The Riemann boundary value problem on non-rectifiable arcs and the Cauchy transform

Boris A. Kats

Abstract. Let Γ be a Jordan arc on the complex plane. We consider distributional derivative $\overline{\partial}F$ of a function F which is holomorphic in a neighborhood of Γ. If arc Γ is rectifiable, then

$$\langle \overline{\partial}F, \phi\rangle = \int_{\Gamma} j_F(t)\phi(t)dt,$$

where $j_F(t) = F^+(t) - F^-(t)$ is jump of the function F on the arc Γ. If Γ is not rectifiable, then distribution $\overline{\partial}F$ is a generalization of weighted integration $\int_{\Gamma} \cdot j_F(t)dt$. Its convolution with $(2\pi i z)^{-1}$ is a generalization of the Cauchy type integral for non-rectifiable arcs.

In the present paper we study this version of the Cauchy transform and its applications for solution of the Riemann boundary value problem on non-rectifiable arcs.

Mathematics Subject Classification (2000). Primary 30E25; secondary 30E20.

Keywords. Non-rectifiable arc, metric dimension, Cauchy transform, Riemann boundary value problem.

Introduction

Let function $F(z)$ be local integrable in domain D of the complex plane. We identify it with distribution

$$F : C_0^{\infty}(D) \ni \phi \mapsto \iint_D F(\zeta)\phi(\zeta)d\zeta d\overline{\zeta},$$

where, as usually, $C_0^{\infty}(D)$ stands for the space of all infinitely smooth functions with compact support in D. Let us consider its distributional derivatives. There are known various spaces of functions F such that their distributional derivatives coincide with some usual functions. But in general they do not.

This work was partially supported by Russian Fund for Basic Researches, grant 11-01-00159-a.

References

[1] L. Hörmander, *The Analysis of Linear Partial Differential Operators I. Distribution theory and Fourier Analysis*, Springer Verlag, 1983.

[2] B.A. Kats, *The refined metric dimension with applications*, Computational Methods and Function Theory 7 (2007), No. 1, 77–89

[3] A.N. Kolmogorov, V.M. Tikhomirov, *$\varepsilon-$entropy and capasity of set in functional spaces*, Uspekhi Math. Nauk, **14** (1959), 3–86.

[4] I. Feder, *Fractals*, Mir Publishers, Moscow, 1991.

[5] B.A. Kats, *The Cauchy transform of certain distributions with application*, Complex Analysis and Operator Theory, to appear.

[6] P. Mattila and M.S. Melnikov, *Existence and weak type inequalities for Cauchy integrals of general measure on rectifiable curves and sets*, Proc. Am. Math. Soc. 120(1994), P. 143-149

[7] X. Tolsa, *Bilipschitz maps, analytic capacity and the Cauchy integral*, Ann. of Math. (2) 162(2005), P. 1243-1304.

[8] J. Verdera, *A weak type inequality for Cauchy transform of finite measure*, Publ. Mat. 36(1992), P.1029–1034

[9] N.I. Muskhelishvili, *Singular integral equations*, Nauka publishers, Moscow, 1962.

[10] F.D. Gakhov, *Boundary value problems*, Nauka publishers, Moscow, 1977.

[11] B.A. Kats, *The Riemann boundary value problem on open Jordan arc*, Izv. vuzov, Mathem., **12** 1983, 30–38

Boris A. Kats
Kazan State University of Architecture and Civil Engineering
Kazan, Zelenaya str. 1, 420043, Russia
e-mail: katsboris877@gmail.com