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Analytical solutions for seepage near material boundaries in dam cores: The Davison–Kalinin problems revisited

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ABSTRACT

Steady Darcian seepage through a dam core and adjacent shells is analytically studied. By conformal mappings of the pentagon in the hodograph plane and triangle in the physical plane flow through a low-permeable dam core is analyzed. Mass-balance conjugation of flow in the core and downstream highly-permeable shell of the embankment is carried out by matching the seepage flow rates in the two zones assuming that all water is intercepted by a toe-drain. Seepage refraction is studied for a wedge-shaped domain where pressure and normal components of the Darcian velocities coincide on the interface between the core and shell. Mathematically, the problem of R-linear conjugation (the Riemann–Hilbert problem) is solved in an explicit form. As an illustration, flow to a semi-circular drain (filter) centered at the triple point (contact between the core, shell and impermeable base) is studied. A piece-wise constant hydraulic gradient in two adjacent angles making a two-layered wedge (the dam base at infinity) is examined. Essentially 2-D seepage in a domain bounded by an inlet constant head segment, an outlet seepage-face curve, a horizontal base and with a straight tilted interface between two zones (core and shell) is investigated. The flow net, isobars, and isotachs in the core and shell are reconstructed by computer algebra routines as functions of hydraulic conductivities of two media, the angle of tilt and the hydraulic head value at a specified point.

1. Introduction

Renewed interest to hydropower stations, dam reservoirs and large water supply schemes drives both civil/geotechncial engineers and applied mathematicians, dealing with movement of water through porous materials, to revisit the legacy of the founders of the specialism of subsurface mechanics: Bear, Casagrande, Cedergren, Dachler, Davison, Charny, Gersevanov, Hamel, Muscat, Numerov, Pavlovsky, Riesenkampf, Polubarinova-Kochina [1]. What unifies these towering figures? All of them 50–100 years ago worked on mathematical problems of seepage through earth dams and their contribution is now in both manuals of geotechnical engineers and applied math books, where the "dam problem" became a shibboleth of a community of mathematicians working with free boundary problems (e.g., [2]). The objective of this paper is to amend solutions to the Laplace equation for the "dam problem" by an analytical study of seepage through dam heterogeneities.

Barrages, dikes, levees, weirs and embankments demarcating reservoirs, ponds, detention pools, canals and other hydraulic and agro-engineering structures involve often an earth-, rock-filled element, which maintains a difference of water levels on two sides of the structure. Dams made of local porous materials are cheap but permeable to water that seeps through the

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