

RINGS WHOSE EVERY RIGHT IDEAL IS A FINITE DIRECT SUM OF AUTOMORPHISM-INVARIANT RIGHT IDEALS

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Abstract: We study the rings R whose every right ideal is a finite direct sum of automorphism-invariant right R -modules. These rings are called right Σ - a -rings. We find a representation in the form of block upper triangular rings of formal matrices for the indecomposable right Artinian right hereditary right Σ - a -rings.

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1. Introduction

All rings are assumed associative and unitary, and all modules are unitary.

A module is *quasi-injective* provided that it is invariant under the endomorphisms of its injective envelope. A module is *automorphism-invariant* provided that it is invariant under the automorphisms of its injective envelope. Automorphism-invariant modules were first considered by Dickson and Fuller in [1] and were systematically studied in [2–13] recently.

Characterization of rings by the properties of their one-sided ideals is one of the important areas in ring and module theories. A ring R is a right a -ring (q -ring, respectively) provided that its every ideal is an automorphism-invariant right R -module (quasi-injective, respectively). The q -rings were studied in many articles (for instance, see [14–19]). Full description of these rings was obtained in [14]. Various properties of a -rings were studied in [20]. In particular, as shown in [20], a right a -ring is regular if and only if it is semiprime, and the structure of the right Artinian nonsingular right a -rings was found.

A ring R is a right Σ - a -ring (Σ - q -ring, respectively) provided that its every right ideal is a finite direct sum of automorphism-invariant right R -modules (quasi-injective, respectively). The Σ - q -rings were introduced and studied in [21]. Some properties of q -rings and Σ - q -rings were reflected in [22] as well. The Σ - a -rings were first introduced in [10], and the question of description of Σ - a -rings was posed there (Question 4, p. 310).

In the present article, we study Σ - a -rings. In Section 2, we give an example of a right Σ - a -ring that is not a right Σ - q -ring (a -ring). In Section 3 we establish some properties of Σ - a -rings. Section 4 is devoted to the structure of the right Artinian right hereditary right Σ - a -rings.

The Jacobson radical (the right singular ideal, respectively) of a ring R is denoted by $J(R)$ (by $Z_r(R)$, respectively). The fact that N is a submodule of a module M is denoted by $N \leq M$. The injective envelope of a module M is denoted by $E(M)$. If M is a module of finite length then its length is denoted by $l(M)$. In the article, we use the standard notions and notations of ring and module theories (for instance, see [23, 24]).

2. Examples of Σ - a -Rings

According to [18, Theorem 2.3], the class of right q -rings coincides with the class of right self-injective rings whose every essential right ideal is an ideal. By [20], a ring R is a right a -ring if and only if R is

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