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II Semester

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Discrete Charge and the Electrostatic Field

In nature we can readily observe ***electromagnetic forces***.
Electromagnetic forces bind electrons to atomic nuclei, bond atoms together to form molecules, mediate the interactions between molecules that allow them to change and organize and, eventually, live.
The energy that is used to support life processes is electromagnetic energy.

The directed observation and study of electricity is quite ancient. It was studied, and written about, at least 3000 years ago, and artifacts that may have been primitive electrical batteries have been discovered in the Middle East that date back to perhaps 250 BCE.

One of the first recorded observations of electrical force was the static electrical force created between amber, charged by rubbing it with wool, and small bits of wool or hair.

However, it took until the Enlightenment (roughly 1600) and the invention of physics and calculus for the scientific method to develop to where systematic studies of the phenomenon could occur, and it wasn't until the middle 1700s that the correct model for electrical charge was proposed

Discrete Charge and the Electrostatic Field

Charge is the fundamental quantity that permits objects to “couple” – affect one another – via the electromagnetic interaction.

Experimentally, objects can carry a (net) charge ***q*** when “***electrified***” various ways (for example by rubbing materials together).

Charge comes in two flavors, **+** and **-**, but most matter is approximately charge-neutral most of the time. Consequently, as ***Benjamin Franklin*** observed, most charged objects end up that way by adding or taking away charge from this neutral base.

Like charges exert a long range (action at a distance) repulsive force on one another. Unlike charges attract.

The force varies with the inverse square of the distance between the charges and acts along a line connecting them.

The “elementary” charge (associated with these elementary particles that are the building blocks of all matter) has experimentally turned out to be discrete and essentially indivisible. Indeed, we characterize elementary particles by a unique signature consisting of their (rest) mass, their charge, and other measurable properties.

The SI unit of charge is called the ***Coulomb*** (C).

Discrete Charge and the Electrostatic Field

Nearly all matter is made up of atoms and hence nothing but protons, neutrons, and electrons.

Nearly all the mobile charge in solid matter is made up of *electrons*. In semiconductors the mobile charge can also be electron “holes” – de facto positive charge carriers consisting of regions of electron deficit that move against an otherwise stationary electronic background.

Franklin, unfortunately, thought that the flavor of mobile charge in ordinary conductors was *positive*. In fact, as noted, it is *negative* – associated with moving electrons.

By choosing some volume ΔV small enough that we can treat it like a volume differential but large enough that it contains a lot of charge, we can define a ***charge density***.

Similarly, we can associate charge densities with two dimensional sheets of matter (for example, a charged piece of paper or metal plate) or one dimensional lines of matter (for example, a wire or piece of fishing line). We summarize this (and define the symbols most often used to represent charge) as:

$$\rho = \frac{dq}{dV} \qquad \sigma = \frac{dq}{dA} \qquad \lambda = \frac{dq}{dx}$$

Discrete Charge and the Electrostatic Field

Insulators.

The charge in the atoms and molecules from which an insulating material is built tends to not be mobile – electrons tend to stick to their associated molecules tightly enough that ordinary electric fields cannot remove them. Surplus charge placed on an insulator tends to remain where you put it. Vacuum is an insulator, as is air, although neither is a perfect insulator.

Conductors.

For many materials, notably metals but also ionic solutions, at least one electron per atom or molecules is only weakly bound to its parent and can easily be pushed from one molecule to the next by small electric fields. We say that these conduction electrons are free to move in response to applied field and that the material conducts electricity.

Semiconductors.

These are materials that can be shifted between being a conductor or an insulator depending on the potential difference at the interfaces between different "kinds" of semiconducting materials.

Coulomb's Law

If one charges various objects (for example, two conducting balls suspended from an insulating string so that they are near to one another but not touching) and measures the deflection of the string when the balls are in force equilibrium, one can verify that:

- The force between the charges is proportional to each charge separately. The force is bilinear in the charge.
- The force acts along the line connecting the two charges.
- The force is repulsive if the charges have the same sign, attractive if they have different signs.
- The force is inversely proportional to the square of the distance between them.

These four experimental observations are summarized as ***Coulomb's Law.***

Coulomb's Law

We can formulate them algebraically. We therefore write the force acting on charge 1 due to charge 2 as:

$$\vec{F}_{12} = k_e q_1 q_2 \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

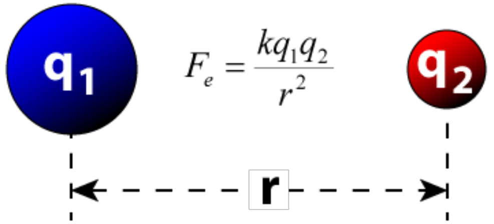
Note that it acts on a line from charge 2 to charge 1, is proportional to both charges, is inversely proportional to the distance that separates them squared, and is repulsive if both charges have the same sign.

The constant:

$$k_e = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{N - m^2}{C^2}$$

effectively defines the "size" of the unit of charge in terms of the already known SI units of force and length, and obviously will vary if we change to a different set of units

We note that Coulomb's law describes *action at a distance*. We'd like there to be a cause for the observed force that is present where the force is exerted, and lacking anything better to do we'll invent the cause and call it the *electrostatic field* just as we similarly defined the gravitational field.



Electrostatic Field

The electrostatic field is the supposed cause of the electrostatic force between two charged objects.

Each charged object produces a field that emanates from the charge and is the cause of the force the other charge experiences at any given point in space. This field is supposed to be present everywhere in space whether or not we measure it.

The fundamental definition of electrostatic field produced by a charge q at position $\vec{r}$ is that it is the electrostatic force per unit charge on a small test charge q_0 placed at each point in space $\vec{r}_0$ in the limit that the test charge vanishes:

$$\vec{E} = \lim_{q_0 \to 0} \frac{F}{q_0}$$

or

$$\vec{E}(\vec{r}_0) = kq \frac{(\vec{r}_0 - \vec{r})}{|\vec{r}_0 - \vec{r}|^3}$$

If we locate the charge q at the origin and relabel $\vec{r}_0 \to \vec{r}$, we obtain the following simple expression for the electrostatic field of a point charge:

$$\vec{E}(\vec{r}) = \frac{kq}{r^2} \hat{r}$$

Electrostatic Field

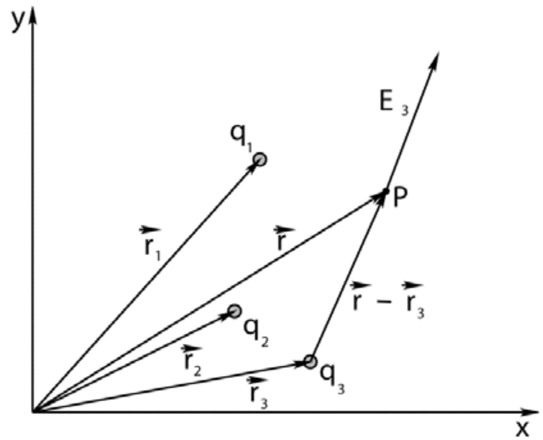
In general, we'll work the other way around. First we'll be given a distribution of charges, from which we must determine the field. With the field known, we can then evaluate the force these charges will exert on another (e.g. test) charge placed placed on the field by means of the following rule:

$$\vec{F} = q\vec{E}$$

We need a way of finding the total field produced by many charges, not just one. Furthermore, that way needs to work for charges counted "one at a time" (when there are only a few and they are enumerable) and it also needs to be useful in the limit of so many charges that a coarse-grained average yields an approximately continuous charge distribution in bulk matter.

Fortunately for all concerned, the fields of many charges simply add right up! This too is a principle of nature (and is related to the linearity of the underlying equations that are the laws of nature). We call it the ***Superposition Principle***.

Electrostatic Field



Geometry needed to evaluate the field of many charges. Only the field of the third charge $\vec{E}_3$ is shown explicitly. Note well the magnitude and direction of the vector $\vec{r} - \vec{r}_3$ – head at $\vec{r}$, tail at $\vec{r}_3$. This is a vector *from* the position of the charge q_3 to the point of observation P at $\vec{r}$.

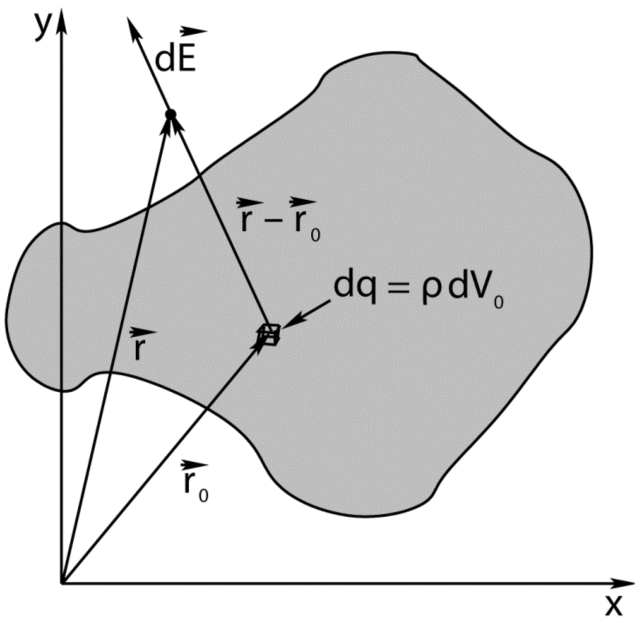
Given a collection of charges located at various points in space, the total electric field at a point is the sum of the electric fields of the individual charges:

$$\vec{E}(\vec{r}) = \sum_i \frac{kq_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

Charge, while discrete, comes in very tiny packages of magnitude *e* such that matter contains order of 10^{27} charges per kilogram, with roughly equal amounts of positive and negative charge so that most matter is approximately electrically neutral most of the time. When we consider macroscopic objects – ones composed of these enormous numbers of atoms and charges – it therefore makes sense to treat the distribution and motion of charge as if it is *continuously* distributed.

Electrostatic Field

In order to find the electrostatic field produced by a charge density distribution, we use the superposition principle in integral form.



The geometry needed to evaluate the field of a general continuous charge distribution. Note well the similarity to the geometry for a collection of charges, except that the many “point charges” are all chunks of differential volume with charge *dq* and the “sum” is now an integral.

$$d\vec{E}(\vec{r}) = \frac{k_e dq(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3}$$

We then use one of the definitions of charge density to convert *dq* into e.g. $dq = \rho\, dV_0 = \rho(\vec{r}_0)d^3r_0$:

$$d\vec{E}(\vec{r}) = \frac{k_e\rho(\vec{r}_0)(\vec{r}-\vec{r}_0)d^3r_0}{|\vec{r}-\vec{r}_0|^3}$$

Finally, we integrate both sides of this equation over the entire volume *V* where $\rho(\vec{r}_0)$ is supported.

Electrostatic Field

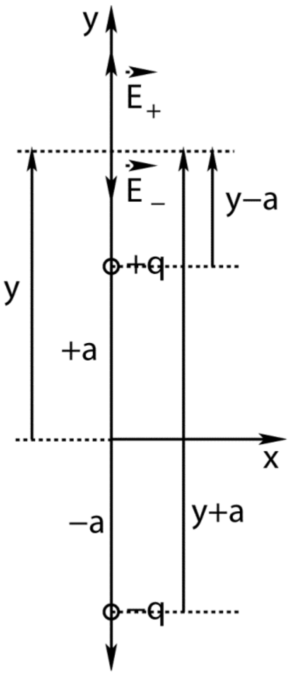
The resulting integral form is:

$\vec{E}(\vec{r}) = k_e \int_V \frac{\rho(\vec{r}_0)(\vec{r}-\vec{r}_0)dV_0}{|\vec{r}-\vec{r}_0|^3}$ for a 3-dimensional (volume) charge distribution,

$\vec{E}(\vec{r}) = k_e \int_S \frac{\sigma(\vec{r}_0)(\vec{r}-\vec{r}_0)dS_0}{|\vec{r}-\vec{r}_0|^3}$ for a surface charge distribution on a surface S, and

$\vec{E}(\vec{r}) = k_e \int_L \frac{\lambda(\vec{r}_0)(\vec{r}-\vec{r}_0)dL_0}{|\vec{r}-\vec{r}_0|^3}$ for a linear charge distribution on a particular line L.

Example: Field of Two Point Charges



Two charges ±*q* on the *y*-axis produce a field that is easy to evaluate at points on the *x* and *y*-axis (and not terribly difficult to *approximately* evaluate at all points in space that are "far" from the origin relative to *a*). This arrangement of charges is called an *electric dipole*.

Suppose two point charges of magnitude −*q* and +*q* are located on the *y*-axis at *y* = −*a* and *y* = +*a*, respectively. Find the electric field at an arbitrary point on the *x* and *y* axis.

The *y*-axis is quite simple. The field due to the positive charge points directly away from it, hence in the positive y direction at a point *y* > *a* and is equal to:

$$\vec{E}_+(0,y) = \frac{k_e q}{|y-a|^2}\hat{y}$$

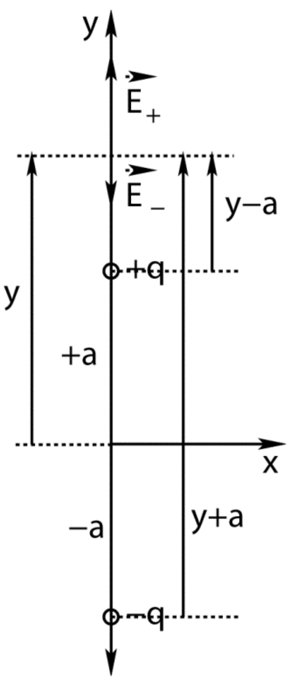
The field of the negative charge points towards it and is equal to:

$$\vec{E}_-(0,y) = -\frac{k_e q}{|y+a|^2}\hat{y}$$

Hence the total field on the y axis is just:

$$\vec{E}_{tot}(0,y) = k_e q\left(\frac{1}{|y-a|^2} - \frac{1}{|y+a|^2}\right)\hat{y}$$

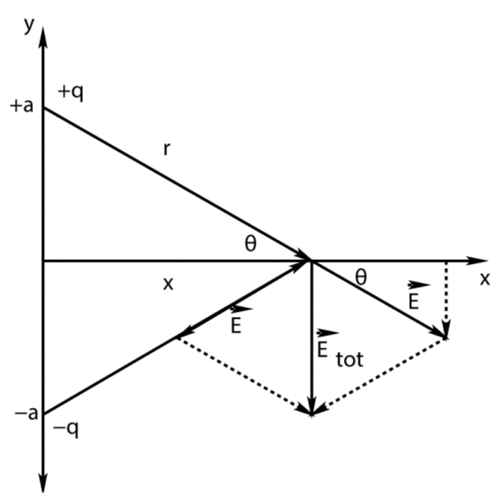
Example: Field of Two Point Charges



The field on the x-axis is a tiny bit more difficult. Here the field produced by each charge has both components. To find the vector field, we must first find the magnitude of the field, then use the geometry of the picture to find its x and y components.

Note that the distance from the charge to the point of observation drawn above is $r = (x^2+a^2)^{1/2}$. Then the magnitude of the electric field vector of either charge is just:

$$\left|\vec{E}(x,0)\right| = \frac{k_e q}{(x^2+a^2)}$$



e formed by x, a and r. By definition:

$$\cos\theta = \frac{x}{r} = \frac{x}{(x^2+a^2)^{1/2}}$$

$$\sin\theta = \frac{a}{r} = \frac{a}{(x^2+a^2)^{1/2}}$$

Example: Field of Two Point Charges

Now we can find the components:

$$E_x = \left|\vec{E}\right| \cos\theta = \frac{k_e q}{(x^2 + a^2)} \cdot \frac{x}{(x^2 + a^2)^{\frac{1}{2}}} = \frac{k_e q x}{(x^2 + a^2)^{\frac{3}{2}}}$$

and

$$E_y = -\left|\vec{E}\right| \sin\theta = -\frac{k_e q}{(x^2 + a^2)} \cdot \frac{a}{(x^2 + a^2)^{\frac{1}{2}}} = -\frac{k_e q a}{(x^2 + a^2)^{\frac{3}{2}}}$$

This is for a single charge (+*q*). The other charge has components that are the same magnitude but its Ex obviously cancels while its E_y obviously *adds*. The total field is thus:

$$\vec{E}_{tot}(x, 0) = -\frac{k_e q a}{(x^2 + a^2)^{3/2}} \hat{y}$$

In terms of the electric dipole moment for this arrangement of charges:

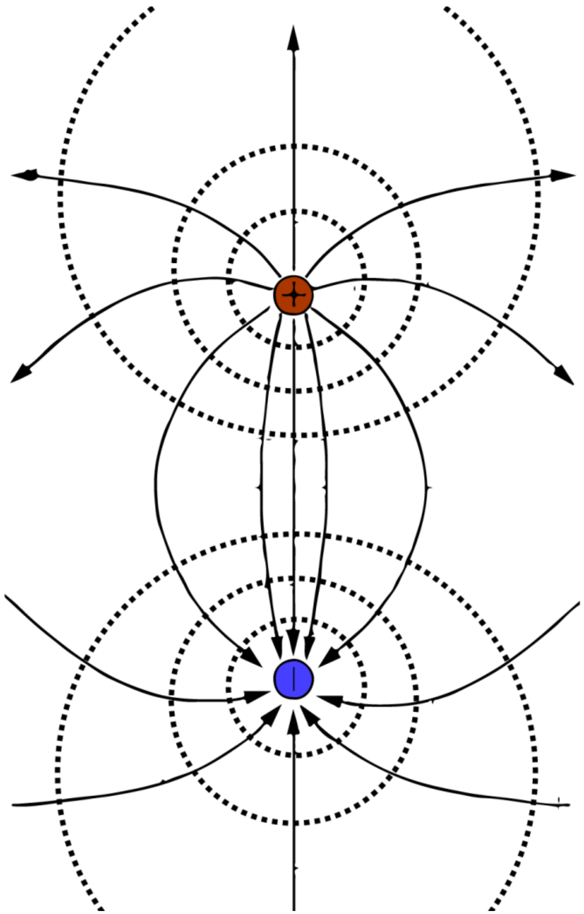
$$\vec{p} = 2qa\hat{y}$$

the field can be expressed as:

$$\vec{E}_{tot}(x, 0) = -\frac{k_e |\vec{p}|}{(x^2 + a^2)^{3/2}} \hat{y}$$

It is worthwhile to look at the general shape of the dipole field. In many cases, the physical dimensions of the dipole (2*a* in this case) will be small compared to x, the distance of the point of observation to the dipole. In this limit, the field or potential produced is that of an ideal dipole, or a point dipole.

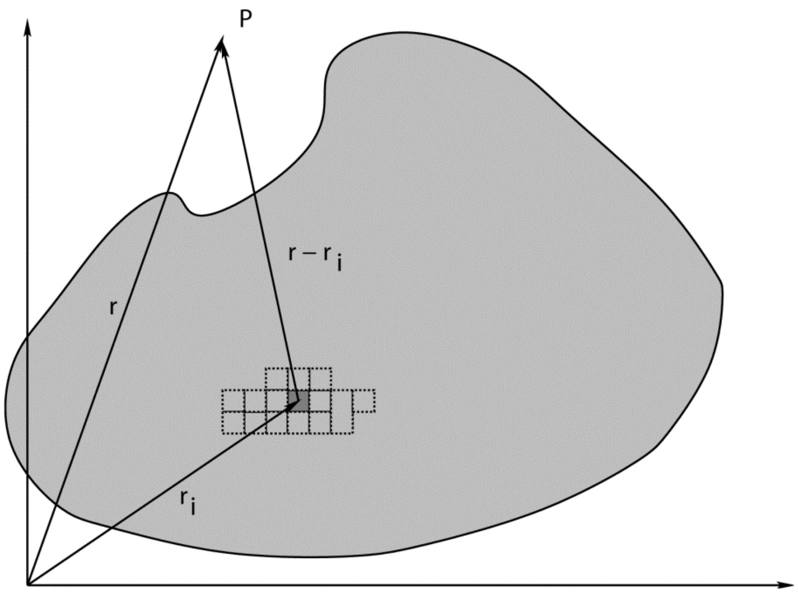
Example: Field of Two Point Charges



The electric field of a classic electric dipole in the vicinity of the charges. Bear in mind that this figure is a plane cross-section of a three-dimensional, cylindrically symmetric field! The dashed lines are the projections into the plane of the equipotential surfaces of this arrangment of charges.

The Field of Continuous Charge Distributions

In natural matter, charges are very, very small compared to the length scales we can directly perceive. If we want to evaluate the electric field produced by a macroscopic piece of matter, we're going to have to do something other than just sum over the $\vec{E}_i$ fields produced by all of these charges. Let's define the average charge density of the object:



$$\rho = \frac{\Delta Q}{\Delta V}$$

We can then compute the field using the superposition principle at the point *P* (position $\vec{r}$) as:

$$\vec{E}_{tot}(\vec{r}) = \sum_i \frac{k\Delta Q_i}{|\vec{r}-\vec{r}_i|^2}\widehat{(\vec{r}-\vec{r}_i)}$$

As there are too many chunks in the blob for us to sum over. So we pretend that the charge is continuously distributed according to:

$$\rho = \lim_{\Delta V \to 0}\frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$

and turn the summation into an integral:

$$\vec{E}_{tot}(\vec{r}) = \sum_i \frac{k\Delta Q_i}{|\vec{r}-\vec{r}_i|^2}\widehat{(\vec{r}-\vec{r}_i)} = \int_V \frac{kq(\vec{r}\,')dV'}{|\vec{r}-\vec{r}\,'|^2}\widehat{(\vec{r}-\vec{r}\,')}$$

The Field of Continuous Charge Distributions

$$\vec{E}_{tot}(\vec{r}) = \sum_i \frac{k\Delta Q_i}{|\vec{r}-\vec{r}_i|^2}\left(\widehat{\vec{r}-\vec{r}_i}\right) = \int_V \frac{kq(\vec{r}\,')dV'}{|\vec{r}-\vec{r}\,'|^2}\left(\widehat{\vec{r}-\vec{r}\,'}\right)$$

where we've used $dQ = \rho dV$

Just as we found the electric field by using the field of a single point charge in its simplest form and then

putting it into suitable coordinates, we'll find it the exact same way, but the point charge in question will be dq and not q. That is:

$$\vec{E} = \frac{kq}{r^2}\hat{r} \Leftrightarrow \overrightarrow{dE} = \frac{kdq}{r^2}\hat{r}$$

We also define all of the charge densities we might need to handle these cases as:

$$\rho = \frac{dq}{dV} \Leftrightarrow dq = \rho dV$$

$$\sigma = \frac{dq}{dA} \Leftrightarrow dq = \rho dA$$

$$\lambda = \frac{dq}{dl} \Leftrightarrow dq = \rho dl$$

the charge per unit volume, per unit area, and per unit length respectively.

The Field of Continuous Charge Distributions

There are thus three steps associated with solving an actual problem:

a) Draw a picture, add a suitable coordinate system, identify the right differential chunk (one you can integrate over) and draw in the vectors needed to express $d\vec{E}$ as given above.

b) Put down an expression for $d\vec{E}$ (or rather, usually $|d\vec{E}|$) in terms of the coordinates, and find its vector components in terms of those same coordinates, using symmetry to eliminate unnecessary work.

c) Do the integral(s), find the field $\vec{E}$ at the desired point.

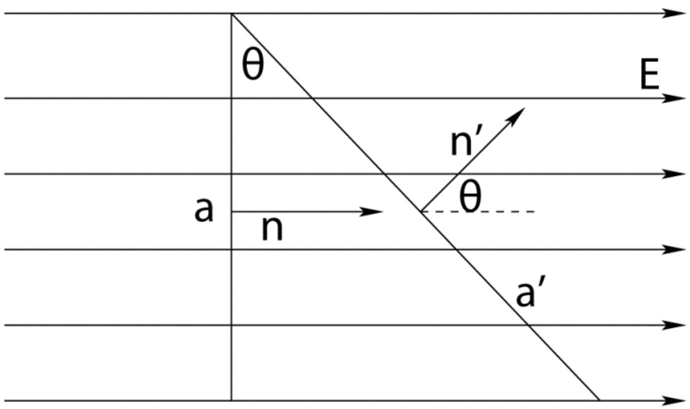
Gauss's Law for the Electrostatic Field

Let's considering the *flux* of the electrostatic vector field through a small rectangular patch of surface ΔS. To compute this, we first must understand what the flux of an arbitrary vector field $\vec{F}$ through a surface S is. Mathematically, the flux of a vector field through some surface is defined to be:

$$\phi = \int_{\Delta S} \vec{F} \cdot \hat{n} dS$$

Note that the word flux means *flow*, and this integral measures the flow of the field *through* the surface. It's mathematical purpose is to detect the *conservation* of flow in the vector field.

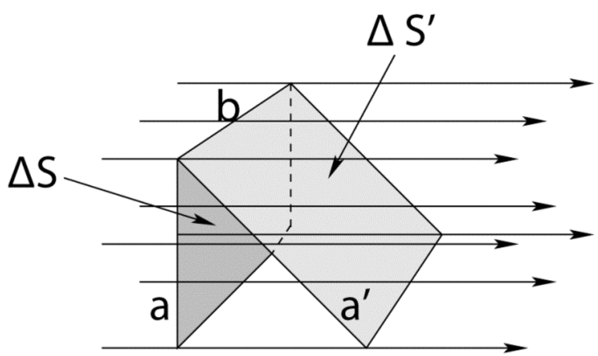
Gauss's Law for the Electrostatic Field



Consider figure, where we show electric field lines flowing through a small $S = ab$ at right angles to the field lines (so that a unit vector $\hat{n}$ normal to the surface is *parallel* to the electric field). ΔS is small enough that the continuous field is approximately uniform across it

Since the field is uniform and at right angles to the field, the flux through just this little chunk is easy to evaluate. It is just:

$$\Delta\phi_e = |\vec{E}|\Delta S = |\vec{E}|ab$$



Gauss's Law for the Electric Field:

$$\oint_{S/V} \vec{E} \cdot \hat{n} dA = 4\pi k_e \int_V \rho dV = \frac{Q_{in} S}{\epsilon_0}$$

or in words, the flux of the electric field through a closed surface S equals the total charge inside S divided by ϵ_0, the permittivity of the electric field.

Using Gauss's Law to Evaluate the Electric Field

a) Draw a closed *Gaussian Surface* that has the symmetry of the charge distribution. The various pieces that make up the closed surface should either be *perpendicular* to the field (which should also be constant on those pieces) or *parallel* to the field (which may then vary but which produces no flux through the surface).

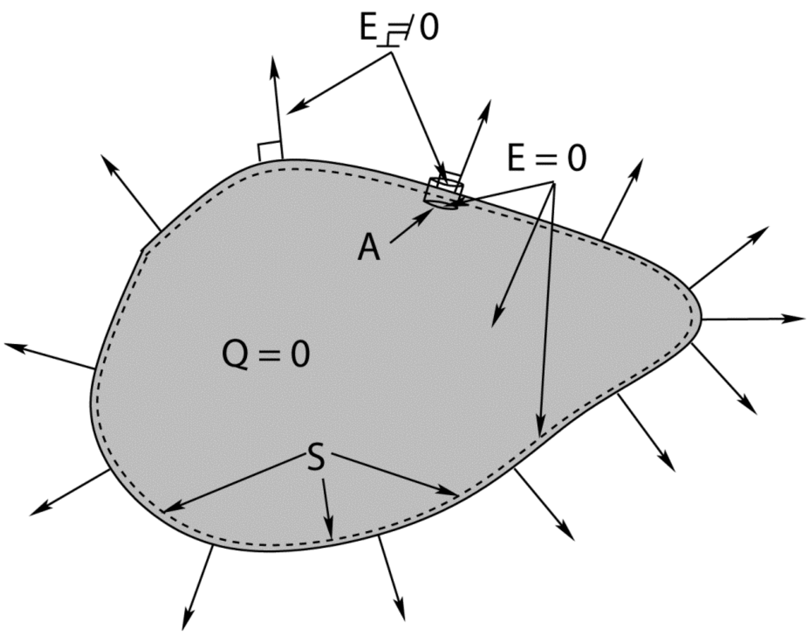
b) Evaluate the flux through this surface. The flux integral will have exactly the same form for every problem with each given symmetry, so we will do this once and for all for each surface type and be done with it!

c) Compute the *total charge inside this surface*. This is the only part of the solution that is "work", or that might be different from problem to problem. Sometimes it will be easy, adding it up on fingers and toes. Sometimes it will be fairly easy, multiplying a constant charge per unit volume times a volume to obtain the charge, say. At worst it will be a problem in integration if the associated density of charge is a function of position.

d) Set the (once and for all) flux integral equal to the (computed per problem) charge inside the surface and solve for $|\vec{E}|$.

Gauss's Law and Conductors

Properties of Conductors



A **conductor** is a material that contains many "free" charges that are *bound to the material* so that they cannot easily jump from the conductor into a surrounding insulating material (where a vacuum is considered an insulator for the time being, as is air) but *free to move* within the material itself if any e.g. electrical field exerts a force on them.

An arbitrary chunk of conducting material in electrostatic equilibrium can have no field inside, or else it wouldn't be in equilibrium. It can have no field tangent to its surface, or it wouldn't be in equilibrium. From these facts we can deduce several useful things about conductors in electrostatic equilibrium using Gauss's Law

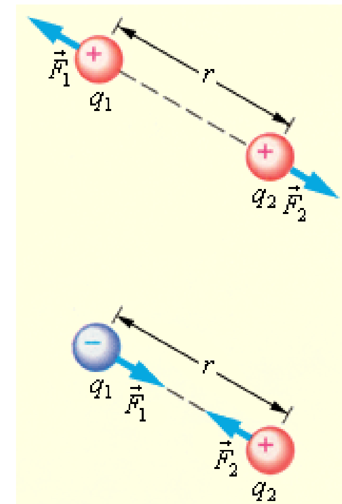
Summary

- **The electrical charge** is a a physical quantity that characterizes the properties of the particles or bodies to engage in the electromagnetic force interactions.
- Objects can carry a (net) charge q when "electrified" various ways. This charge comes in two flavors, **+** and **-**.
- **Like** charges exert a long range (action at a distance) repulsive force on one another. **Unlike** charges attract. The SI unit of charge is called the **Coulomb** (**C**).
- **Charge Conservation:** Net charge is a conserved quantity in nature.

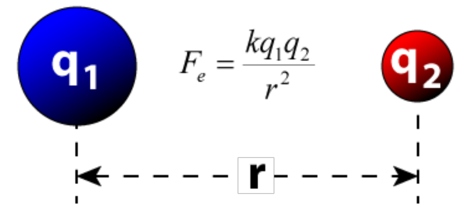
$$q_1 + q_2 + q_3 + \ldots + q_n = const$$

- From a modern point of view, the charge carriers are elementary particles. All objects are composed of atoms, which contain positively charged *protons*, negatively charged *electrons* and neutral particles - *neutrons*. Protons and neutrons are part of the atomic nuclei, electrons form the electron shell of atoms.
- The electric charges of the proton and electron in absolute value are exactly the same and equal to the elementary charge *e*.

$$e = 1{,}602177 \cdot 10^{-19}\ C \approx 1{,}6 \cdot 10^{-19}\ C$$



Summary



- **Coulomb's Law:**

From performing many careful experiments directly measuring the forces between static charges and from the consistent observations of many other things such as the electric structure of atoms, the conductivity of metals, the motion of charged particles, and much, much more, we infer that for any two stationary charges, the experimentally verified electrostatic force acting on charge 1 due to charge 2 is:

$$\vec{F}_{12} = k_e \frac{|q_1||q_2|}{r^2}$$

Note that it acts on a line from charge 2 to charge 1, is proportional to both charges, is inversely proportional to the distance that separates them squared, and is repulsive if both charges have the same sign.

The constant:

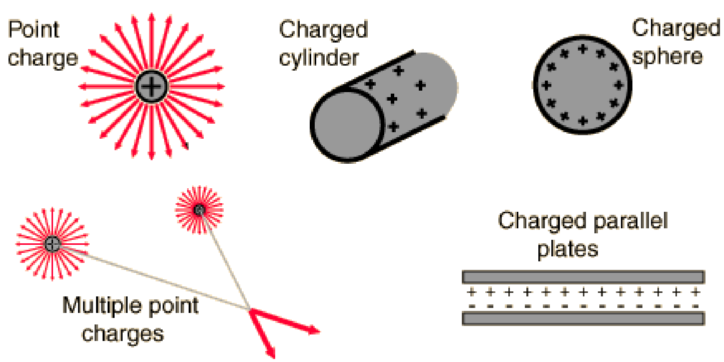
$$k_e = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{N-m^2}{C^2}$$

effectively defines the "size" of the unit of charge in terms of the already known SI units of force and length, and obviously will vary if we change to a different set of units

Summary

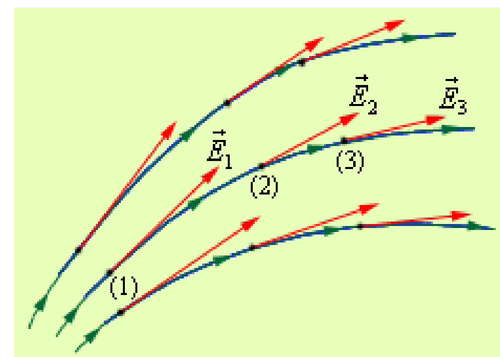
- **Electrostatic Field**

The fundamental definition of electrostatic field produced by a charge q at position $\vec{r}$ is that it is the electrostatic force per unit charge on a small test charge q_0 placed at each point in space $\vec{r}_0$ in the limit that the test charge vanishes:



$$\vec{E} = \frac{\vec{F}}{q}$$

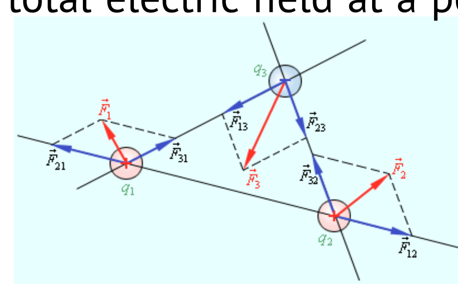
$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$



- **Superposition Principle**

Given a collection of charges located at various points in space, the total electric field at a point is the sum of the electric fields of the individual charges:

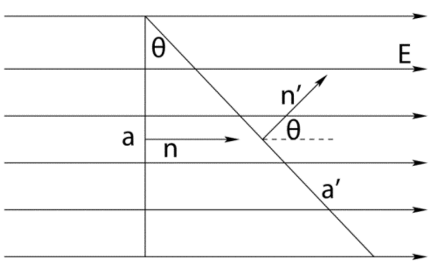
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$



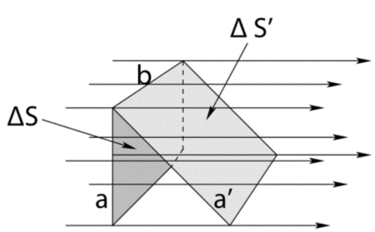
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elefie.html#c1

Gauss's law

The electric ***flux*** through an area is defined as the electric field multiplied by the area of the surface projected in a plane perpendicular to the field. Electric flux is proportional to the number of electric field lines going through a normally perpendicular surface. If the electric field is uniform, the electric flux passing through a surface of vector area S is



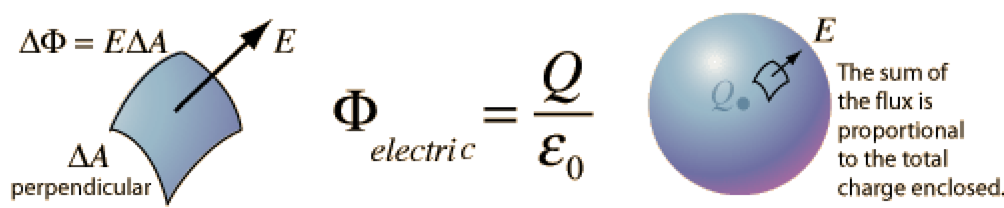
$$\Delta\phi = |\vec{E}|\Delta S\cos\theta = |\vec{E}_n|\Delta S$$



Gauss's law:

The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

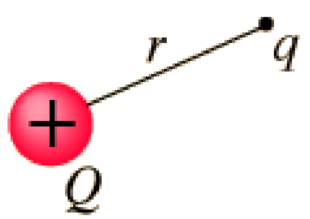
$$\phi = \frac{1}{\varepsilon_0}\sum q_i$$



http://hyperphysics.phy-astr.gsu.edu/hbase/electric/gaulaw.html

Electric Potential Energy

Potential energy can be defined as the capacity for doing work which arises from position or configuration. In the electrical case, a charge will exert a force on any other charge and potential energy arises from any collection of charges. For example, if a positive charge Q is fixed at some point in space, any other positive charge which is brought close to it will experience a repulsive force and will therefore have potential energy. The potential energy of a test charge q in the vicinity of this source charge will be:



$$U = \frac{kQq}{r}$$

where k is Coulomb's constant.

In electricity, it is usually more convenient to use the electric ***potential energy per unit charge***, just called ***electric potential or voltage***.

Voltage is electric potential energy per unit charge, measured in joules per coulomb (= volts). It is often referred to as "electric potential", which then must be distinguished from electric potential energy by noting that the "potential" is a "per-unit-charge" quantity. Like mechanical potential energy, the zero of potential can be chosen at any point, so the difference in voltage is the quantity which is physically meaningful. The difference in voltage measured when moving from point A to point B is equal to the work which would have to be done, per unit charge, against the electric field to move the charge from A to B. When a voltage is generated, it is sometimes called an "***electromotive force***" or ***emf***.

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elewor.html