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Automorphisms of Order Structures of Abelian Parts of Operator Algebras and Their Role in Quantum Theory

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Abstract It is shown that any order isomorphism between the structures of unital associative JB subalgebras of JB algebras is given naturally by a partially linear Jordan isomorphism. The same holds for nonunital subalgebras and order isomorphisms preserving the unital subalgebra. Finally, we recover usual action of time evolution group on a von Neumann factor from group of automorphisms of the structure of Abelian subalgebras.

Keywords JB algebra · Abelian subalgebras · Jordan isomorphism

1 Introduction and Preliminaries

Let A be a mathematical structure representing quantum system. In operator algebraic approach to quantum mechanics, A is a JB algebra or a C^*-algebra. We refer the reader to [7, 15] for nice explanation of the role of Jordan and C^*-structures in quantum theory. In order to capture uncertainty principle (existence of observables that cannot be measured simultaneously) A has to be nonassociative as a Jordan algebra or noncommutative as a C^*-algebra. These commutative properties distinguish between quantum and classical system. Bohr viewed quantum system as a pasting of local classical subsystems. His doctrine says that, roughly speaking, quantum system can only be seen through classical one. Recently this idea has been revived considerably by many authors (see e.g. [3, 4, 11]). One of these approaches, so called topos approach to foundations of quantum theory, is based on the structure $Abel(A)$ of Abelian unital C^*-subalgebras of a C^*-algebra A ordered by set theoretic inclusion. This structure embodies Bohr' idea of classical contexts through which

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