===== **ROBOTICS** =====

The Energy-Optimal Motion of a Vibration-Driven Robot in a Medium with a Inherited Law of Resistance

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Abstract—The rectilinear motion of a two-mass system consisting of a spherical body and a movable internal mass in a liquid is considered. In addition to quadratic in velocity viscous forces, resistance forces also include those dependent on the history of the motion of Basset forces and inertial forces of added mass. The task is to find the periodic law of motion of the internal mass that minimizes the work of the resistance forces during the period of motion of the system for a fixed period of oscillations and the given average velocity of the shell. The dependence of the optimal modes of the dimensionless oscillation period that characterizes the ratio of Basset forces to viscous forces is investigated.

INTRODUCTION

The investigated mechanical system simulates a vibration-driven robot, which is a mobile device that moves without moving external parts in a resisting medium. The motion of the system as a whole is ensured by the periodic oscillations of the internal propulsion (internal mass) relative to the shell. Vibration-driven robots have a number of advantages over traditional mobile devices. They are simple in design, their shell can be made hermetic, and it can contain no protruding parts, which makes it possible to use them in confined spaces.

For the first time, the question of the optimal motion of a system by moving the inner body was raised by F.L. Chernous'ko [1, 2], who considered the linear motion of the rigid body with a cavity containing a movable internal mass in the horizontal plane in the presence of the Coulomb friction between the plane and the body. In recent years, this problem has been widely discussed in the literature both for other idealized laws of resistance and for the multidimensional motion of the internal mass [3−8]. A very common case was considered in [9], where the resistance force should only be a monotonically increasing function of the body velocity. In [10], the resistance law was selected based on the known experimental data for the case of motion of a spherical body in a viscous liquid [11]. In this case, the resistance crisis makes the dependence of the resistance force on the velocity nonmonotonic. However, until now studies have been limited to quasi-stationary laws, when the resistance force is uniquely determined by the shell velocity.

In fact, the resistance forces of hydrodynamic shell motion in a viscous liquid are defined by the flows which have been formed by the body in the liquid for the entire time of movement. In the general case, they cannot be described solely in terms of the instantaneous velocity but should be determined by the entire history of the movement. In hydrodynamics the history is taken into account by the Basset resistance force which is nonlocal in time. In this paper, the Basset force is set in its simplest classical form with a rigorous justification only in the case of a slow body movement. However, the adopted formulation is useful in the study of the vibration-driven robot motion in a viscous liquid for the following two reasons: as a necessary first step in considering more realistic laws [12, 13] for the inherited resistance forces as a means of the qualitative assessment of the limits of applicability of the quasi-stationary approximation.

In this paper, the optimization problem is put in the energy formulation proposed in [8, 10]. It consists in determining the periodic law of motion of the internal mass that minimizes the work of the resistance forces for the period of the system's motion in the case of a fixed period of oscillations and the given average velocity of the shell's motion.

1. FORMULATION OF THE PROBLEM

Let us consider a system of two bodies. The body of a spherical shape (shell) with mass M is in a viscous liquid, and a shell with mass m (hereinafter, the internal mass) moves inside it. The longitudinal periodic movements of the internal mass relative to the shell, in which the whole system moves as a whole, are

investigated. Let us denote the body velocity through u and the movement and velocity of the internal mass relative to the shell as x and $v = \dot{x}$. The basic equation which describes the motion's velocity $u(t)$ of the shell under the given law $x(t)$ of the motion of the internal mass has the form

$$(m + M)\dot{u} + R = -m\ddot{x}. \tag{1.1}$$

Here, R is the liquid resistance force to the shell's movement, which depends, in general, not only on the current values of the velocity $u(t)$ but also on its history $\{u(\tau), \tau < t\}$. In (1.1), $x(t)$ plays the role of the kinematic control.

Let us denote with the angled brackets

$$\langle \cdot \rangle = T^{-1} \int\limits_0^T (\cdot)dt,$$

the average over the period T and determine the average power

$$N[u(t)] = \langle uR[u(t)] \rangle,$$

spent on overcoming the forces of resistance. This value is equal to [8] the power of the forces imparted to the body by the internal mass. The formulation of the optimization problem proposed in [8] consists of finding such a periodic law $x(t)$ of internal mass oscillations which for a fixed period T of oscillations and given average velocity $\langle u \rangle = U$ of the shell movement will minimize the power of the internal propulsion $N[u]$.

The convenience of this formulation is that the original problem is split into two consecutive tasks: the first task determines the optimal law $u(t)$ of the shell movement, while the second restores the dependence $x(t)$ of the movement of the internal mass on time according to the optimal law $u(t)$. The possibility of splitting is associated with the fact that the only condition imposed on the function $x(t)$ is the periodicity condition, which can be expressed in terms of u. Indeed, (1.1), which is regarded as the problem of finding a periodic function $x(t)$ at a predetermined periodicity of the left side, has a solution if and only if $\langle R \rangle = 0$. Therefore, for any periodic law $u(t)$ that satisfies the constraint $\langle R \rangle = 0$, the periodic function $x(t)$ can be found from (1.1) by simple integration. The original problem is thus reduced to the problem of finding the periodic function $u(t)$ with period T that provides a minimum for function $N[u]$ within the constraints $\langle u \rangle = U$ and $\langle R \rangle = 0$.

Until now the problem posed above and similar problems were considered [8, 10] in the quasi-stationary formulation when resistance R is completely determined by the current shell velocity. In the case of the motion of the spherical vibration-driven robot in the viscous liquid

$$R(u) = \frac{1}{2}C_x(\mathrm{Re})\pi\rho a^2 |u|u, \quad \mathrm{Re} = \frac{2au}{\nu}. \tag{1.2}$$

Here, a is the radius of the sphere, while ν and ρ are kinematic viscosity and liquid density. The resistance coefficient C_x is considered a known function of the instantaneous Reynolds number Re. For $C_x = \mathrm{const}$, the optimization problem was solved in [8], and for dependence $C_x(\mathrm{Re})$ given according to the results of the experiments, it was solved in [10].

When the shell moves in the viscous liquid, the condition of quasi-stationarity of the hydrodynamic resistance forces is applicable only for small accelerations of the shell. Therefore, it inevitably breaks with the increasing oscillation frequency of the internal mass. Unfortunately, until now the only strictly valid expression for the hydrodynamic forces acting on the sphere is the formula obtained in the limit of infinitesimal Reynolds numbers [14]:

$$R = 6\pi a\rho\nu u + 6\pi\rho\nu a^2 \int\limits_{-\infty}^t \frac{du/d\tau}{\sqrt{\pi\nu(t-\tau)}}d\tau + \frac{2}{3}\pi a^3\rho\frac{du}{dt}. \tag{1.3}$$

The first term in it describes viscous resistance forces, the second term denotes Basset forces, and the third term refers to the inertia forces of the added mass. Note that for any periodic motion law $u(t)$ the average over the period from Basset forces is zero, and the Basset operator

$$R_H[u(t)] = \int_{-\infty}^{t} \frac{\dot{u}(\tau)d\tau}{\sqrt{t-\tau}}$$

is positively defined

$$\langle R_H \rangle = 0, \quad \langle uR_H \rangle > 0.$$

With this in mind, it is clear that (1.3) does not allow the linear motion of the vibration-driven robot. Indeed, when the average of both sides is taken, using the periodicity condition u and by satisfying the restriction $\langle R \rangle = 0$ we find that $\langle u \rangle = 0$. Thus, for the motion to be possible, it should not be under infinitesimal but under finite Reynolds numbers.

A natural and widely used generalization of (1.3) for the case of finite Reynolds numbers in practice [15] is obtained by replacing the viscous forces in (1.3) with a dependence of form (1.2) with an empirically determined resistance coefficient. This approach is subject to justifiable criticism [13, 16]. Nevertheless, in our opinion, it is a useful first step in the study of the motion of a vibration-driven robot in the presence of hydrodynamic drag forces. If we exclude additional inertial forces in (1.3) due to the increase in (1.1) of the apparent mass of the main shell in the associated mass $M_0 = 2\pi\rho a^3/3$ (half the mass of the liquid displaced by the shell), we come to the expression for the resistance forces used in this paper:

$$R = \frac{1}{2}C_x\pi\rho a^2 |u| u + 6\rho a^2 \sqrt{\pi\nu} R_H[u]. \tag{1.4}$$

As can be seen, (1.4) differs from the quasi-stationary approximation (1.2) only by the additional account for Basset forces.

Let us restrict ourselves to the consideration of the important special case of $C_x = \text{const}$ of the quadratic resistance. It corresponds to moderate Reynolds numbers lying in the range $800 < \text{Re} < 3 \cdot 10^5$. Within this range, C_x varies in the range 0.4–0.5 [11]. By normalizing velocity u on U and time t on period T, we write down the problem of the optimal control of the shell movements in the following form:

$$N_{\min} = \min_u \left(N_V[u] + sN_H[u]\right), \tag{1.5}$$

$$\langle u \rangle = 1, \tag{1.6}$$

$$\langle u|u| \rangle = 0, \tag{1.7}$$

$$N_V = \left\langle |u|^3 \right\rangle, \quad N_H = \langle uR_H \rangle.$$

The minimization in (1.5) is carried out on a set of periodic functions with a unit period that satisfy constraints (1.6) and (1.7). When writing (1.7), it is further taken into account that $\langle R_H \rangle = 0$ for any periodic function u. The only dimensionless parameter of problem (1.5)

$$s = \frac{12}{C_x U}\sqrt{\frac{\nu}{\pi T}} \tag{1.8}$$

sets the degree of the nonstationarity of the shell's motion by characterizing the ratio of Basset forces to viscous forces.

As a result of solving (1.5)−(1.7), the optimal dependences $u(t)$ and $R[u]$ are determined, and the problem of finding the optimal internal mass motion control is solved. By normalizing the relative coordinate x of the internal mass ξa we write this problem in the following form:

$$\ddot{x} = -p\dot{u} - p^2 R, \tag{1.9}$$

$$p = \frac{1}{qs^2}, \quad q = \frac{\pi C_x \xi_2}{108} \cdot \frac{Ua}{\nu}, \quad \xi = \frac{4\xi_2\xi_1}{3C_x}.$$

Parameters ξ_1, ξ_2 characterize the mass ratio

$$\xi_1 = \frac{m + M + M_0}{m}, \quad \xi_2 = \frac{m + M + M_0}{M_0}.$$

Note that in contrast to parameter s the dimensionless parameter q introduced here does not depend on the oscillation period T. Within a constant factor this is the Reynolds number constructed from the average velocity of the shell's motion.

The main resulting integral characteristics of the optimal motion found in the solutions of (1.5)−(1.7) and (1.9) are the energy coefficient $\eta(s)$ and the range of oscillation of the internal mass $l(s, q)$

$$\eta = \frac{1}{N_{\min}}, \quad l = \max x - \min x.$$

The energy coefficient η describes the effectiveness of the internal propulsion. It is defined in [8] as the ratio of energy consumption in the case of the uniform motion of the shell to energy costs when moving the shell by means of the internal mass with the same average velocity.

2. OPTIMAL SHELL MOTION

If $s = 0$ the problem (1.5)− (1.7) of the optimal control by the shell's motion is reduced to an algebraic one. Its solution in [8] indicates that the shell's motion with maximum energy coefficient $\eta_0 = 0.079$ is implemented when its velocity takes two values: $u_+ = 1.278$ and $u_- = -6.743$. In this case, the alternation of periods of forward and backward motion can be of any kind, but the total time of each motion type on the periodicity interval should be $2\Delta = 0.035$ and $(1 - 2\Delta) = 0.965$, respectively. The only (*basic* in the notation of [8]) among the whole set of optimal laws of the shell's motion stands out by the requirement stating that motion phases should change only once on the periodicity interval (forward−backward−forward).

We imposed the same requirements in the general case of nonzero values of parameters s. It will make it possible not to consider the set of local high frequency minima of the functional capacity $N[u]$. The presence of such minima is easy to understand, having noted, for example, that if $u\left(t; \sqrt{2}s\right)$ is the solution of (1.5)−(1.7), then $u\left(2t; \sqrt{2}s\right)$ along with $u(t; s)$ will also deliver a local minimum to the functional $N[u(t; s)]$. The values of the functionals in these minima are $N_{\min}(\sqrt{2}s)$ and $N_{\min}(s)$, respectively. The first is knowingly greater because of the positive definiteness of the Basset operator.

By introducing Lagrange multipliers λ and μ that meet constraints (1.6) and (1.7) and by varying the Lagrange functional obtained from (1.5), (1.6), and (1.7)

$$L = \left\langle |u|^3 \right\rangle + s\left\langle u \int_{-\infty}^{t} \frac{\dot{u}(\tau)d\tau}{\sqrt{t - \tau}} \right\rangle + \lambda\left(\langle u|u\rangle\right) - \mu(\langle u\rangle - 1)$$

we find

$$\delta L = \left\langle \left(3u|u| + \lambda|u| - \mu + s\int_{-\infty}^{t} \frac{\dot{u}(\tau)d\tau}{\sqrt{t - \tau}}\right)\delta u \right\rangle + s\left\langle u \int_{-\infty}^{t} \frac{d\delta u}{d\tau} \frac{d\tau}{\sqrt{t - \tau}} \right\rangle.$$

By integrating by parts the last term in this equation and by using the fact that the operation of taking the average gives the same result at any time shift, one can show that

$$\left\langle u \int_{-\infty}^{t} \frac{d\delta u}{d\tau} \frac{d\tau}{\sqrt{t - \tau}} \right\rangle = -\left\langle \delta u \int_{t}^{\infty} \frac{\dot{u}(\tau)d\tau}{\sqrt{\tau - t}} \right\rangle.$$

By taking this into consideration and using randomness δu, we obtain the following equation for determining $u(t)$:

$$3|u|u + s\int_{-\infty}^{\infty} \dot{u}(\tau)\frac{\text{sign}(t-\tau)}{\sqrt{|t-\tau|}}d\tau + \lambda|u| - \mu = 0. \tag{2.1}$$

Because of invariance (2.1) with respect to the shift and time inversion it is possible to consider problem (1.6), (1.7), and (2.1) in the half-period $0 < t < 1/2$ and set symmetry conditions at its ends:

$$\dot{u}(0) = \dot{u}(1/2) = 0. \tag{2.2}$$

Further simplification of the resulting problem is associated with the removal of constraints (1.6) and (1.7) because of the selection of Lagrange multipliers. Let us integrate (2.1) over the period. Because of constraint (1.7) the mean of the first term on the left side of (2.1) is zero. The mean of the second term is also zero. This follows directly from the properties of the zero mean for the Basset operator of periodic functions. As a result of averaging we obtain $\mu = \lambda\langle|u|\rangle$. The remaining Lagrange multiplier λ can be selected arbitrarily by taking, for example, $\lambda = 1$. The thus obtained solution $\overline{u}(t;\overline{s})$ will be normalized to $\langle\overline{u}\rangle$ and will meet both (2.1) and all additional conditions in the case of a simple translation of $s = \overline{s}/\langle\overline{u}\rangle$.

Problem (2.1) and (2.2) for fixed values of λ and μ was solved numerically. The sampled on a uniform grid

$$t_k = kh, \quad h = \frac{1}{2n}, \quad k = \ldots, 0, 1, \ldots n, \ldots$$

(2.1) takes the form

$$3|u_k|u_k + s\left(\sum_{j=-\infty}^{\infty}(u_j - u_{j-1})h^{-1}\text{sign}(k-j+1/2)\int_{t_{j-1}}^{t_j}\frac{d\tau}{\sqrt{|t_k-\tau|}}\right) = \langle|u|\rangle - |u_k|. \tag{2.3}$$

By calculating the integrals in (2.3) using the conditions of periodicity and symmetry of the grid function and by passing to (2.3) to the estimated interval, we obtain

$$3\gamma_k|u_k|u_k + s\sum_{j=0}^{n}A_{kj}u_j = f_k, \quad k = \overline{0,n}; \tag{2.4}$$

$$f_k = \gamma_k(\langle|u|\rangle - |u_k|), \quad k = \overline{0,n}; \quad \gamma_k = \begin{cases} 1/2, & k = 0, n, \\ 1, & 0 < k < n. \end{cases}$$

Matrix A with elements A_{kj} appearing in (2.4) is symmetric, positive semi-definite, and the sum of the elements in each row is zero. The nonlinear finite-difference equation (2.4) is solved using Newton's method with the lowering of the right side of (2.4) to the previous iteration

$$6\gamma_k|u_k^{(s)}|\delta u_k + s\sum_{j=0}^{n}A_{kj}\delta u_j = f_k^{(s)} - 3\gamma_k|u_k^{(s)}|u_k^{(s)} - s\sum_{j=0}^{n}A_{kj}u_j^{(s)}, \quad k = \overline{0,n},$$
$$u^{(s+1)} = u^{(s)} + \xi\delta u. \tag{2.5}$$

The relaxation parameter ξ in all calculations was set to 0.4. The reversion of the symmetric positive definite matrix in (2.5) was carried out by direct methods. A grid containing 4097 nodes was used in the calculations. The iteration loop is exited when the maximum residual of 10^{-5} is reached on the right side of (2.5). In this case, depending on parameter s and the accuracy of the initial approximation the number of iterations varied from a few dozen to a few hundred.

As noted above, the solution of this problem is not unique. In order to obtain basic solutions (i.e., solutions without other extreme points except for 0 and 1/2 on the marked half-cycle) corresponding to the global minimum (1.5), the initial approximation should fall in the vicinity of these solutions. In the numerical procedure it was guaranteed by the gradual continuation of parameter s, when $u(t;s)$ was selected as the initial approximation for $u(t;s+ds)$. The presence of the basic analytical solution for $s=0$ was used at the initial step.

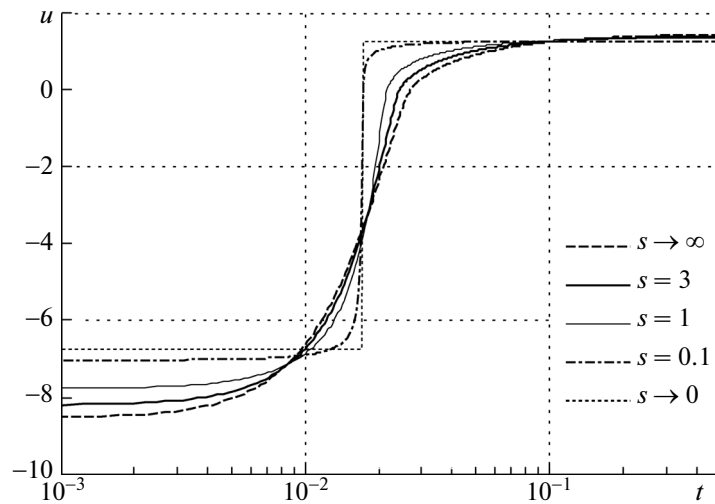**Fig. 1.** The optimal motion law $u(t)$ of the shell on the half-cycle at various values of the parameters.
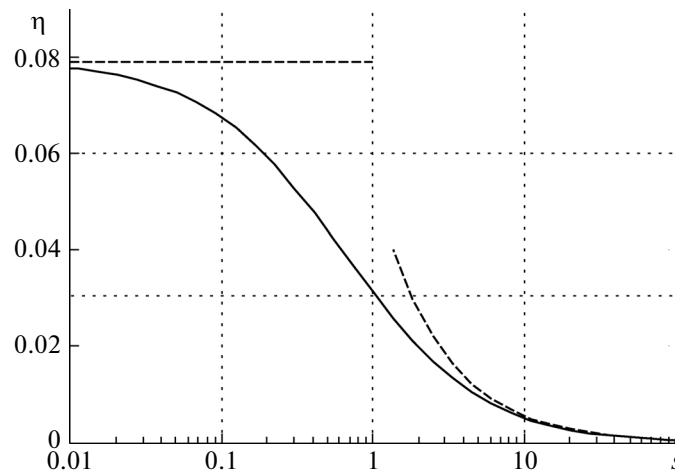

Fig. 2. The dependence of the energy coefficient η of the parameter s (solid line), and the asymptotics at limiting values of s (dashed lines).

The calculation results are presented in Figs. 1 and 2, in which, for clarity, the logarithmic scale is used on the abscissa. In Fig. 1, optimal dependences $u(t)$ for different values of s are shown on half of the period $(0, 1/2)$. As can be seen, with the growth of s the nature of the quasi-stationary optimal motion law of the shell is preserved, but the jump of velocities at $t = \Delta$ is gradually smoothed out. The maximum shell velocity increases with the increase of s from $u_+ = 1.286$ at $s = 0$ to $u_+ = 1.436$ at $s = \infty$, the minimum velocity decreases from $u_- = -6.653$ to $u_- = -8.562$, and the duration of the backward motion phase increases from $2\Delta = 0.035$ to $2\Delta = 0.054$. Figure 1 shows that the most significant change in the optimal motion law $u(t; s)$ occurs in the range of s from 0.1 to 3. If s is less than 0.1, the law of motion is close to the quasi-stationary $u_0(t)$ obtained by neglecting the Basset forces. At s larger than 3, in contrast, viscous friction forces can be ignored. Here $u(t; s)$ practically coincides with $u_\infty(t) = u(t; \infty)$.

In Fig. 2 the solid line shows the dependence of the main integral characteristic, which is the energy coefficient η, on parameter s. The dashed lines in this figure indicate $\eta_0 = 0.079$ and the asymptotic $\eta(s) = \eta_\infty s^{-1}$ ($s \rightarrow \infty$). $\eta_\infty = 0.056$ is calculated by the power of Basset forces for $u_\infty(t)$ according to the formula $\eta_\infty = (N_H[u_\infty])^{-1}$. As might be expected, the energy coefficient decreases monotonically with an
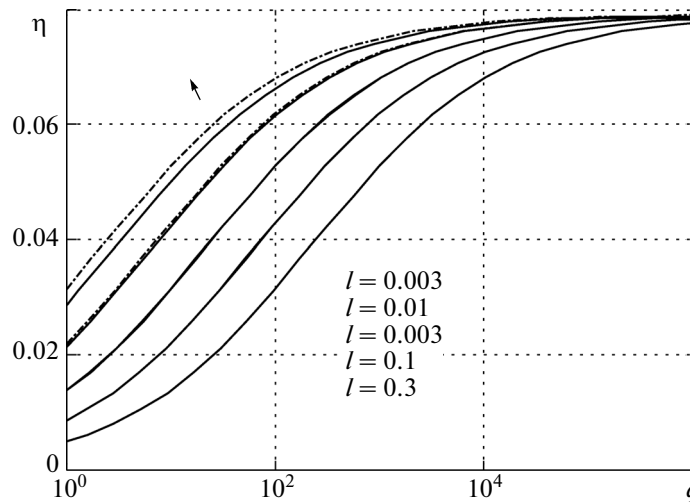
Fig. 3. Curves of the level of the oscillation amplitude of the internal mass $l(\eta, q)$: the solid line indicates the general case, and the dashed-dotted line refers to the case on the assumption of the inertial mode of motion.

increase of parameter s, which corresponds to additional power losses of the propulsion to overcome Basset forces.

3. OPTIMAL MOTION OF THE INTERNAL MASS

After solving the minimization problem, the optimal motion law of the internal mass is found by integrating (1.9). Let us restrict ourselves to the representation of the results relating to the calculation of the dimensionless amplitude of oscillations l of the internal mass. It is convenient to express l directly in terms of the optimal law $u(t)$ of shell movements. To do this, by introducing the function $\Phi(t)$

$$\dot{\Phi} = R, \quad \langle \Phi \rangle = 0$$

and by integrating (1.9) once, we obtain

$$\dot{x} = -p(u-1) - p^2\Phi. \tag{3.1}$$

Periodicity $\Phi(t)$ is guaranteed by the condition $\langle R \rangle = 0$, and periodicity x is guaranteed by conditions $\langle \Phi \rangle = 0, \langle u \rangle = 1$. As shown by the numerical calculations, the function $x(t)$ has only two extrema in the periodicity interval, high and low. Therefore, by taking the module from both sides of (3.1) and by integrating the result over the period, we arrive at the desired expression

$$2l = p\langle |(u-1) + p\Phi| \rangle. \tag{3.2}$$

Given that $p = q^{-1}s^{-2}$, and the optimal law $u(t)$ is defined by parameter s, which in turn is uniquely associated with the energy coefficient η, the oscillation amplitude l is a function of the energy coefficient η and the Reynolds number q. In Fig. 3, the solid lines show the curves of the level of this function.

Let us note two important points. Firstly, for a fixed value of parameter q the function $l(s)$ decreases monotonically with the increase of its argument from infinity at $s = 0$ to zero at $s = \infty$. Therefore, $l(s)$ is reversible, and each l can be associated with the corresponding s. Secondly, as will be seen below, the main physical interest is the range $l < 0.1$, $q > 10$ of the parameter values. In this range there is the so-called inertial mode of the internal mass [8], when the second term in the right-hand side of (3.1) and (3.2) can be neglected. This is illustrated in Fig. 3 by the proximity of the curves of the level of the function $l(\eta, q)$ constructed by the general formula (3.2) (solid lines) and on the assumption

$$2l = p\langle |u-1| \rangle \tag{3.3}$$

of the inertial mode (dashed lines). Relation (3.3) implies that

$$lqs^2 = b, \tag{3.4}$$

where $b(s) = \langle |u - 1| \rangle / 2$. The calculations show that when s changes from zero to infinity $b(s)$ increases from 0.275 to 0.335. A slight change in b makes it possible to consider this value constant, with $b \approx 0.305$ in practice.

4. RESULTS AND DISCUSSION

The wording of the original problem involves fixing the diameter of the shell of the vibration-driven robot, its average motion velocity, and oscillation period of the internal mass. In dimensionless variables it was consistent with the setting of parameters s and q. After minimizing the power integral the range l of oscillation of the internal mass was calculated. Instead of fixing the oscillation period of the internal mass, the range of oscillations could be fixed by counting the corresponding period after the solution of the respective minimization problem. In both cases, the result would have been the same. Figure 3 would have been interpreted as a series of diagrams of the dependence of the energy coefficient η (and, hence, the dimensionless oscillation period s uniquely associated with it) on the dimensionless average velocity q of the shell's motion for different values of l.

Further, it is convenient to use this representation. The fact is that as opposed to the period the magnitude of the oscillation range has a purely structural limitation. It cannot exceed the diameter of the shell. In dimensionless variables we have $l = 2r\xi^{-1}$. Here, the safety factor $r \leq 1$ is the ratio of the oscillation range to the diameter of the shell. In the case of $r = 1$, $C_x = 0.45$, zero mass of the shell, and neutral buoyancy of the vibration-driven robot ($2M_0 = M + m$) we have $l = 0.15$. The clarification of rough estimates for the safety factor and shell mass can lead to a significant decrease of l.

If $C_x = 0.45$, in the case of neutral buoyancy the dimensionless velocity q is related to the Reynolds number built by an average velocity of the shell motion by the ratio $q = 0.02\,\mathrm{Re}$. The range of the quadratic law of the viscous resistance in terms of q can be written as $16 < q < 6 \cdot 10^3$.

In specified ranges of variation of l and q (Fig. 3), the inertial motion mode of the internal mass is obviously implemented. In the inertial mode, (3.4) holds for optimal movements. This formula can be given the following simple form in dimensional variables

$$UT = \frac{3.28m}{m + M + M_0}\, ra. \tag{4.1}$$

It associates the motion UT of the vibration-driven robot for one period with its structural characteristics. By setting the average velocity of the translational motion of the shell U, the shell diameter $2a$, the ratio of masses of the body and the internal propulsion, and fixing the amplitude of oscillations, according to (4.1) it is possible to determine the optimal duration of the period T of oscillations of the system. Next, by calculating the parameter s, according to (1.8) and using Fig. 2, we can find the energy coefficient that characterizes the optimal motion efficiency.

In the inertial mode the velocity of the internal mass $v(t)$ is determined equally simply by the optimal velocity $u(t)$ of the shell movements. By neglecting the second term in the right-hand side in (3.1) we obtain

$$v(t) = \frac{m + M + M_0}{m}(U - u(t)).$$

As can be seen, the maximum velocity of the internal mass is several times higher than the shell velocity.

CONCLUSIONS

In this paper, the problem of optimal motion control of the vibration-driven robot in a viscous liquid was posed when the resistance forces include not only viscous forces but also the inherited Basset forces, which corresponds to a more realistic description of the forces acting on the sphere. In the course of solving the problem the following facts were established.

Firstly, the energy formulation of the problem of optimizing the control of the motion of the vibration-driven robot and approaches to its solution proposed in [8] can be used in laws of resistance to motion from the external environment that are more complex than the quasi-stationary laws. In particular, they can be transferred directly to nonlocal time dependences that take into account the motion's history.

Secondly, the accounting for inherited effects does not lead to a qualitative change in the optimal modes of motion of the shell and internal mass obtained under the assumption of quasi-stationary nature of the resistance law. Optimum movements are still biphasic; the durations and velocities of forward and backward phases of the shell's motion differ from the quasi-stationary ones for about the first ten percent.

Thirdly, the energy coefficient $\eta_0 = 0.079$ calculated for the quasi-stationary case ($T = \infty$) that characterizes the motion efficiency is in the general case $T < \infty$ of the upper bound for η. With the reduction of the oscillation period T the energy coefficient of the optimal motion is reduced because of the additional power losses of the propulsion to overcome the Bass forces.

Fourthly, in the practically interesting range of changes of the parameters of the problem the inertial mode of motion of the internal mass is implemented. In this case, the vibration-driven robot movement for one period in the optimal motion is determined completely by the structural characteristics of the vibration-driven robot by (4.1).

Obviously, the last three conclusions were made within the considered formulation of the problem. It is too early to directly extend them to the case of the vibration-driven robot motion in a viscous liquid at moderate Reynolds numbers. The restricted formulation is mainly due to the excessive schematization of the inherited forces. Their more realistic accounting based on semi-empirical models, such as [12, 13], and the direct numerical simulation of the hydrodynamic problem will be carried out in the future.

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