LONG-PERIOD NUTATIONAL-PRECESSIONAL MOVEMENTS OF THE INSTANTANEOUS POLE OF THE EARTH'S ROTATION

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Abstract

The influence of the rotational regime on real long-period nutational-precessional motions of the instantaneous pole of the Earth's rotation is considered. The solar-magnetospheric (through changes of the angular velocity of rotation) mechanism of action is discussed. The observed 12-month oscillations of the instantaneous axis of rotation are explained by the influence of the variable centrifugal force arising when the angular velocity changes and causing deformation of the figure that is not uniform in different longitudinal zones, without involving the baric-circulational processes as an intermediate link. Thus, both 14-month free (Chandler) and forced 12-month fluctuations of the pole location are considered from a single point of view – as a result of the variability of the rotational regime of the Earth. The analysis of heliogeophysical data for 1900-2017 confirms the validity of the proposed relationship mechanism.

Keywords: Earth rotation, nutation, precession, instantaneous pole of rotation, Earth figure deformation, pole position fluctuations, free oscillations, forced oscillations, solar-magnetospheric control.

1. Introduction

As known, the instantaneous axis of the Earth's rotation experiences nutational-precessional oscillations relative to its average position, the main periods are 14-month (the Chandler period) and 12-month (forced variations) ones. In addition, these fluctuations experience significant long-term changes, the causes of which have not yet been fully identified. If for the Chandler variations the fact of their relationship with the unevenness of the Earth's rotation can be considered established (according to the results by Eigenson [1] the correlation between the annual variation of the Earth's rotation period (*T*), on the one hand, and the amplitude (*A*) and the duration of the Chandler's period (*P*), on the other hand, are: $r_{TA} = 0.875$ and $r_{AP} = 0.910$), most researchers consider the 12-month forced variations as a consequence of changes in the global baric circulation regime, putting forward as an argument the observed correlation of pole movements and solar activity (see, for example, book [2] and references there). Besides, tectonic hypotheses are used to explain the long-term changes in the forced part of the oscillations of the Earth's instantaneous axis of rotation (see [3] and references there).

The objective of this work is to show the role of the Earth's rotation regime in long-term real oscillations of its instantaneous axis of rotation, and also to propose a mechanism that qualitatively explains the observed long-term forced oscillations of the instantaneous axis of rotation due to deformations of the Earth's figure when the angular velocity changes [4].

In Sect. 1, basing on an analysis of heliogeophysical data, we show the relationship between real long-period poles' motions and changes of the angular velocity of the Earth's rotation, as well as, basing on the hypothesis of the solar-magnetospheric control of the rotation regime [5], [6], with fluctuations of the Wolf numbers (*W*) and global magnetic disturbance [index $M = 10(\sum K_p - 10)$]. Further, for the elastically deformable Earth, we obtain the relations that relate the period of nutational-precessional pole movements and the deformation of the Earth's body with the angular velocity of rotation (Sect. 2). Here we also represent the results showing the correlation in cycles of different durations of the parameters of free (Chandler) variations of the Earth's rotation pole with changes of heliogeophysical factors, that can be as an indirect confirmation of the hypothesis proposed by Belashov [5].

In Sect. 3 we propose a "deformational" mechanism determining the influence of changes of the Earth's rotational movement on the forced part of the nutation-precession oscillations of the rotation axis. The provided results of the analysis of heliogeophysical data confirm the validity of the alleged relationships.

2. Real movements of the instantaneous pole of rotation of the Earth

As it was found by Maksimov [7], a wave with an average period of 11 years is observed in the oscillations of the pole location, which negatively correlates with the average annual Wolf numbers, that was explained by the acceleration of the troposphere circulation with increasing *W*. However, later it was shown [2] that such a parameter as the size of the major axis of the ellipse of forced oscillations positively correlates with the average, for every 11 years, anomalies of the Wolf numbers, that was explained by a weakening of the climatic contrast between summer and winter at the era of maximum *W*, that is, by decreasing of seasonal transfer. However, in the latter case, the amplitude of the forced oscillations should decrease, that contradicts the results of data analysis [2]. This leads to the idea that the hypothesis of the determining influence of seasonal meteorological phenomena on forced fluctuations of the pole location is not justified. For free oscillations, a simultaneous change of the ellipticity of the pole trajectory with changes of the day duration was established in [2].

To substantiate the idea of the influence of the variability of the rotational regime (as of consequence of changes of the global magnetospheric disturbance [5], [6] that is, ultimately, of fluctuations of solar activity) on the real movements of the instantaneous pole of the Earth's rotation, the data on *X*- and *Y*-components of the radius vector of the pole for the period from 1900 to 1985 (according to the data of the International Latitude Service, ILS[1]) were analyzed in [3], the curves of real pole movements by epochs were plotted separately for even and odd 11-year cycles of solar activity taking into account the reference phases: the *W* minima, *W* maxima, and *M* maxima. The results of our analysis of the data series for 1985-2017 confirm the conclusions made in [3] regarding the fact that the 7-year cycle is present in the fluctuations of the pole location (the cycle period varies from 6 to 9 years over the indicated period, that is, this cycle cannot be considered a purely classic 7-year cycle [8], that is a consequence of the imposition of forced 12-month and free 14-month fluctuations of the place of the pole). There is a delay in the extrema of these cycles relative to the reference phases in time (on average, about a year) and, most importantly, the extrema of 7-year cycles in pole oscillations show a clear connection with the extrema of previously identified [5] 7-year cycles in changes of the day duration, moreover, with a corresponding delay of about one year, that leads to the idea of a causal relationship between these two phenomena. The observed delay in the fluctuations of the magnitude of the radius vector of the pole relative to changes in the day duration enables us the latter to be considered as the cause, and the former as the consequence. The results of the analysis are summarized in table 1.

Thus, it should be noted that not only oscillations with a Chandler period, but also real movements of the instantaneous pole of the Earth's rotation occur quite synchronously with changes of the angular velocity of the Earth's rotation.

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[1] International Earth Rotation Service (IERS) – from 1988.

Table 1: Extremes of variations of *W*, *M*, $\Delta\tau$ and *R* in the even and odd cycles of solar activity

Cycles		Even				Odd	
W	–	+		–	+		–
M	–		+	–		+	–
$\Delta\tau$	+	–	+	–	+	–	+
R	+	–	+	–	+	–	+

Note. Extremes: + maximum, – minimum; *W* are the Wolf numbers, *M* is the magnetic disturbance index, $\Delta\tau$ is the day duration, *R* is the value of the pole's radius vector.

3. Free rotation of the elastically deformable Earth

Considering the polar compression of the planets of the Solar system, their angular velocities of rotation and average densities, we can conclude that the degree of compression of the planet mainly depends on its rotation speed [9], and therefore, the change in the rotation regime of the planet should primarily affect on a change of polar compression (according to estimates made in [9] it can reach ~ 0.09% of the average Earth compression for real fluctuations of the day duration).

To describe the rotation of a body of a changing shape, the Liouville equations which take into account the exchange of momentum between different parts of the body are usually used. However, due to the difficulty of taking into account the influence of the momentum moments associated with the redistribution of density in the subcrustal layer at the figure deformation (we assume that this deformation is conjugated, that is, occurring without changing the volume), we will use the way of successive approximations choosing as the first (the most of the rough) approximation the rotation of the absolutely solid undeformable Earth, that is described by the Euler equations in the form [10]:

$$
\begin{aligned}
&A\frac{dp}{dt} + rq(C-B) = L,\\
&B\frac{dq}{dt} + pr(A-C) = M,\\
&C\frac{dr}{dt} + pq(B-A) = N
\end{aligned}
\qquad (1)
$$

where *L*, *M* and *N* are the moments of external forces relative to the main axes of inertia; A, *B* and *C* are the moments of inertia; *p*, *q* and *r* are the components along the coordinate axes *x*, *y* and *z* fixing the position of the Earth relative to the axis of rotation. Considering the free rotation of the ellipsoid (abstracting from the influence of tidal forces) and neglecting the equatorial ellipticity (terms of small order), and also taking into account the relations accepted by ILS: $X = p/\omega$, $Y = q/\omega$, we obtain from (1) $r = \text{const} \approx \omega$ and

$$
\frac{dX}{dt} - \alpha\omega Y = 0, \qquad \frac{dY}{dt} - \alpha\omega X = 0 \qquad (2)
$$

where $\alpha = (C-A)/A$ is the dynamic compression. Integrating (2) we obtain:

$$X = R\cos(\alpha\omega t - \sigma), \qquad Y = R\sin(\alpha\omega t - \sigma) \tag{3}$$

where σ is the Lagrange parameter. Thus, for the model of an absolutely solid non-deformable Earth, the pole moves along a circular path with the period

$$\tau_0 = 2\pi / \alpha\omega. \tag{4}$$

In (3), the integration constants R and σ are found from observations, the value of dynamic compression is determined astronomically, $\alpha = 1/305$ [10]. Now, consider the next approximation when the Earth is an elastically deformable body (the Hooke's body) [4]. We introduce the parameter w by the equations [10]

$$D = -Yw\alpha A, \quad E = -Xw\alpha A, \quad F = 0$$

where $D = \int_M YZdm, \quad E = \int_M ZXdm, \quad F = \int_M XYdm$ are the inertia products. Taking into account that equation (4) in this case takes form $\tau = 2\pi/[\alpha\omega(1-w)]$, we can write $w = 1 - \tau_0/\tau$. For hydrostatic equilibrium $C - A = \frac{2}{3}(e - q/2)Ma^2$ (M is the mass of the Earth, a is the equatorial semi-axis, e is the eccentricity) using simple transformations, we obtain:

$$\tau = \tau_0 \frac{2e - q}{2e - q(1+k)} \tag{5}$$

where $q = f_1[\omega(t)] = \omega^2 a / 2g$, $e = f_2[\omega(t)]$, k is the compression module.

Formula (5) expresses the Chandler period of the pole motion and shows that any change of the angular velocity of rotation, as well as the associated deformation of the Earth's figure [4], which, according to estimates made in [11], can reach 10 % of the tidal, will cause a change of the period of free (Chandler) movements of the Earth's rotation pole.

Now, we note that if the hypothesis of solar-induced (through the magnetosphere) changes of the angular velocity of the Earth's rotation [5, 6] is true, then there should be a relationship between the variations of solar activity and magnetic disturbances, on the one hand, and with the free oscillations of the Earth's rotation axis, with another hand. To illustrate this, in [3] and here, the analysis of changes of solar activity (the Wolf numbers [12, 13]), magnetic disturbance (index M [14, 15]), sizes of the major semi-axis of the ellipse of the distinguished free motion of pole $V_{\max}$, and of the X- and Y-coordinates of the free movement of the Earth's pole for the period from 1900 to 2017 for cycles of different durations was made. Figure 1 shows the most typical example of comparing changes in the index W and $V_{\max}$ (variations with periods of less than 20 years were filtered by the method of moving average). One can see that, in general, the antiphase behavior of the curves is observed with a shift of the opposite phases W and $V_{\max}$ an average of about 4 years, that seems natural because with an increase of the level of solar activity, the angular velocity of the Earth's rotation decreases (with phase shift ~ 3-4 years) [5] and, therefore, the range of free oscillations of the pole of rotation should decrease.

Figure 2 shows the most typical example of comparing the course of the M index and the oscillations of the X-coordinate of the distinguished free motion of pole for the period January 1955 – October 1956. One can see the similar course of the curves. The absence in changes of X_P in figure 2 the waves with periods lesser than the Chandler wave which present in the oscillations of M, can apparently be explained by the large inertia of the change of α in comparison with the fluctuations of ω in formula (3), as a result of which the waves of a shorter period are “masked” by the 14-month wave significantly exceeding them in amplitude.

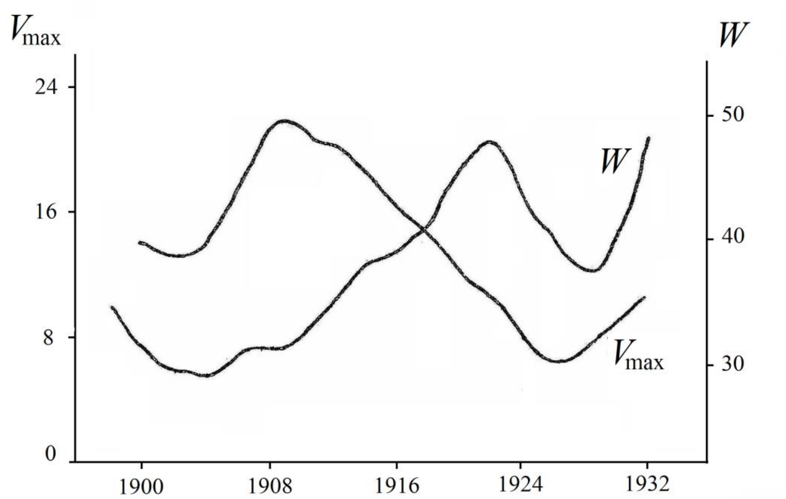

Figure 1: Change of average values of V_{max} in hundredths of an arc second in comparison with the variations of W in 22-year cycles, 1900-1932.

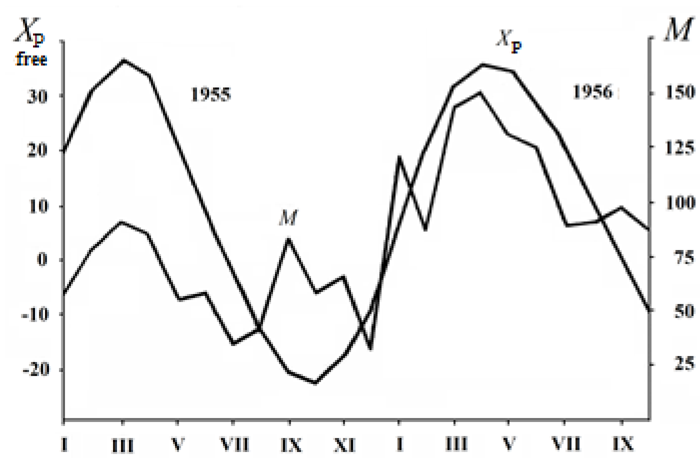

Figure 2: Change of magnetic disturbance $M = 10(\sum K_p - 10)$ in comparison with pole coordinate X_p values, January 1955 – October 1956.

The Fourier analysis carried out by [16], however, enabled to isolate in the changes of latitude, along with the previously known semi-annual wave, the wave with a period equal to half one the Chandler wave. Figures 1 and 2 show that the relationship between solar activity, magnetic disturbance, and the elements of free motion of the instantaneous pole of the Earth's rotation is really observed.

4. Forced movements of the instantaneous pole of rotation of the Earth

As it was mentioned above (see Introduction), most researchers consider seasonal changes of the atmospheric circulation and baric field to be the cause of the forced 12-month variations of the Earth’s instantaneous rotation pole. However, in our opinion [5, 6], the energetic effects of atmospheric processes are hardly sufficient to violate the stability of the Earth's rotation in real limits. From the results obtained in [2] it follows that the amplitude of variations with the 12 months' period increases with increase of solar activity, that suggests the genetic relationship of these two phenomena.

Our investigations of the forced pole location's variations (extracted forced component *X*), the changes of angular acceleration of the Earth's rotation ε [17-23] and the magnetic disturbance index *M* showed (figure 3) that these parameters change in a similar way. In particular, it can be seen from the figure that the projection of the radius vector of the pole on the *X*-axis (0 – 180°) is maximum at the maximum angular velocity of the Earth's rotation, that is, the variations along the *X*-axis increase with increasing ε. The noted similarity in the variations of these elements allows us to put forward a hypothesis about the deformation nature (due to a change in the angular velocity of rotation) of the forced variations of the pole.

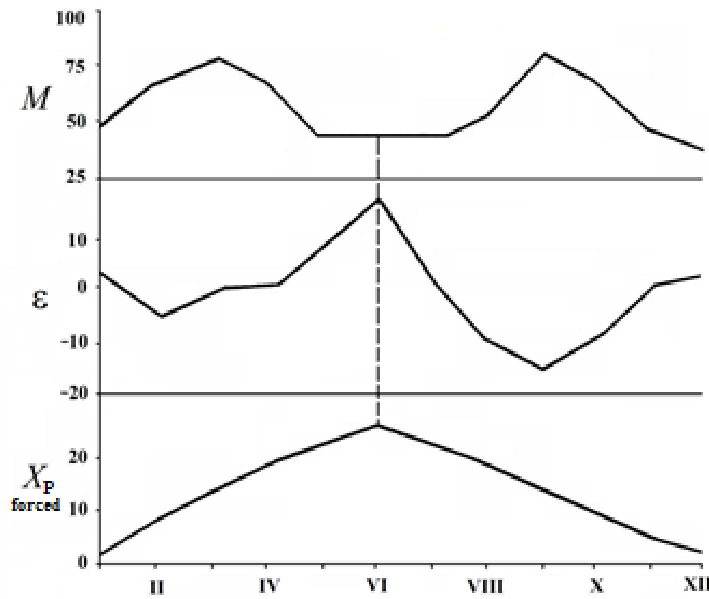

Figure 3: Averaged cyclic curves of the index *M*, angular acceleration ε and the projection of the radius vector of the pole on the *X*-axis (0 – 180°) (forced wave).

In order to obtain an analytical solution of the problem of dependence of the forced nutation-precession oscillations of the rotation axis on the deformation of the Earth's body or, more exactly, on the movement of subcrustal masses in the Earth's body and the corresponding density redistribution occurring [9] due to the variability of the planet's rotation regime, it is necessary, as the next approximation (see Sect. 2), to go from the Euler equations to the Liouville equations, taking into account the influence of the relative angular momentum on the axes associated with the Earth's mantle [10]:

$$h_i = \int_V \rho \varepsilon_{ijk} x_j u_k dV \qquad (6)$$

where $\varepsilon_{ijk} = 0$ at $i = j$ or $i = k$, and $\varepsilon_{ijk} = 1$ for right order of indices: 1, 2, 3, 1, …, and $\varepsilon_{ijk} = -1$ for out of order indices: 1, 3, 2, 1, …. However, in (6) it is necessary to know exactly the function, i.e., the density at each point of the Earth as a function of time, which in itself currently seems to be an unsolvable problem. Therefore, here we put forward a hypothesis and make an attempt to substantiate it from a qualitative standpoint, that is, by logical basing and analysis of the available factual material.

In some approximation, the Earth is a triaxial ellipsoid of revolution. The ellipticity of the equator indicates that the masses in the Earth's body along the longitudinal zones (i.e., the density over these zones) are nonuniformly distributed. This is indicated by direct geodetic measurements of the geoid level, and gravimetric measurements, and, finally, measurements of deviations of satellite orbits from ideal ellipses. The Lambek and Gaposhkin model [10], which is in good agreement with direct measurements, gives a picture of the geoid levels shown in figure 4. From the figure it can be seen that the Earth in the equatorial plane is compressed in the direction of $80°E - 100°W$ and in the direction of $0 - 180°$ it has a convexity. In our opinion, this fact should be explained by the fact that heavier, denser masses lie in longitude zones $40 - 130°E$ and $40 - 140°W$, which cause a positive anomaly of gravity in these zones and, as a result, geoid compression in the direction of $80°E - 100°W$. Accordingly, in longitudinal zones $40°E - 40°W$ and $130°E - 140°W$ less dense masses lie, and, due to the negative anomaly of gravity, the convexities are observed in these zones. We supplement this important point with the fact that masses of lower density naturally have greater mobility than denser masses.

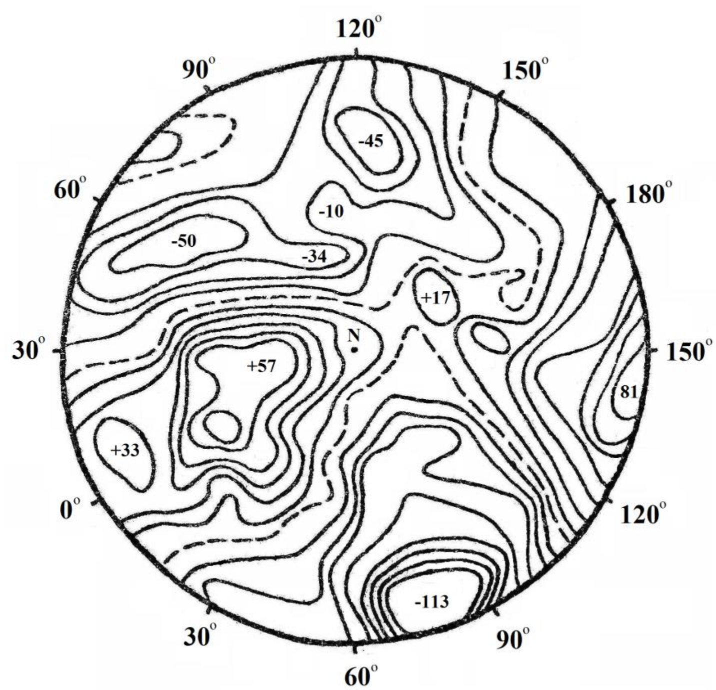

Figure 4: Curves of the geoid ($e = 1/298.225$) level plotted after 10 m, zero level is shown by dotted line. The Lambek and Gaposchkin model (Melchior, 1972) (northern hemisphere).

We now consider the possibility of the occurrence of forced oscillations of the axis of rotation of the Earth. As was shown in [9] and [4], when the angular velocity of rotation changes, a deformation of the Earth's figure arises, which results in the "flow" of subcrustal masses from the polar regions to the equatorial ones (with decreasing the polar diameter of the Earth, that is, with increasing the angular velocity of rotation) and vice versa (with decreasing ω, when the Earth tends to gravitational equilibrium at the already new (smaller) value of the deforming centrifugal force).

In this case, it is natural to assume that, due to greater mobility, with increasing the angular velocity of rotation, subcrustal masses in the zones corresponding to the equatorial convexity (that is, in the direction $0 - 180°$) will initially start to move, which will cause precession-nutational oscillations of the rotation axis, because the moment of inertia of the Earth will change, and the projection of the major axis of the ellipsoidal movements of the instantaneous pole of rotation of the Earth on the *X*-axis (that is, in the direction $0 - 180°$) will increase.

The analysis of changes of the angular velocity of the Earth's rotation (angular accelerations ε) and the projection of the radius vector of the pole on the *X*-axis confirms our assumption (See figure 3).

Decreasing the angular velocity of the Earth's rotation causes, correspondingly, a reverse movement of masses, and the variations of the radius vector of the pole along the *X*-axis decrease. The absence of a six-month cycle (at least, cycle of significant amplitude) in the changes of X_P (like in the ε and M oscillations), in our opinion, can be explained by the delay in alignment of the figure to gravitational equilibrium when the angular velocity of the Earth's rotation changes.

In cycles of long duration (22 years and secular cycle) the major axis of the ellipse of forced variations of the pole's place lies in the direction of the small equatorial axis of the Earth's rotation ellipsoid [2], that is, according to figure 4 towards 80°*E* – 100°*W*. This, in our opinion, is explained by the fact that in cycles of longer duration the main role in changing the moment of inertia of the Earth plays movement (also occurring in long cycles, together with cycles of long duration in changes of ω) of denser sub-crustal masses lying in longitudinal zones 40 – 130°*E* and 40 – 140°*W*, while long cycles in movements of masses of lower density are less significant due to compensation of polar-equatorial and equatorial-polar movements (occurring in cycles of shorter duration) on a long time interval. A positive correlation of changes of the amplitude of variations of the major axis of the ellipse of forced oscillations with solar activity in 22-year cycles [2], taking into account the observed correlation in 22-year cycles of the angular velocity of rotation of the Earth and the Wolf numbers [5, 6] (in light of the hypothesis presented there and taking also into account the results obtained in [4]) is explained by the presence of a similar cycle in global deformation processes.

Figure 5 shows the curves of the distinguished forced motions of the instantaneous pole of rotation of the Earth averaged over 7-year cycles, as well as the changes during this period of the ratio of projections of the major axis of the ellipse of forced variations of the pole ($V = V_x / V_y$), of solar activity (*W*), magnetic disturbance (*M*), and day duration ($\Delta\tau$)[2] [17-23]. The figure shows that the forced pole motion (in particular, elements such as ellipticity, the length of the major axis and the direction of the major axis of the ellipse of forced variations) change in accordance with the angular velocity of the Earth's rotation, and also correspond to the reference phases of cycles of the Wolf numbers and index *M*. The oscillations along the *X*-axis reach a maximum on the growth branch of ω, and then (approximately 0.5 ÷ 1 year to the maximum of ω) a decrease of the ellipticity of the pole trajectory is observed, that can be explained by the establishment of dynamic equilibrium with a new ratio of moments. In changes of the elements of the pole trajectory, a secular cycle is also visible.

The obtained correspondence in the oscillations of *W*, *M*, $\Delta\tau$ and *V* once again confirms our assumption about the solar-induced (through the angular velocity of the Earth's rotation and deformation of the ellipsoid caused by the rotation velocity changes) forced nutation-precession oscillations of the Earth's instantaneous axis of rotation.

5. Discussion and Conclusions

In the paper, free 14-month (Chandler) and forced 12-month oscillations of the Earth's instantaneous axis of rotation are considered from a single point of view – as a consequence of the variability of the Earth's rotational regime (due, according to the hypothesis by Belashov [5, 6], to changes in solar activity indirectly through oscillations global magnetic disturbance). We have attempted to explain (at a qualitative level) the forced part of real variations of the pole's location by the alternating Earth's rotational regime arising due to the polar pulsations of the figure, and also by uneven, along the longitudinal

[2] The 11-year variations of *W* and *M*, as well as the 7-year variations of the change in the length of the day, were obtained by smoothing using moving averages by analogy with [5]. The parameter *V* characterizes the ellipticity of the trajectory of the pole and the direction of the major axis of the ellipse.

zones, movement of the subcrustal masses in the planetary body. The reality of the movement of subcrustal masses, their “flowing” from the polar regions to the equatorial ones and vice versa, at change of the rotational regime and the density redistribution in the Earth’s body follows, first of all, from the fact that the substances in the subcrustal layer are in liquid or at least semi-liquid state due to the colossal pressures and temperatures [10]. Further, based on the fact that the density in the Earth’s body is nonuniformly distributed, we hypothesized the “deformational” nature of the forced nutational-precessional wave in the oscillations of the instantaneous axis of the Earth’s rotation and analyzed the available heliogeophysical data for 1900-2017.

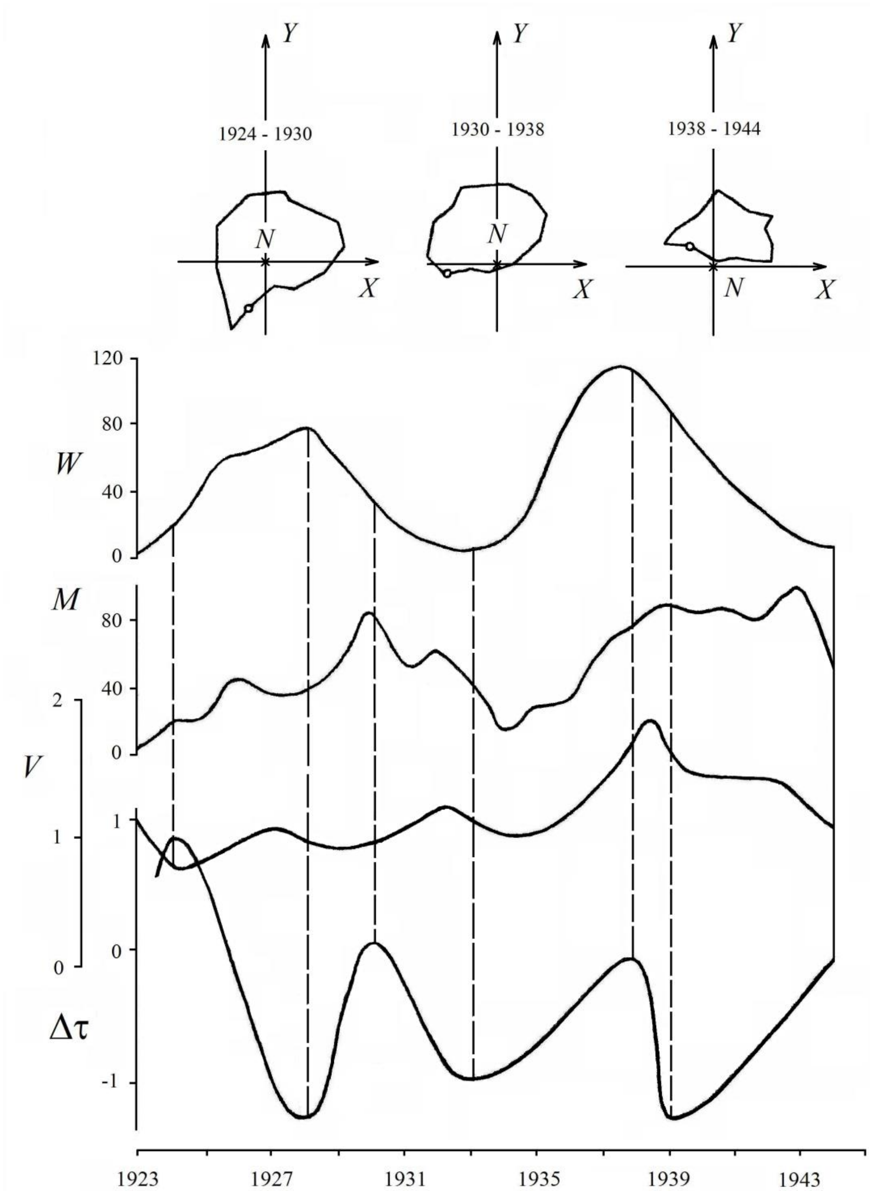

Figure 5: Forced motion of the instantaneous pole of rotation of the Earth and the changes of *V, W, M* and $\Delta\tau$ for the same period.

The main conclusions can be formulated as follows.

1. Real fluctuations of the location of the instantaneous pole of rotation occur quite synchronously with fluctuations of the angular velocity of the Earth's rotation, and with some delay relative to the latter,

that allows us to put forward the variability of the planet's rotation regime as the cause of long-period precession-nutation movements of the Earth's rotation axis.

2. A deforming force (of a centrifugal nature) arising when the angular velocity changes, causes a deformation of the figure nonuniform over the longitudinal zones (due to the nonuniform distribution of density in the planet's body). With such a nonuniform deformation, the moment of inertia of the Earth changes, that causes oscillations of its instantaneous axis of rotation, that is, nutation and precession of the axis (forced wave).

3. An analysis of the available factual material confirms the validity of the assumption about the decisive role of changes of the angular velocity of the Earth's rotation both for free 14-month (Chandler) and forced 12-month oscillations of instantaneous axis of the Earth's rotation.

4. The observed correlation of cyclic changes of solar activity, magnetic disturbance, and oscillations of instantaneous axis of the Earth's rotation is explained, taking into account the hypothesis by Belashov [5, 6], by cyclic changes of the rotation regime, without involving as an intermediate link, the baric circulation processes which have relatively low energy efficiency.

5. It is natural to consider changes of the baric circulation regime as a consequence of the variability of the deflecting force of the Earth's rotation (of the Coriolis force) both in magnitude and direction. The Coriolis force vector itself undergoes changes, since the vector of the angular velocity of the Earth's rotation changes in time.

Thus, the solar-caused (through the magnetosphere) variability of the Earth's rotational regime is put forward as a single reason for the nutational-precessional oscillations of the instantaneous axis of rotation, and it is fundamentally new to consider the variability of the rotational regime as the cause of forced movements of the planet's rotation pole.

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