

**MATHEMATICAL ANALYSIS AND NUMERICAL
SIMULATION OF THE GUIDED MODES
OF THE WEAKLY GUIDING OPTICAL FIBERS**

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A mathematical formulation for the guided modes of the weakly guiding optical fibers is derived from the two-dimensional Helmholtz equation. The original problem is reduced to a spectral problem for the Helmholtz equation in a bounded domain with nonlocal boundary condition. This formulation leads to a nonlinear eigenvalue problem for a family of self-adjoint compact operators. The main spectral properties of these operators are established. Then the min-max principle provides an expression of the nonlinear dispersion relation, which connects the propagation constants of guided modes with the frequency. Various existence results are proved and complete description of dispersion curves (monotonicity, asymptotic behavior, existence of cutoff values) is carried out. A finite element scheme follows from the operator formulation of the problem. This scheme leads to a nonlinear eigenvalue problem for a family of self-adjoint matrix operators. The existence results for this problem are proved. For its solution we propose an efficient method. Practical opportunities of this algorithm are shown by the results of numerical experiments.