# The jump problem for holomorphic functions on non-rectifiable arc and $\overline{\partial}-$equation

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The jump problem is the following well known boundary value problem for analytical function. Let $\Gamma$ be a given arc on the complex plane $\mathbb{C}$. It is required to find analytical in $\overline{\mathbb{C}} \setminus \Gamma$ function $\Phi(z)$ such that

$$\Phi^+(t) - \Phi^-(t) = f(t), t \in \Gamma, \tag{1}$$

where $\Phi^+(t)$ and $\Phi^-(t)$ are limit values of $\Phi(z)$ at point $t \in \Gamma$ from the left and from the right correspondingly, and $f(t), t \in \Gamma$, is given function. If arc $\Gamma$ is piecewise–smooth, then a solution of this problem is given by the Cauchy integral

$$\Phi(z) = \frac{1}{2\pi i} \int_\Gamma \frac{f(t)dt}{t - z}.$$

It is convolution of distributions $E(z) \equiv (\pi i z)^{-1}$ and $\int_\Gamma f(t) \cdot dt$, where

$$\left\langle \int_\Gamma f(t) \cdot dt, \phi \right\rangle \equiv \int_\Gamma f(t)\phi(t)dt, \phi \in C_0^\infty(\mathbb{C}).$$

As $E$ is fundamental solution of differential operator $\overline{\partial} \equiv \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$, since the differential equation

$$\overline{\partial}\Phi = \int_\Gamma f(t) \cdot dt \tag{2}$$

is distributional version of the jump problem.

If the arc $\Gamma$ is not rectifiable, then both the Cauchy integral and the distribution $\int_\Gamma f(t) \cdot dt$ lose their certainty. In the present work we construct generalization of $\int_\Gamma f(t) \cdot dt$ for non-rectifiable arcs such that solution of the equation (2) with this generalization in the right side is also solution of the problem (1) on non-rectifiable arc.