Heat conduction in a solid substrate with a spatially-variable solar radiation input: Carslaw-Jaeger solution revisited

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Abstract

Temperature distributions recorded by thermocouples in a solid body (slab) subject to surface heating are used in a mathematical model of 2-D heat conduction. The corresponding Dirichlet problem for a holomorphic function (complex potential), involving temperature and heat stream function, is solved in a strip. The Zhukovskii function is reconstructed through singular integrals, involving an auxiliary complex variable. The complex potential is mapped by the Schwartz-Christoffel formula onto an auxiliary half-plane. The final heat conduction flow net (orthogonal isotherms and heat lines) is compared with the known Carslaw-Jaeger solution and shows a puzzling topology of energy fluxes for simple temperature-boundary conditions.

Key words: Laplace's equation, topology of heat lines, complex potential, conformal mappings.

1. Introduction

Analytical solutions to potential field problems, where the intricate topology of 2-D flow nets (stream lines and constant potential lines) was controlled by heterogeneity of the flow domain, but the boundary conditions were uniform (constant potentials on the inlet and outlet of a standard flow tube) were presented in [1], [2]. In this paper we study the effect of non-uniform boundary conditions, although assume that the medium, through which flow takes place, is homogeneous. Analytical solutions for steady 2-D heat conduction in solid bodies are needed in different engineering designs involving heat transfer [3]. A powerful technique to solve these problems is based on the theory of boundary-value problems for holomorphic functions (e.g., [4], [5]). In this paper we average the diurnal temperature swings, recorded by thermocouples on the surface of a concrete slab, and show that the corresponding explicit analytical solution gives a computer-algebra-visualized topology of heat lines, which is counterintuitive and puzzling.

2. Mathematical Model

We consider a vertical cross-section of the slab of a thickness b and thermal conductivity k, and a thermal barrier E_1OE_2 (practically, strip-type shading against solar radiation). Fig.1a depicts a vertical cross-section and Cartesian coordinates. Far from the barrier (the rays AE_1 and E_2B), the slab temperature is the same as the ambient air temperature, T_0 =constant. Along AOB, we have experimental data of temperature obtained by thermocouples and we take the daily averages of these values. The x-distribution of this average temperature is a single-minimum function f(x). We assume that this function is symmetric f(-x)=f(x) and $f(x) \to T_0$ at $x \to \pm\infty$ (this is confirmed by experiments). We introduce F(x) as:

$$f(x)=T_0 - F(x), \quad \text{at } y=0 \qquad (1)$$

where F(x) is a single-maximum ($T_M = T_0 - T_m$) function shown in Fig.1b. We assume that along the internal surface (DC in Fig.1a) temperature is constant, T_c:

$$T=T_c, \quad \text{at } y=-b \qquad (2)$$