lens, P [diopters].

Solution Unit conversations:

$f = 10$ [cm] = 0.1 [m].

The power of the lens is:

$P = \frac{1}{f} = \frac{1}{0.1\ \text{m}} = 10$ [diopters].

Answer: 10 diopters.

Example 5.4.

A light beam is incident from a substance 1 (refractive index $n_1 = 2.5$) upon a substance 2 (refractive index $n_2 = 1.5$) normally to the boundary surface between the substances. Calculate the reflectivity of the boundary surface between the substances, R [%].

Solution The reflectivity of the boundary surface is

$R = \left|\frac{n_2 - n_1}{n_2 + n_1}\right| \cdot 100\ \% = \left|\frac{1.5 - 2.5}{1.5 + 2.5}\right| \cdot 100\ \% = 25$ [%].

Answer: 25 %.

Example 5.5.

A film has the thickness $d = 100$ nm and a refractive index $n = 1.5$. A monochromatic light (wavelength in air $\lambda = 600$ nm) is incident normally upon this film. Can we see the reflection (maximum) of this light?

Solution The angle of incidence is 0°, and in this case: $\sin(0°) = 0$, $\cos(0°) = 1$. Thus, the maximum reflection condition for thin-film interference is

$2 \cdot n \cdot d = (m - 0.5) \cdot \lambda$,

where $m = 0, \pm 1, \pm 2, \pm 3, \ldots$

$m = \frac{2n \cdot d}{\lambda} + 0.5 = \frac{2 \cdot 1.5 \cdot (100\ \text{nm})}{600\ \text{nm}} + 0.5 = 1.$

Answer: $m = 1$, yes.

Example 5.6.

You pass a laser beam $\lambda = 600$ nm through a narrow slit and observe the diffraction pattern on a screen $D = 6$ m away. The distance on the screen between the centers of the first minima outside the central bright fringe is 25 mm. How wide is opening of the slit a ?

Solution Unit conversations:

$\lambda = 600\ \text{nm} = 600 \times 10^{-9}$ [m]

25 mm = 25×10^{-3} [m]

Condition for the first minima diffraction pattern from a single slit is m = 1:

$a = \frac{m\cdot\lambda\cdot D}{y}$,

where a is the width of the slit, λ is the wavelength, D is the distance from the slit to the screen, y is the distance on the screen between centers of the first minima and the center of image symmetry, units are m for all variables.

$a = \frac{1\cdot(600\times10^{-9}\,\text{m})\cdot(6\text{ m})}{0.5\cdot25\times10^{-3}\text{ m}} = 2.88\times10^{-4}$ [m].

Answer: 2.88×10^{-4} m.

Example 5.7.

A gamma-ray photon has the energy of $E = 2.209\times10^{-13}$ J. Calculate the wavelength λ of this electromagnetic radiation.

Solution The energy of an individual photon is

$E = h\cdot f = \frac{h\cdot c}{\lambda}$,

where E is the energy of photon, units are J, Planck constant $h = 6.626\times10^{-34}$ J·s, f is the frequency of electromagnetic radiation, units are Hz, the speed of light in vacuum $c = 3\times10^{8}$ m/s, λ is the wavelength, units are m.

$\lambda = \frac{h\cdot c}{E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\cdot(3\times10^{8}\text{ m/s})}{2.209\times10^{-13}\text{ J}} = 9\times10^{-13}$ [m].

Answer: 9×10^{-13} m.

6. TASKS FOR INDEPENDENT SOLUTION

Mechanics

6.1.1. The speed of a body is 18 km/h. Convert km/h to m/s. (Ans.: 5 m/s)

6.1.2. The lengths of the vectors $\vec{A}$ is 2 and $\vec{B}$ is 1, the angle between them is 60º. Calculate their scalar product. (Ans.: 1)

6.1.3. A body moves along a straight line with a constant acceleration ($a > 0$). The acceleration is 1 m/s^2, the initial speed is 50 m/s, the time interval of acceleration is 10 s. Calculate the final speed. (Ans.: 60 m/s)

6.1.4. The body moves along a straight line with a constant acceleration and passed 100 m in 4 s. The acceleration is 10 m/s^2. Calculate the initial speed. (Ans.: 5 m/s)

6.1.5. Two forces, F_1 = 100 N and F_2 = 20 N, act on a body in opposite directions toward each other. The mass of a body is 2 kg. Calculate the acceleration of the body. (Ans.: 40 m/s^2)

6.1.6. A mass m (20 kg) hangs on a spring that has spring constant k (20 N/m). Calculate the extension of the string, when the mass attached to the string is at rest and at static force equilibrium. (Ans.: 10 m)

6.1.7. A body (mass m is 5 kg) is at the height h = 5 m relative to the surface of the Earth. Calculate its potential energy. (Ans.: 250 J)

6.1.8. A body moves along a straight line at the constant speed υ (10 m/s) under the action of the force F = 2 N. The directions of velocity and force coincide. Calculate the power related with this process. (Ans.: 20 W)

6.1.9. Two particles are at a distance r_1 under the gravitational attraction F_1. Calculate the force of gravitational attraction F_2 between the same particles, if the distance between them is decreased 3 times. (Ans.: $F_2 = 9 \cdot F_1$)

6.1.10. A point particle (mass 2 kg) moves on a circle trajectory with the radius of 3 m under the angular acceleration of 10 rad/s^2. Calculate the torque associated with this motion. (Ans.: 180 N·m)

6.1.11. A child (mass m is 20 kg) slides down from the height of 5 m and reaches its end with the speed of 8 m/s. Calculate the thermal energy generated in this process due to friction. (Ans.: 360 J)

6.1.12. The first ball (mass m_1 is 2 kg) moving east at the speed of 10 m/s collides head-on with the second ball (mass m_2 is 2 kg) that was at rest. Calculate the velocities of each ball after perfectly inelastic collision. (Ans.: 5 m/s, east)

Molecular physics

6.2.1. A force, $F = 20$ N, acting normally on a surface with the area $A = 2\times10^4$ cm^2. Calculate pressure at the surface. (Ans.: 10 Pa)

6.2.2. A hydraulic lift has two pistons with cross sectional areas $A_1 = 0.1$ m^2 and $A_2 = 0.2$ m^2, which support masses $m_1 = 10$ kg and m_2, respectively. Calculate m_2. (Ans.: 20 kg)

6.2.3. The speed of flow in a circular pipe is $\upsilon = 1$ m/s. Diameter of the pipe is $D = 0.1$ m. The density of the liquid is $\rho = 1000$ kg/m^3. The viscosity of the liquid is $\eta = 0.2$ Pa·s. Calculate Reynolds number of the flow. (Ans.: 500)

6.2.4. A system received a heat $Q = 1000$ J and performed a work $W = 50$ J. Calculate the change of energy of the system. (Ans.: 950 J)

6.2.5. An ideal gas (the amount of substance 10 mol) is in the vessel of $V = 100$ liters at the temperature of $T = +27$ ºC. Calculate the gas pressure in the vessel. (Ans.: 249300 Pa)

6.2.6. Calculate an oceanic pressure at depth $L = 0.10$ km. The atmospheric pressure is $P_0 = 10^5$ Pa. The density of water is $\rho = 1000$ kg/m^3. (Ans.: 11×10^5 Pa)

6.2.7. The diameter of syringe\`s cylinder is $D_1 = 2.0$ cm. The diameter of syringe\`s needle is $D_2 = 1.0$ mm. A nurse moves the syringe\`s plunger with the speed $\upsilon_1$. Calculate the speed of flow of the liquid from the syringe\`s needle. (Ans.: $\upsilon_2 = 400\cdot\upsilon_1$)

6.2.8. A fluid flows laminar through a pipe under the pressure difference $\Delta P = 10^4$ Pa. The viscosity of the fluid is $\eta = 0.314$ Pa·s. The length of the pipe is $L = 12.5$ m, the radius of the pipe is $R = 50$ cm. Calculate a volumetric flow rate Q. (Ans.: 62.5 m^3/s)
6.2.9. An initial pressure of the ideal gas is P_i. The temperature of the gas decreased twofold as a result of an isovolumetric (isochoric) process. Calculate the final pressure of the gas. (Ans.: $P_f = P_i/2$)
6.2.10. A heat engine consumes from a hot reservoir the heat $Q_h = 100$ J per each thermodynamic cycle, converts some of the consumed heat to a work and transfers the rest of the heat $Q_c = 20$ J to a cold reservoir. Calculate the efficiency $\in$ of this heat engine [%]. (Ans.: 80 %)
6.2.11. A liquid is at the height of $h = 0.1$ m in a capillary tube, the capillary radius is $R = 2$ mm. The density of the liquid is $\rho = 800$ kg/m^3. Calculate the surface tension of the liquid. (Ans.: 0.8 N/m)
6.2.12. The density of ice is $\rho_{ice} = 900$ kg/m^3, that of water is $\rho_{water} = 1000$ kg/m^3. What fraction of iceberg (by volume – V_{above}/V_{whole}) is above the surface of the water? (Ans.: 0.1)

Electricity and magnetism

6.3.1. A point charge $q = -1\ \mu$C is surrounded by a sphere with the radius $r = 1$ m. Calculate the electric flux Φ through the sphere`s surface. (Ans.: -1.13×10^5 N·m/C)
6.3.2. Calculate the equivalent capacitance, when three capacitors $C_1 = 50$ nF, $C_2 = 150$ nF, and $C_3 = 75$ nF are connected in series. (Ans.: 25 nF)
6.3.3. Calculate the electric potential energy for the charge $q = +1$ mC at the distance of $r = 9$ cm from the charge $Q = +10$ C. (Ans.: 10^9 J)
6.3.4. A wire has the diameter $d = 2$ mm, its length $l = 314$ m, and its resistivity $\rho = 2\times10^{-8}\ \Omega$·m. Calculate the resistance of the wire. (Ans.: 2 Ω)

6.3.5. Calculate the equivalent resistance, when the three resistors $R_1 = 50$ kΩ, $R_2 = 150$ kΩ, and $R_3 = 75$ kΩ are connected in parallel. (Ans.: 25 kΩ)

6.3.6. Calculate the work of electrical forces A in uniform electric field with magnitude $E = 1$ N/C upon displacement $L = 1$ km of charge $q = +1$ mC in the direction of the field. (Ans.: 1 J)

6.3.7. Calculate the rate of energy dissipation in the resistor $R = 100$ Ω, if current $I = 1$ mA. (Ans.: 10^{-4} W)

6.3.8. A proton moves within a uniform magnetic field with the magnitude $B = 5$ T. Proton\`s speed $\upsilon = 2\times10^5$ m/s. Proton\`s velocity and direction of the magnetic field constitute the angle $\alpha = 0°$. Calculate the magnitude of Lorentz force. (Ans.: 0 N)

6.3.9. Two parallel straight superconducting cables lay 4.5 mm apart and carry equal currents of $I = 1.5$ kA. The length of each cable is $L = 1$ m. Calculate the magnitude of the interaction force between these cables. (Ans.: 100 N)

6.3.10. A solenoid consists of a helical winding of a wire on a cylinder. The solenoid has $n = 10^7$ turns of the wire per meter and carries the current $I = 4$ A. Calculate the magnitude of magnetic field B at the middle of the solenoid\`s length. (Ans.: 50.27 T)

Sound and waves

6.4.1. The speed of sound at 20 °C is $\upsilon = 344$ m/s. The frequency of a sound wave is 344 Hz. Calculate its wavelength λ. (Ans.: 1 m)

6.4.2. Calculate the underdamped response of an oscillator x at the moment $t = 2$ s if damping coefficient is $g = 1$ s^{-1}, the amplitude is $A = 10$ m, the period is $T = 1$ s, the starting phase is $\alpha = 0$ on the cosine fashion. (Ans.: 1.35 m)

6.4.3. The distance to the point source of sound has increased 3 times. Calculate how many times the intensity of sound has decreased. (Ans.: 9)

6.4.4. The intensity of sound is $I = 2$ W/m^2. Calculate the power of the sound passing through the window of 1 m^2. (Ans.: 2 W)

6.4.5. The speed of sound in ideal gas at T_1 = -223 °C is υ_1 = 300 m/s. Calculate the speed of sound at T_2 = -73 °C. (Ans.: 600 m/s)
6.4.6. The intensity of sound is $I = 0.1 \cdot I_0$. Calculate the loudness of sound. (Ans.: -10 dB)
6.4.7. A car traveling at the speed υ_s = 180 km/h creates a sound with the frequency f_s = 1 kHz and approaches a standing person. Calculate the frequency of the sound that this person hears. (Ans.: 1170 Hz)

Optics

6.5.1. The speed of light in a substance is $\upsilon = 1\times10^8$ m/s. Calculate the refractive index n of the substance`s material. (Ans.: 3)
6.5.2. The distance between an object and a thin lens is o = 12 cm. The distance from the lens to the object`s image is i = 4 cm. Calculate the focal length of the lens. (Ans.: 3 cm)
6.5.3. The focal length of a thin lens is f = 5 cm. Calculate the power of the lens. (Ans.: 20 diopters)
6.5.4. Light beam is incident from a substance 1 (refractive index n_1 = 1) upon a substance 2 (refractive index n_2 = 1.5) normally to the boundary surface between the substances. Calculate the reflectivity of the boundary surface between the substances. (Ans.: 20 %)
6.5.5. A thin film has a thickness d = 200 nm and a refractive index n = 1.5. A monochromatic light (λ = 600 nm) is incident normally upon this film. Can we see the reflection of this light? (Ans.: no)
6.5.6. You pass a laser beam light λ = 600 nm through a narrow slit and observe the diffraction pattern on a screen D = 6 m away. The distance on the screen between the centers of the second minima outside the central bright fringe is 25 mm. How wide is opening of the slit? (Ans.: 5.76×10^{-4} m)
6.5.7. A gamma-ray photon has energy of $E = 1.103\times10^{-13}$ J. Calculate the wavelength λ of this electromagnetic radiation. (Ans.: 18×10^{-13} m)

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