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TOPOLOGY OF STEADY HEAT CONDUCTION IN A SOLID SLAB SUBJECT TO A NONUNIFORM BOUNDARY

CONDITION: THE CARSLAW–JAEGER SOLUTION REVISITED. R. G. KASIMOVA, 1 and YU. V. OBNOSOV, 2

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Abstract

Temperature distributions recorded by thermocouples in a solid body (slab) subject to surface heating are used in a mathematical model of two-dimensional heat conduction. The corresponding Dirichlet problem for a holomorphic function (complex potential), involving temperature and a heat stream function, is solved in a strip. The Zhukovskii function is reconstructed through singular integrals, involving an auxiliary complex variable. The complex potential is mapped onto an auxiliary half-plane. The flow net (orthogonal isotherms and heat lines) of heat conduction is compared with the known Carslaw–Jaeger solution and shows a puzzling topology of three regimes of energy fluxes for temperature boundary conditions common in passive thermal insulation. The simplest regime is realized if cooling of a shaded zone is mild and heat flows in a slightly distorted "resistor model" flow tube. The second regime emerges when cooling is stronger and two disconnected separatrices demarcate the back-flow of heat from a relatively hot segment of the slab surface to the atmosphere through relatively cold parts of this surface. The third topological regime is characterized by a single separatrix with a critical point inside the slab, where the thermal gradient is nil. In this regime the back-suction of heat into the atmosphere is most intensive. The closed-form solutions obtained can be used in assessment of efficiency of thermal protection of buildings.

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1. Introduction

Analytical solutions for heat conduction in solid bodies are needed in different engineering designs involving heat transfer [3]. Steady conduction can be quantified

1 German University of Technology in Oman, Muscat, Sultanate of Oman;

e-mail: rouzalia.kasimova@gutech.edu.om.

2 Institute of Mathematics and Mechanics, Kazan Federal University, Kazan, Russia;

e-mail: yurii.obnosov@gmail.com.