142. Measuring the free-fall acceleration. Reversible pendulum

Free-fall acceleration g is the acceleration with respect to the Earth at which a body *begins* to fall. This acceleration is defined by the sum of gravitational attraction to the Earth and the centrifugal inertia force.
The value of g can be found with the aid of a physical pendulum. Physical pendulum is a perfectly rigid body which can swing around a fixed horizontal axis. If there are no friction forces, the pendulum's equation of motion looks like

$$I\frac{d^2\varphi}{dt^2} = -mga\sin\varphi, \qquad (1)$$

where m is the mass of the body, I is its moment of inertia with respect to the point of suspension, a is the distance from the point of suspension to the centre of mass of the pendulum, and φ is the angle at which the pendulum declines from the equilibrium position. In the case of small oscillations we can substitute φ for $\sin\varphi$ in the above expression. This gives the equation for harmonic oscillations having the period

$$T = 2\pi\sqrt{\frac{I}{mga}}. \qquad (2)$$

A particular case of a physical pendulum is the so-called simple pendulum – an idealized pendulum with all its mass concentrated in one point. In this case we can simplify Eq. (2) because $I = ml^2$, and $a = l$ (the length of the pendulum), and finally get for g:

$$g = \frac{4\pi^2 l}{T^2}. \qquad (3)$$

Thus, an idea is clear how the free-fall acceleration may be measured: to do this, the length and the period of oscillations of a simple pendulum should be measured.
Comparing Eqs. (2) and (3) we can see that a physical pendulum oscillates in the same way (with the same period) as a simple pendulum having the length of $l = I/(m\cdot a)$. This value is called the *reduced* pendulum length. The point lying on the line passing through the centre of mass at the distance l from the point of suspension is called the centre of oscillation. If a pendulum is hung at its centre of oscillation, its period of oscillations will not change (this statement is called the Huygens' theorem).
It is important to note that the same period can be achieved, generally speaking, when the pendulum is hung at any of the points from a certain infinite manifold of points, which are called *respective points of suspension*. According to this definition, the point of suspension and centre of oscillations are the respective points, but they are not the only possible pair of such points. That is why the distance between respective points (which can be easily found by checking that the periods are equal) is not always the reduced length. *The distance between respective points is equal to the reduced pendulum length only if these points lie on the same line together with the centre of mass and on different sides from the centre of mass*.

Reversible pendulum

Reversible pendulum is a device used to determine the free-fall acceleration. It consists of a steel rod with two fixed supporting prisms P_1 and P_2 made of steel and a steel lentil A between them (see Fig. 1). The other lentil B is placed on one of the rod's ends (not between the prisms); it can be moved along the rod and fastened in a required place. By moving this prism one can make the two periods of oscillations equal when the points of suspension are the edges of the prisms P_1 and P_2. When the equality of periods is achieved, the prisms' edges will be the respective points, and they will be asymmetric with respect to the centre of mass C. Hence, when the equality of periods is achieved, the distance between the prisms is the reduced length l of the physical pendulum. Having measured its period of oscillations T, we can calculate g from Eq. (3).

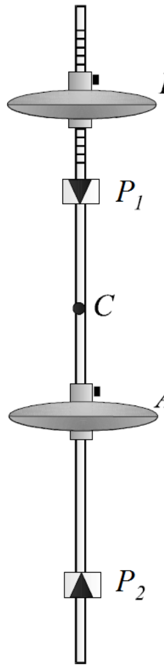

Figure 1.

Algorithm of measurements

1. Measure the distance l between the prisms with a ruler.
2. Hang the pendulum by one of the prisms and decline it by a small angle.
3. Count several total oscillations (the more the better) and find the corresponding time t using the stopwatch; calculate the period of oscillations T_1.
4. Hang the pendulum by the other prims and find the period T_2.
5. Repeat steps3-4 (find the values T_1 and T_2) for 7–10 different positions of the lentil B.
6. Plot the dependences of the periods T_1 and T_2 on the position of the lentil B on a single diagram.
7. Find the point of intersection of the plots and determine corresponding period for the respective points (where $T = T_1 = T_2$).
8. Calculate g using Eq. (3).

Questions

1. What are “inertia forces”?
2. Newton's law of gravity.
3. Force of gravity, free-fall acceleration.
4. Draw a physical point in a reference frame bound to the rotating Earth. Which forces act on it? Estimate the contribution of the centrifugal force to the free-fall acceleration.
5. The concept of “weight.”
6. Describe the method of finding the value of g used in this work. Derive the formulas. Which approximations were made and how are they reflected in the construction of the experimental setup?

Note

This table demonstrates the accuracy of the assumption that an angle and its sine value are close to each other if the angle is “small.”

Angle α, °	α, rad	sinα	Rel. diff.	Angle α, °	α, rad	sinα	Rel. diff.
3	0.05236	0.05234	0.05%	15	0.26180	0.25882	1.15%
5	0.08727	0.08716	0.13%	18	0.31416	0.30902	1.66%
8	0.13963	0.13917	0.33%	24	0.41888	0.40674	2.99%
10	0.17453	0.17365	0.51%	30	0.52360	0.50000	4.72%
12	0.20944	0.20791	0.73%	35	0.61087	0.57358	6.50%